1. If $x \neq 8$, find the simplified value of $\frac{4x}{x-8} - \frac{32}{x-8}$.

2. Find the value of $\left|7-2\left|3-\left|14-28\right|\right|\right|$.

3. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   If x is 20% of y, then y is what percent of x?

   A) 120%
   B) 220%
   C) 180%
   D) 500%
   E) 420%
   F) 2000%

   Note: Be certain to write the correct capital letter as your answer.

4. The product of two consecutive negative integers is 144 more than the smaller of the two integers. Find the sum of the two consecutive negative integers.

5. The repeating decimal $0.68\overline{3}$ (where only the 3 repeats) can be written as $\frac{k}{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

6. Bob enters a department store and says: “Give me as much money as I have now, and I will spend $12 in your store.” The manager agrees, and Bob spends $12. Bob immediately enters a candy store and says: “Give me as much money as I have now, and I will spend $12 in your store.” The manager agrees, and Bob spends $12. Bob then immediately enters an ice cream store and says: “Give me as much money as I have now, and I will spend $12 in your store.” The manager agrees, and Bob spends $12. Then Bob has no money left. Find the number of **cents** that Bob started with.

7. If $a=4$, $b=3$, and $c=5$, find the value of $3(a+2)(c-b)$.

8. The smaller of two consecutive positive integers is an integral multiple of 23, and the larger of the two consecutive positive integers is an integral multiple of 29. If the smaller integer is less than 2000, find the sum of all distinct possibilities for the smaller integer.

9. Find the value of k if both x and y are positive integers in the consistent system:
$$\begin{cases} 7x+11y=2295 \\ 6x+ky=5901 \end{cases}.$$

10. Change 334_{six} to base five. Be certain to express your answer in base five.

11. Let $x<y$ and let x and y be two unequal positive integers. If $\frac{x}{8}+\frac{y}{8}=1$, find the sum of all possible values for x.

12. Let *a*, *b*, and *c* represent single digit positive integers such that $f(x)=ax^2+bx+c$. If $f(4)=63$ and $f(10)=273$, find the value of $a+2b+3c$.

13. Bob is 6 years older than Judy who is 8 years older than Marnie who is 12 years older than Christie who is $\frac{3}{5}$ of the age of Judy. Find the number of years in the age of Marnie.

14. Let y be a positive integer such that $y=1.065x$. If $x=\frac{k}{w}$ where k and w are positive integers, find the smallest possible value of $(k+w)$.

15. Find the slope of a line that is perpendicular to the line that contains the points $(48,0)$ and $(0,-24)$.

16. Benny illegally "cancelled" the sixes when trying to reduce the common fraction $\frac{26}{65}$. Surprisingly, the $\frac{2}{5}$ he got is a fraction equivalent to the original fraction, $\frac{26}{65}$, a case of dumb luck. Bonnie caught his error and pointed out that a similar situation occurs when the sixes in $\frac{16}{64}$ are illegally "cancelled" to yield a correct equivalent fraction, $\frac{1}{4}$. If the "cancelled" digit n must be the units digit in either the numerator or denominator and the tens digit in the other part of the common fraction, a third example of this type is the illegal "cancelling" of the digit n yielding a fraction equivalent to $\frac{1}{5}$. Find the value of the "cancelled" digit n.

17. Let k be the smallest integer and let w be the largest integer such that both k and w are values for x that satisfy the inequality $|2x-1| < 15$. Find the value of $(3k+2w)$.

18. Let k represent a positive integer. Let k be divided into 4 parts such that each part is a positive integer and such that the sum of the 4 parts is k. If the first part were increased by 4, the second part decreased by 4, the third part divided by 4, and the fourth part multiplied by 4, then the 4 results would be equal. Find the smallest possible value of k.

19. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

If $0 < a < b < c < d$, which of the following **might** be true?

A) $a + d = c$
B) $a + b + c < d$
C) $b - c = a$
D) $2d < b + c$
E) $a > c$

Note: Be certain to write the correct capital letter as your answer.

20. For all positive integers x and y such that $x < 100$ and $y < 100$, let $x \oplus y = (x+3)(y+2) - 5k$. If $x \geq y$, then $k = x$. If $x < y$, then $k = y$. If $x \oplus y = 175$, find the sum of all distinct possible values of x.

2009 RA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 4

2. 15

3. D **(Must be this capital letter)**

4. –23

5. 101

6. 1050 **("cents" optional.)**

7. 36

8. 2346

9. 29

10. 1010 (**or** 1010_5 **or** 1010_{five})

11. 6

12. 25

13. 42 **("years" optional)**

14. 201

15. –2

16. 9

17. –4

18. 50

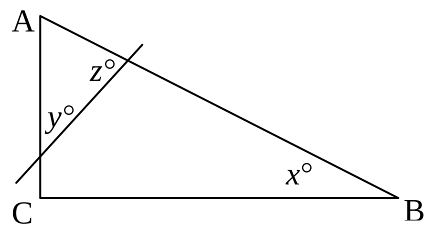
19. B **(Must be this capital letter.)**

20. 185

1. A triangle with base $\overline{AB}$ has a height to base $\overline{AB}$ whose length is $\frac{3}{x}$ units, $x \neq 0$. The area of this triangle is $\frac{24}{x}$ square units. Find the number of units in the length of the base $\overline{AB}$.

2. In which quadrant does $(-5,11)$ lie? For your answer, **write a Roman numeral—either I, II, III, or IV—whichever is correct.**

3. In the figure, if $x = 3y$, $\angle ACB = 90°$, and $\angle BAC = 27°$, find the value of z.



4. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

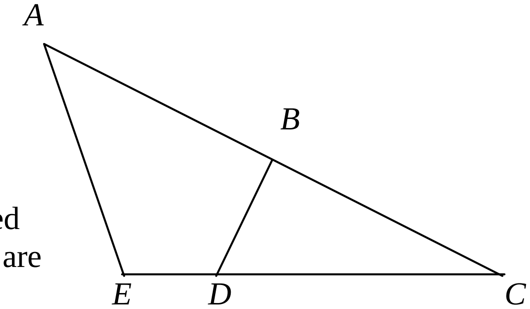
   If a line is perpendicular to a plane, then that line is perpendicular to every line that lies in the plane.

5. A circle has an equation of $(x-8)^2 + y^2 + 10y = -13$. Find the length of a radius of that circle.

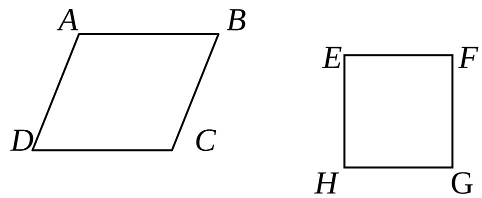
6. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never—**whichever is correct.

   If a line that is perpendicular to the plane that contains Δ ABC intersects the plane at point A, then the line is perpendicular to exactly two sides of Δ ABC.

7. In the diagram, B lies on $\overline{AC}$, and D lies on $\overline{EC}$. If $\frac{AB}{BC} = \frac{4}{5}$ and $\frac{ED}{DC} = \frac{1}{7}$, then the ratio of the area of ΔBDC to the area of ΔAEC can be expressed in the form of $k:w$ where k and w are positive integers. Find the smallest possible value of $(k+w)$



8. In the diagram, $ABCD$ is a parallelogram which is not a rhombus. $EFGH$ is a square. $\overline{AB} \cong \overline{EF}$. If two of the eight sides are selected at random (without replacement), find the probability the two sides selected are congruent. Express your answer as a common fraction reduced to lowest terms.

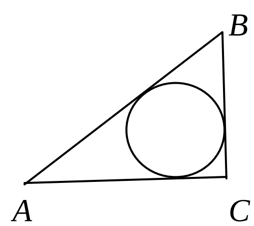


9. Located inside equilateral triangle PQR is a point T such that $TP = 23$, $TQ = 30$, and $TR = 35$. Rounded to the nearest integer, find PQ.

10. A parallelogram that is **not** a rectangle has an area of 184. The two diagonals of the parallelogram divide the parallelogram into 4 triangles. Find the smallest possible area of any one of these 4 triangles.

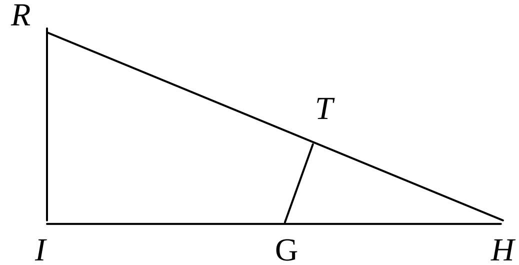
11. Find the sum of the degree measures of the exterior angles, one per vertex, of a convex polygon that has 9 sides.

12. In the diagram, $\overline{AB}$ is the hypotenuse of ΔABC and has a length of 20. The radius of the inscribed circle has a length of 4. Find the perimeter of ΔABC.

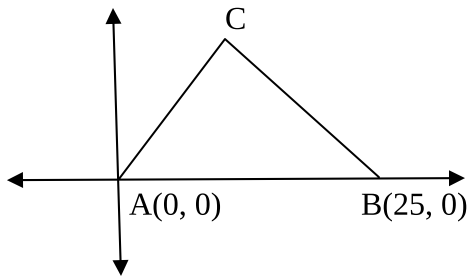


13. The length of the hypotenuse of a right triangle is 97, and the sum of the lengths of the two legs of this right triangle is 137. Find the length of the smaller leg.

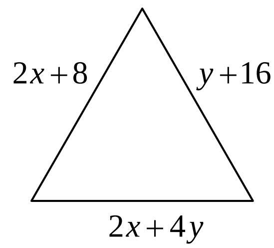
14. In the diagram, G lies on $\overline{HI}$, T lies on $\overline{RH}$, $\overline{GT} \perp \overline{RH}$, and $\overline{RI} \perp \overline{HI}$. If $RI = 9$, and $RT = 8$, then the area of quadrilateral $RTGI$ can be expressed, in simplest radical form as $\frac{141\sqrt{7}}{14}$. Find HT.



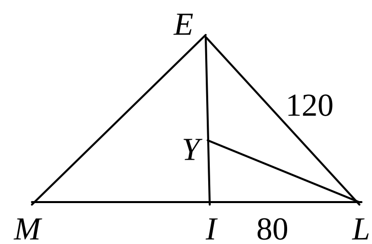
15. In the diagram with coordinates as shown, $AC = 34$, and $BC = 39$. Find the **ordered pair** that represents point C. Express your answer as an **ordered pair** with **each member** of the ordered pair expressed as an improper fraction reduced to lowest terms. (Note: C lies in the first quadrant.)



16. Find the perimeter of the equilateral triangle with side-lengths as shown.



17. In the diagram, M, I, and L are collinear, and E, Y, and I are collinear. Ray $\overrightarrow{LY}$ bisects $\angle ELM$. $\overline{EI}$ is an altitude of ΔEML. $EL = 120$, and $IL = 80$. Find the exact value of $(EY - YI)$.



18. A square has a side whose length is 5.2 units. How many more square units are there in the area of this square than there are units in the perimeter of this square? Express your answer as a **decimal**.

19. A right triangle is inscribed in a circle. The side-lengths of this right triangle are all positive integers. If the area of the circle is 1056.25π, find the **sum** of all possible distinct areas of the right triangle.

20. In rhombus $ABCD$, $\angle DAB = 60°$. A circle passes through vertices A, B, and D, and intersects diagonal $\overline{AC}$ at E. If $EC = 12$, find the exact area of this circle.

2009 RA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 16

2. II **(Must be this Roman Numberal.)**

3. 132 **(Degrees optional.)**

4. NEVER **(Must be the whole word.)**

5. $2\sqrt{3}$

6. ALWAYS **(Must be the whole word.)**

7. 107

8. $\frac{4}{7}$ **(Must be this reduced common fraction.)**

9. 50

10. 46

11. 360 **(Degrees optional.)**

12. 48

13. 65

14. 4

15. $\left(\frac{26}{5},\frac{168}{5}\right)$ **(Must be this ordered pair of reduced improper fractions.)**

16. 54

17. $8\sqrt{5}$ **(Must be this exact simplified radical.)**

18. 6.24 **(Must be decimal answer.)**

19. 3192

20. 144π **(Must be this exact answer.)**

1. Is the sequence $\{1,4,16,64,256\}$ an arithmetic sequence, a geometric sequence, or neither type of sequence? For your answer, write the whole word **arithmetic**, **geometric**, or **neither type**, whichever is correct.

2. Let $A=\left\{-\frac{1}{4},\frac{2}{3},-2,17\right\}$. If one of the four members of A is selected at random and substituted for x, find the probability that $x>x^3$. Express your answer as a common fraction reduced to lowest terms.

3. **(Always, Sometimes, or Never)** For your answer, write the _whole word_ **Always, Sometimes,** or **Never—**whichever is correct.

$$|3x|=3x$$

4. The focus of the parabola whose equation is $9x^2-24x+72y+16=0$ is $(\frac{4}{3},k)$. Find the value of k.

5. Let k represent a positive integer. If the roots of $x^2+10x+k=0$ are non-real, find the smallest possible value of k.

6. Let k, w, and p each represent positive integers such that $k>2$, $w>2$, and $p>2$. Each of k cases contains w boxes, and each box contains p "items." If $w\neq p$ and the total number of "items" in the k cases is 90, find the smallest possible value of $(w+p)$.

7. When 2^{2010} is written as an integer, how many digits are there in the integer?

8. If the speed of the current remains constant at 5.5 mph, a boat can travel 246 miles downstream in the same time it can travel 180 miles upstream. Find the number of miles per hour in the speed of the boat in still water. Express your answer as a **decimal**.

9. By substituting 1, 2, 3, 4, 5, and 6 for x, in that order, in a polynomial function in x, the first six terms are respectively: 22, 57, 132, 268, 489, and 822. If $P(x)$ is the polynomial function of lowest degree satisfying the given, find $P(25)$.

10. Find the smallest positive integer that has exactly 44 distinct positive divisors.

11. If $x^4 - 1 = 624$, then find the largest possible value of $x^3 + 2x^2 - 3x + 7$.

12. The sum of the squares of the roots for x of $3x^3 + 81x^2 + kx - 287 = 0$ is 785. Find the value of k.

13. Find the value of x such that $\log_2(x+3) - \dfrac{1}{\log_5(2)} + \log_2(3) = 4$. Express your answer as an improper fraction reduced to lowest terms.

14. Let x and n represent two-digit positive integers for which the tens digit is non-zero and for which $n > x$. Find the value of x if the sum of the consecutive positive integers from 1 through $(x-1)$ is the same as the sum of the consecutive positive integers from $(x+1)$ through n.

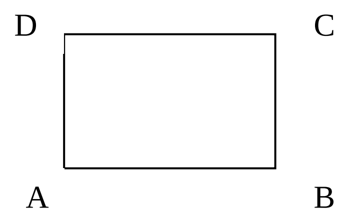
15. Find the degree of the following polynomial: $3x(x+5)+6x^2(x^2-7)^3+44x-179$

16. A quadratic function $f(x)=ax^2+bx+c$ has zeroes of 3 and 5. The graph of $f(x)$ passes through the point $(2,-9)$. If a, b, and c represent integers, find the largest possible value of $(a+b+c)$.

17. The eleven positive integers from 1 through 11 are placed in a hat and drawn out at random one at a time without replacement. Find the probability that the odd integers were drawn out as the first six draws. Express your answer as a common fraction reduced to lowest terms.

18. If $x=1+\cfrac{3}{1+\cfrac{3}{1+\cdots}}$, then $x=\dfrac{k+\sqrt{w}}{p}$, where k, w, and p are positive integers. Find the value of $(k+w+p)$

19. In the diagram, *ABCD* is a rectangle with $AB=45$ and $BC=k$. Let $i=\sqrt{-1}$. If $|45+ki|=53$, find the perimeter of the rectangle.



20. A circle has the line $x-2y+4=0$ tangent at the point $(0,2)$ and $y=2x-7$ tangent at $(3,-1)$. Find the exact area of this circle.

2009 RA
Algebra II

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

_______ Correct X **2** pts. ea. = [ ]

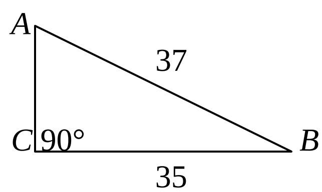
Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. Geometric (Must be this word.)
2. $\frac{1}{2}$ (Must be this reduced common fraction.)
3. Sometimes (Must be this word.)
4. -2
5. 26
6. 8
7. 606
8. 35.5 (Must be this decimal, "mph" optional.)
9. 86284
10. 15360
11. 167
12. -84
13. $\frac{71}{3}$ (Must be this reduced improper fraction.)
14. 35
15. 8
16. -24
17. $\frac{1}{462}$ (Must be this reduced common fraction.
18. 16
19. 146
20. 5π (Must be this exact answer.)

1. Let $E = \{19,\ 29,\ 39,\ 49,\ 59,\ 684,\ 979794,\ 897,\ 923687541,\ 6876532\}$. If one of the members of E is selected at random, find the probability that the number selected is an integral multiple of nine. Express your answer as a common fraction reduced to lowest terms.

2. The first three terms of a geometric sequence are respectively: 1200, 720, 432. Find the fourth term of this geometric sequence. Express your answer as an **exact** decimal.

3. In the diagram with measures as shown, find $\sin(\angle ABC)$. Express your answer as a common fraction reduced to lowest terms.



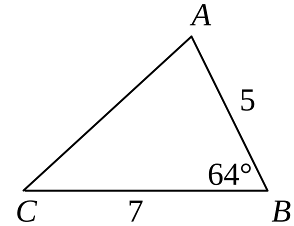
4. **(Multiple Choice)** The conditions for a circle are that it passes through the point $(5, 6)$ and is tangent to the two lines whose equations are $3x - 4y + 1 = 0$ and $4x + 3y = 7$. Exactly one of the statements A, B, or C is true. For your response, write the capital letter of the true statement:

   A) No circle is determined.
   B) Exactly one circle is determined.
   C) More than one circle is determined.

   Note: Be certain to write the correct capital letter as your answer.

5. If $(a^2 + 2b)^{14}$ is expanded and completely simplified, find the numerical coefficient of the term in which the exponent of a is 8.

6. In ΔABC with measures as shown, $(AC)^2 = 7^2 + 5^2 - k\cos(64°)$. Find the value of k.



7. Expressed in interval form, the range of values of x for which the series $\sum_{n=1}^{\infty}\left(\left(\frac{x+1}{3}\right)^{n}\right)$ is convergent is (k,w). Find the value of $(2k+9w)$.

8. A committee of four is to be chosen from five Whigs and six Tories. How many fewer possibilities are there if there must be two persons from each party on the committee than if there must be at least one person from each party on the committee?

9. Let the brackets $[\ ]$ represent the greatest integer function. If $6[\frac{x}{8}]^3-19[\frac{x}{8}]^2-116[\frac{x}{8}]=-84$, then the set of all possible values for x can be denoted by $\{x:k\le x<w\}$ for some integers k and w. Find the value of $(k+w)$.

10. Urn A contains two orange marbles and one blue marble. Urn B contains two orange marbles and two blue marbles. An urn is selected at random and then one of the marbles is selected at random from that urn. Find the probability that the marble selected is orange. Express your answer as a common fraction reduced to lowest terms.

11. The sum of the reciprocals of two numbers is 20. The sum of the two numbers is 10. Find the product of the two numbers.

12. When $(3x^2-2x+5)(2x-3)^{15}$ is expanded and completely simplified, find the coefficient of the x^{12} term.

13. Let $(x-1)$, $(x-2)$, and $(x+17)$ be factors of the polynomial x^3+kx^2+wx+p where k, w, and p represent integers. Find the value of $(k+p)$.

14. Lee and Cindy have \$200 in their joint checking account. One day each writes a check, both amounts randomly selected from the interval $(0, 200]$. Find the probability that each wrote a check for more than \$60 and that the sum of their two checks is less than \$200. Express your answer as a common fraction reduced to lowest terms.

15. The fifth term of an arithmetic sequence is 12, and the tenth term is 20. Find the third term. Express your answer as a **decimal**.

16. Find the value of k in the determinant equation: $\begin{vmatrix} 5 & 24 \\ 25 & k \end{vmatrix} = \begin{vmatrix} 6 & 40 \\ 13 & 5 \end{vmatrix}$.

17. Let e be the base for natural logarithms and let $\ln$ be the symbol for natural logarithm. If $p = \ln 4$, then the value of x such that $e^{(4x)} - e^{(2x-3)} = 12e^{(-6)}$ can be expressed as $\frac{p-k}{w}$. If k and w are positive integers, find the smallest possible value of $(2k + 5w)$.

18. Let $i = \sqrt{-1}$ and let k and w represent real numbers. If $(k + wi)(6 - i) = 15 + 16i$ then find the value of $(k + w)$.

19. The points $(-6,15)$, $(9,15)$, and $(-6,7)$ are the vertices of a triangle. Rounded to the nearest degree, find the degree measure of the smallest angle of this triangle.

20. $A = \{1,3,11,8,x\}$ where the set consists of 5 **different** positive integers. Let $k = \sigma_x$, the total population standard deviation of A. Let w be the median of A. For how many distinct values of x is $|k - w| < 1$?

2009 RA

Pre-Calculus

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{3}{10}$ (Must be this reduced common fraction.)
2. 259.2 (Must be this exact decimal.)
3. $\frac{12}{37}$ (Must be this reduced common fraction.)
4. C (Must be this capital letter.)
5. $1{,}025{,}024$
6. 70
7. 10
8. 160
9. 104
10. $\frac{7}{12}$ (Must be this reduced common fraction.)
11. $\frac{1}{2}$ or 0.5 or $.5$
12. $-2{,}946{,}198{,}528$
13. 48
14. $\frac{2}{25}$ (Must be this reduced common fraction.)
15. 8.8 (Must be this decimal.)
16. 22
17. 16
18. 5
19. 28 (Degrees optional.)
20. 8

NO CALCULATORS

1. Express $(27)^{\left(-\frac{2}{3}\right)}$ as a common fraction reduced to lowest terms.

2. For what value(s) of m does $(1,2)$ lie on the line whose equation is $mx - y + 3 = 0$?

3. The equation of the line containing all points in the plane that are equidistant from the points $(4,2)$ and $(-2,-3)$ can be expressed in the form: $y = mx + b$. Find the value of $(120m + 300b)$.

4. Find the value of $\sqrt{(331-7)(618+7)}$.

5. Let $f(x) = \dfrac{x^2 + kx + w}{x + p}$ have zeroes for x of 3 and 20 and be undefined when $x = -10$. Find the value of $(2k + 3w + 4p)$.

6. The lengths of the three sides of a triangle are 52, 73, and 75. Find the length of the altitude drawn to the longest side.

7. If $x \neq 2y$, find the maximum possible **number of distinct values of** x such that $x(y - \frac{1}{2}x) = 0$?

8. Find the units digit of $3^5 + 4^6 + 5^7 + 6^8 + 7^9 + 8^5 + 9^5 + 100^5 + 1011^5 + 2022^5$.

NO CALCULATORS

NO CALCULATORS

9. The lengths of the sides of a **scalene** triangle are in the ratio of $17:32:k$. If k is an integer, find the smallest possible value of k.

10. Find the number of seconds that will elapse between the first time after 5:25 that the **minute** hand and the **second** hand will form a 118° angle and the second time after $5:25$ that the **minute** hand and the **second** hand will form a 118° angle.

11. Point $A(19,25)$ lies on a line with a slope of 0.75. There are two distinct points on this line that are at a distance of 25 from point A. Find the sum of the ***x*-coordinates** of these two points.

12. Let a, b, c, d, and e be five prime positive integers such that $a<b<c<d<e$. If the product of these five prime positive integers is a four-digit number, find the largest possible value of e. **Note: 1 is not a prime integer.**

13. In a room of only boys and girls with at least one of each, there are a total of 120 people. When expressed as a common fraction reduced to lowest terms, the fractional part of the people in the room that are boys has a denominator less than 14. Find the largest possible number of boys that are in the room.

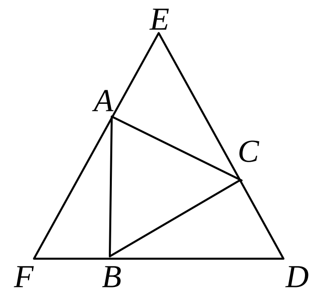
14. The point $A(5,17)$ is rotated 270° clockwise about the point $(10,5)$ to point B. Find the exact distance from point A to point B.

15. If $\frac{x}{a+b}=-1$, find the value of $(x+a+b)$.

NO CALCULATORS

NO CALCULATORS

16. In the diagram, ΔDEF is equilateral, points A, B, and C lie on $\overline{EF}$, $\overline{DF}$, and $\overline{DE}$ respectively. $\overline{EA} \cong \overline{FB} \cong \overline{DC}$. $EF = 1$, and $EA = \frac{3}{8}$. Find the ratio of the area of ΔABC to the area of ΔDEF.
Express your answer as a common fraction reduced to lowest terms.

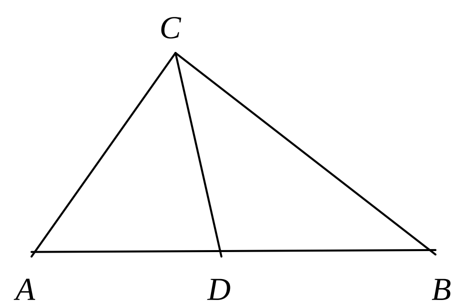


17. From the set $\{15, 24, 72, 80\}$, a number is selected at random as the number of sides of a regular polygon. Find the probability that an exterior angle of that regular polygon has a degree measure that is an integral number of degrees. Express your answer as a common fraction reduced to lowest terms.

18. If the area of a square whose side has a length of x is 7, find the area of a square whose side has a length of $4x$.

19. Let $17249760 = xy$ where x and y are positive integers such that $x < y$. Find the number of distinct ordered pairs (x, y) that exist.

20. In the diagram D lies on $\overline{AB}$. $\angle ACD \cong \angle BCD$. $AB = 84$, $BC = 75$, and the area of ΔABC is 1890. Find the area of ΔACD.



2009 RA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{9}$ **(Must be this reduced common fraction.)**
2. -1
3. 66
4. 450
5. 174
6. 48
7. 1
8. 7
9. 16
10. 40 **(Seconds optional.)**
11. 38
12. 47
13. 110
14. $13\sqrt{2}$ **(Must be this exact answer.)**
15. 0
16. $\frac{19}{64}$ **(Must be this reduced common fraction.)**
17. $\frac{3}{4}$ **(Must be this reduced common fraction.)**
18. 112
19. 120
20. 765

NO CALCULATORS

1. $\begin{bmatrix} 7 & 3 & -2 \\ 8 & 6 & 12 \end{bmatrix} - \begin{bmatrix} 11 & -3 & 3 \\ 1 & 7 & -5 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. Find the value of $acd - 2cf$.

2. Let $A = (\lambda - a)(\lambda - b)(\lambda - c)(\lambda - d)\cdots(\lambda - x)(\lambda - y)(\lambda - z)$, where A has 26 factors arranged alphabetically. If $\lambda = p$, find the value of A.

3. Find the product of the distinct **rational** roots of $x^4 + 5x^3 - 17x^2 - 15x + 42 = 0$.

4. A parabola has a directrix whose equation is $x = -2$ and has its focus at $(2, 2)$. The equation of this parabola can be expressed in the form $kx = (y-2)^2$. Find the value of k.

5. Find the value of k such that $4^k = 32^{\left(-\frac{3}{5}\right)}$. Express your answer as a **decimal**.

6. Let $i = \sqrt{-1}$. If $(k + wi)(7 - 16i) = 94 + 3i$, find the value of w.

7. Let $f(x) = 2x^2 - 15$ and let $g(x) = \sqrt{\dfrac{x+15}{2}}$. Find $f\left(g\left(36\right)\right)$.

8. A box contains orange marbles and blue marbles. You draw two of these marbles without replacement. If the probability of selecting an orange marble and a blue marble in that order is $\frac{17}{60}$, and the probability of drawing an orange marble on the first draw is $\frac{2}{3}$, find the probability of drawing a blue marble on the second draw if you know the first marble drawn was orange. Express your answer as a common fraction reduced to lowest terms.

NO CALCULATORS

NO CALCULATORS

9. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

   If h, k, and j are real numbers such that $h > k$ and $k > j$, then $hk > kj$.

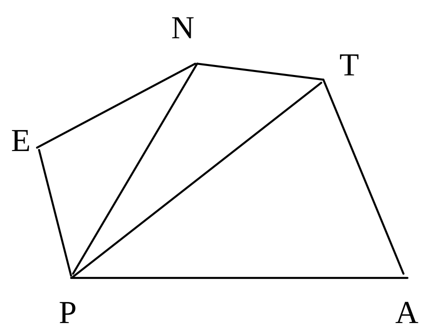
10. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

    Which of the following lines are asymptotes of the graph of $y = \frac{x+7}{x-5}$?

    A) $x = 5$ only.
    B) $x = -7$ only.
    C) $y = 1$ only.
    D) $x = 5$ and $x = -7$.
    E) $x = 5$ and $y = -7$.
    F) $x = 5$ and $y = 1$.

    Note: Be certain to write the correct capital letter as your answer.

11. The perimeter of pentagon $PENTA$ is 40. $PN = 10$, $PT = 11$, and $PA = 12$. All sides of pentagon $PENTA$ have lengths which are integers. If 4 of the sides of $PENTA$ are congruent, then $\sin(\angle PAT) = \frac{k\sqrt{w}}{f}$ where k, w, and f are positive integers. Find the smallest possible value of $(k + w + f)$.



12. The average SAT score of a group of students was 600, and the standard deviation was 100. If each score had been increased by 20, find the sum of the new arithmetic mean and the new standard deviation.

13. Let n be a positive integer. For all $n \geq 1$, $t_{(n+1)} = 3t_n + 7$. If $t_5 = 163$, find the value of t_2. Express your answer as an improper fraction reduced to lowest terms.

NO CALCULATORS

NO CALCULATORS

14. Find the sum of all distinct values of x such that the three terms $x-2$, $x+1$, and $4x-8$ in that order will form a geometric progression.

15. All ages in this problem are in whole numbers of years. A father has three daughters whose ages form an arithmetic progression. The father is now 6 years older than the sum of the daughters' ages. Six years ago, the sum of the daughters' ages at that time was half of the father's present age and the youngest daughter was then $\frac{1}{3}$ as old as the oldest daughter was then. Find the number of years in the present age of the oldest daughter.

16. If $0° < x < 180°$, find the sum of all distinct values of x such that $\sin(6x)° + \cos(4x)° = 0$.

17. A parabola has its vertex at $(0,0)$, has its focus on the *y*-axis, and passes through the point $(-8,4)$. The equation of this parabola can be expressed in the form $(x-h)^2 = py$. Find the value of $(h+p)$.

18. A circle has the equation $(x-4)^2 + (y+2)^2 = k$. If the area of this circle is 324π, find the value of k.

19. When $37!$ is computed, it ends in 8 zeroes. Find the digit that immediately precedes these zeroes.

20. Let $i = \sqrt{-1}$. Let w and f be integers such that the 3 roots for x of $x^3 - 12x^2 + cx + dix + k + hi = 0$ are $4-i$, $3+wi$, and f. Find the value of $(c+d+k+h)$.

NO CALCULATORS

2009 RA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 310
2. 0
3. -14
4. 8
5. -1.5 (Must be decimal answer.)
6. 5
7. 36
8. $\frac{17}{40}$ (Must be this reduced common fraction.)
9. Sometimes (Must be the whole word.)
10. F (Must be this capital letter.)
11. 19
12. 720
13. $\frac{8}{3}$ (Must be this reduced improper fraction.)
14. 6
15. 18 (Years optional)
16. 495 (Degrees optional.)
17. 16
18. 324
19. 4
20. -21

1. If $7.348x = 16.853$, find the value of $2.333x$.

2. The product of two real numbers is 10 and the difference of their squares is 24. Find the least possible value of the smaller of the two numbers.

3. Find the value of a number that is 67.84% more than 146.2.

4. Find the slope of a line that is perpendicular to a line that contains the points $\left(24.15, 876.1\right)$ and $\left(102.6, 875.4\right)$.

5. A manufacturer needs a closed right-cylindrical can to contain 1000 cubic centimeters of product. What radius (in centimeters) of the base circle will provide the can of desired volume that uses the minimum amount of material? (Neglect overlaps, waste and presume the same material is used for all parts of the can.)

6. Determine the minimum **vertical** distance between the line $y = 5.874x - 9.211$ and the parabola $y = 3.000(x + 4.000)^2 + 14.01$.

7. Assume that 1 gallon is equal to 3.785 liters. Based on this assumption of equality, if 4.185 quarts are removed from 3.847 gallons, how many liters would be left?

8. Amber is at $A(18,30)$, and Barry is at $B(10,0)$. A rectangular building has vertices at $(80,70)$, $(380,70)$, $(380,350)$, and $(80,350)$. Amber and Barry have connected their ankles with a partially folded elastic cord. If the ankles were moved so that the cord first formed a straight-line segment (is fully extended without stretching), the length from ankle knot to ankle knot would be 120. The elastic cord is capable of stretching to a length of 300 from ankle knot to ankle knot (not necessarily in a straight line). Amber is moving her ankle with the cord attached at a constant rate of 9.168 units per second in the positive y-direction. Barry is moving his ankle with cord attached at a constant rate of 8.457 units per second in the positive x-direction. Find the *y*-coordinate of Amber's ankle when the cord is stretched to its limit of 300. **Note: the cord will not be in a straight line because of the building.**

9. When $x = 4.12$, $y = 9.888$. If y varies directly as x, find the value of x when $y = 2.9616$.

10. The lengths of two sides of a triangle are respectively 5.684 and 4.981. Find the length of the third side that will enable such a triangle to have the largest possible area.

11. Assume that 87.13% of all mathletes at the ICTM State Math Contest chew gum. From a random group of mathletes at the ICTM State Math Contest, 50 students are selected at random. Calculate the standard deviation for the number of these mathletes who chew gum. Express your answer as a **decimal** rounded to 4 significant digits.

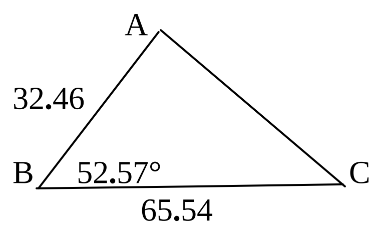
12. Assuming that $x > 0$, find the value of x such that $\log_3(5x+7)+\log_3(2x)=2$.

13. If $5.8x+1.4y=173.448$ and $y = x^3 - 142x^2 - 8x + 977.820264$, find the largest possible value for y.

14. In ΔABC, $AC = 24.68$, $\angle ABC = 27.00°$, and $\angle ACB = 84.00°$. Find BC.

15. The **second, third**, and **fourth** terms of a geometric progression are respectively 2.6781, 3.026253, and 3.41966589. Find the sum of the first 37 terms of this geometric progression.

16. In the diagram (not necessarily drawn to scale) with measures as shown, find the degree measure of the largest angle of ΔABC.



17. Harry is in a lake and is 422.1 yards from the perfectly straight shoreline. He wishes to reach a point on the shoreline that is 947.3 yards down the shoreline. He can swim at the rate of 2.886 yards per second. Once he gets to the shoreline, he can walk along the shoreline at 3.434 yards per second. For the smallest possible number of seconds for Harry to get from his present spot in the lake to the point that is 947.3 yards down the shoreline, Harry needs to aim at a point on the shoreline that is k yards from his destination point. Find the value of k.

18. It is said that Mia scored an average of 0.79 goals in every soccer game she played. If Mia played in 3186 soccer games, find the total number of goals that Mia scored. Round your answer to the nearest integer and express your answer as that **integer**. Do **not** use scientific notation.

19. Karen owns x shares of Champaign, Inc. and owns y shares of Urbana, Inc. Presently, each share of Champaign, Inc. is worth \$19, and each share of Urbana, Inc. is worth \$31. The total value of her x and y shares is presently \$16,030. If x and y are positive integers, find the largest possible value of $(x+y)$. Express your answer as an **integer**.

20. An ellipse has foci at $(0,5+\sqrt{2})$ and $(0,5-\sqrt{2})$ and has a minor axis of length 2. Find the absolute value of the shortest possible total distance from $(5,11)$ to a point on the ellipse and then from that point on the ellipse to a point on the graph of $x^2-18x+y^2-24y+221=0$.

2009 RA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 5.351 or 5.351×10^{0}
2. -5.256 or -5.256×10^{0}
3. 245.4 or 2.454×10^{2}
4. 112.1 or 1.121×10^{2}
5. 5.419 or 5.419×10^{0} **(Centimeters optional.)**
6. 43.84 or 4.384×10 or 4.384×10^{1}
7. 10.60 or 1.060×10 or 1.060×10^{1} **(Trailing "0" necessary.)**
8. 226.6 or 2.266×10^{2}
9. 1.234 or 1.234×10^{0}
10. 7.558 or 7.558×10^{0}
11. 2.368 or 2.368×10^{0}
12. 0.4790 or $.4790$ or 4.790×10^{-1} **(Trailing "0" necessary.)**
13. 134.0 or 1.340×10^{2} **(Trailing "0" necessary.)**
14. 50.75 or 5.075×10 or 5.075×10^{1}
15. 1659 or 1.659×10^{3}
16. 98.06 or 9.806×10 or 9.806×10^{1}
17. 292.7 or 2.927×10^{2} **(Yards optional.)**
18. 2517 **(Must be this integer.)**
19. 838 **(Must be this integer.)**
20. 14.48 or 1.448×10 or 1.448×10^{1}

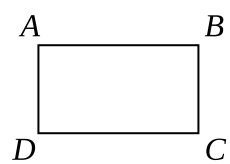
1. Let $(x+2)(9-4)=25(153-138)$. A regular polygon is inscribed in a circle, and the number of degrees of a minor arc between two consecutive vertices of the polygon is 24. Let y be the number of sides of the regular polygon. Find the value of $(x+y)$.

2. Let k be the sum of all distinct **positive integral** values of p such that the roots for x of the equation $2x^2-7x+3p=0$ are real. Let $f(x)=x^3+3x^2-2x-8$. Find the value of $(f(4)+k)$.

3. Let k be the area of a right triangle with a leg of length 36 and hypotenuse of length 85. Let w be the area of a square whose diagonal has a length of $\sqrt{242}$. Find the value of $(k+w)$.

4. Assume that x and y are real numbers. Let k be the minimum value of the expression $2x^2-8x-5$ and let w be the maximum value of the expression $\sqrt{9-y^2}$. Find the value of $(k+w)$.

5. The area of rectangle $ABCD$ is 13, and the perimeter of rectangle $ABCD$ is 20. Find the value of $(AB)^2+(BC)^2$.



6. A rectangle has an area of k. If a new rectangle were created by increasing the length of the original rectangle by 20% and by increasing the width of the original rectangle by 25%, the new area would be wk. Let $f(x)=3.246x+1.783$. Let $f(p)=61.834$. Find the value of $(w+p)$.

7. Abby drove 453 miles in 490 minutes; Bill, 589 miles in 640 minutes; Carol 254 miles in 290 minutes; Dan, 389 miles in 410 minutes; Emily, 402 miles in 430 minutes. Express, as a **decimal rounded to the nearest tenth**, the absolute value of the difference between the highest and lowest average mph. of the five.

8. Let k be the **smallest** integer that is a member of the solution set of $|3x-5| < 671$. Let w be the **largest** integer that is a member of the solution set of $x^2-8x<33$. Find the value of $(k+w)$.

9. Players stand in a circle. Player 1 stays in. Player 2 is knocked out. Player 3 stays in. Player 4 is knocked out. This process continues, knocking every other Player out, until only one Player remains. Let k be the number of the last Player remaining with 8 Players; let w be the number of the last Player remaining with 12 Players. Find the value of $(2k+3w)$.

10. ΔPHW is a right triangle with right angle vertex at H. The medians of ΔPHW meet at point G. Let k be the length of $\overline{HG}$ and w be the area of ΔPWG. Find the simplified ratio $k:w$.

2009 RA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		Score (to be filled in by proctor)
1. 88		______
2. 99		______
3. 1507		______
4. –10		______
5. 74		______
6. 20		______
7. 4.4	**(Must be this decimal, mph optional.)**	______
8. –211		______
9. 29		______
10. 1:12	**(Must be this simplified ratio.)**	______

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. 2637

12. 41

13. 31

14. 28

15. 51.34 **(Must be this decimal.)**

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. If x represents a positive integer such that $x^2 < 267$, find the **sum** of all distinct possible values for x.

2. A 2007 recorded message on Waste Management's phone line claimed every aluminum can recycled produced enough energy to power a TV for 3 hours. Further, they claimed they recycled 30,000 tons of aluminum cans to provide enough energy to power 6 billion hours of TV in the past year. Using these figures, 1 aluminum can weighs k ounce, where k is a proper fractional part of an ounce. Find k in simplified form.

3. Among the transformations required to produce $y = -7x^2 + 196x + 208$ from the graph of $y = x^2$, one is a vertical shift of k units. Let $\log_2\left(\log_3\left(\log_3\left(w\right)\right)\right) = 1$. Find the value of $\left(k + w\right)$.

4. In the following relation, by how much does the **sum** of the distinct numbers in the domain exceed the **sum** of all the distinct numbers in the range: $\left\{(4,-3),(6,2),(9,5),(43,1),(57,6),(19,37)\right\}$?

5. Let k be the sum of all integers x such that $y = \dfrac{2x+5}{x+2}$ is also an integer. The largest interval for x in which $y < 0$ is $\left(w, p\right)$. Find the value of $\left(k + w + p\right)$ as a decimal rounded to the nearest tenth.

6. Let k be the number of terms in the arithmetic sequence: $3, 4, 5, \cdots, n+2$. Let w be the number of terms in the arithmetic sequence: $7, 8, 9, \cdots, n+6$. There are two ordered pairs $\left(k, w\right)$ such that the sum of the k terms of the first arithmetic sequence is equal to the sum of the w terms of the second arithmetic sequence. One such ordered pair is $\left(2,1\right)$ since $3 + 4 = 7$. Find the other **ordered pair** $\left(k, w\right)$.

7. If the sides of a triangle have lengths of $3a$, $2b$, and c, then $9a^2 = qb^2 + rc^2 - wbc\cos(A)$ where $3a$ is the length of the side opposite $\angle A$. Find the value of $\left(q + r + w\right)$.

8. From the following four vectors, $\left(3,-2\right)$, $\left(8,-4\right)$, $\left(13.28,-6.64\right)$, and $\left(8.2,-4.1\right)$, two vectors are selected at random. Find the probability that both vectors selected are perpendicular to the vector $\left(12, 24\right)$. Express your answer as a common fraction reduced to lowest terms.

9. Let k be the first term of the arithmetic sequence whose 7th term is 16 and 24th term is 67. Find the sum of the infinite geometric series $\displaystyle\sum_{n=1}^{\infty} k\left(\frac{1}{3}\right)^{n-1}$.

10. A quadratic polynomial $P(x)$ generates the first five terms 6, 8, 16, 30, 50 when 1, 2, 3, 4, and 5 respectively is substituted for x. A five-person committee is to consist of one boy and one girl co-chairpersons and then must have at least 2 additional girls. If there are 5 girls and 4 boys from whom to choose, k such committees are possible. Find the sum $(k+P(10))$.

2009 RA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer		Score (to be filled in by proctor)
1.	136		
2.	$\frac{12}{25}$	**(Must be this reduced common fraction.)**	
3.	21263		
4.	90		
5.	-8.5	**(Must be this decimal.)**	
6.	$(15,12)$	**(Must be this ordered pair.)**	
7.	9		
8.	$\frac{1}{2}$	**(Must be this reduced common fraction.)**	
9.	-3		
10.	680		

TOTAL SCORE: ____________

(*enter in box above)

Extra Questions:

11. -368

12. 23

13. 6

14. 57340

15. 23

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

Oral Competition
Division A - 2009 ICTM Regional Competition
Voting Theory Questions

1. In the 2002 Illinois Gubernatorial Democratic Primary the election results were as follows:

Candidate	Popular Vote
Rod Blagojevich	457,197
Paul Vallas	431,728
Roland Burris	363,591

a. Assuming the Blagojevich vote was unchanged, what is the minimum percentage of the Burris votes would Vallas have needed in order to have won a plurality of the votes? (Give your answer to the nearest hundredth of a percent.)

b. Suppose the election had been decided by a runoff between the top two finishers if none of the candidates received a majority of the votes. In a runoff election between Blagojevich and Vallas, what is the minimum percentage of Burris supporters that would have needed to vote for Vallas in order for Vallas to have won the election if all original voters turned out and those who voted for Blagojevich and Vallas voted the same way? (Give your answer to the nearest whole percent.)

2. In a Research Illinois Poll conducted from October 20 through October 23, 2008 a total of 800 likely voters who vote regularly in state elections were interviewed statewide by telephone. Those interviewed were asked, "What do you consider the single most important issue facing Illinois?" The results were as follows:

A education funding
B health care
C balancing the budget
D rebuilding infrastructure
E preventing tax increases

	Democrats	Republicans	Independents
rank	339	214	247
1	B	E	A
2	A	C	B
3	C	A	C
4	D	B	D
5	E	D	E

a. Is there a Condorcet winner for this poll? If so, what is it?
b. Does the plurality method of picking a winner violate the Condorcet Winner Criterion in this election? Explain.
c. Which issue wins under Borda count?

3. The Jefferson High School math club is having its end of the year pizza party. In order to decide what kind of pizza to order, the group has decided to have each member prepare a preference list. The results are summarized as follows:

A veggie
B pepperoni
C sausage and mushroom
D ham and pineapple

Number of Voters

rank	**5**	**4**	**2**	**6**	**9**
1	D	C	C	B	A
2	B	D	B	C	D
3	C	B	D	D	B
4	A	A	A	A	C

a. If the club uses the Hare system to determine the winner, which type of pizza does the group choose?

b. Suppose the group of 2 voters with the preference ranking C changes its ranking to B
B C
D D
A A

If the club now uses the Hare system to determine the winner, which type of pizza will be chosen?

c. What desirable property of voting systems has been violated in this example? Explain.

Oral Competition
Division A - 2009 ICTM Regional Competition
Voting Theory SOLUTIONS

1. a. Vallas would have needed 457,198 votes in order to have a plurality, i.e., he would have needed 25,470 additional votes, or 7.01% of the Burris votes.

b. The total number of votes cast was 1,252,516, of which Blagojevich received only 36.5%. Thus, a runoff would have been required. Let x be the number of Burris voters supporting Vallas in the runoff. We require

$$431,728 + x > 457,197 + (363,591 - x)$$
$$431,728 + x > 820,788 - x$$
$$2x > 389,060$$
$$x > 194,530$$

Vallas would have needed 194,531 or 54% of the Burris votes. The final result would have then been

Vallas	Blagojevich
626,259	626,257

2. a. To determine if there is a Condorcet winner, we must perform head-to-head comparisons.

A to B — number preferring A: 214 + 247 = 461
number preferring B: 339
outcome: A wins

A to C — number preferring A: 339 + 247 = 586
number preferring C: 214
outcome: A wins

A to D — number preferring A: 339 + 214 + 247 = 800
number preferring D: 0
outcome: A wins

A to E — number preferring A: 339 + 247 = 586
number preferring E: 214
outcome: A wins

Since A wins every head-to-head comparison, A (education funding) is the Condorcet winner.

b. Yes, the plurality method of picking a winner violates the Condorcet Winner Criterion. Under the plurality method, B (health care) was the most important issue facing Illinois. The plurality method failed to pick as winner the Condorcet winner.

c. Using 4 points for a first place ranking, 3 points for a second place ranking, 2 points for a third place ranking, 1 point for a fourth place ranking, and 0 points for a fifth place ranking, we have the following Borda point scores:

issue	**computation**	**total points**
A	$339(3)+214(2)+247(4)$	2433
B	$339(4)+214(1)+247(3)$	2311
C	$339(2)+214(3)+247(2)$	1814
D	$339(1)+214(0)+247(1)$	586
E	$339(0)+214(4)+247(0)$	856

A (education funding) is the most important issue facing Illinois if we make that determination by using the Borda count method.

3. a.

Number of Voters

rank	**5**	**4**	**2**	**6**	**9**
1	D	C	C	B	A
2	B	D	B	C	D
3	C	B	D	D	B
4	A	A	A	A	C

The numbers of first place votes are: A=9, B=6, C=6, D=5. Since no choice received a majority of the first place votes, D is eliminated. The preference ranking is now:

Number of Voters

rank	**5**	**4**	**2**	**6**	**9**
1	B	C	C	B	A
2	C	B	B	C	B
3	A	A	A	A	C

The numbers of first place votes are: A=9, B=11, C=6. Since no choice has a majority of first place votes, C is eliminated. The preference ranking is now:

Number of Voters

rank	**5**	**4**	**2**	**6**	**9**
1	B	B	B	B	A
2	A	A	A	A	B

The numbers of first place votes are: A=9, B=17. B is the winner. The club will order pepperoni pizza.

b. With the change in the group of 2 voters' ranking, the preference ranking becomes:

	Number of Voters			
rank	**5**	**4**	**8**	**9**
1	D	C	B	A
2	B	D	C	D
3	C	B	D	B
4	A	A	A	C

round 1: C is eliminated

	Number of Voters			
rank	**5**	**4**	**8**	**9**
1	D	D	B	A
2	B	B	D	D
3	A	A	A	B

round 2: B is eliminated

	Number of Voters			
rank	**5**	**4**	**8**	**9**
1	D	D	D	A
2	A	A	A	D

The number of first place votes is: A=9, D=17. D is the winner. The club will order ham and pineapple pizza.

c. Monotonicity has been violated. Under the original ranking, pepperoni was the winner. Changing two ballots to place pepperoni in first place rather than second place caused pepperoni to lose.

Oral Competition
Division A - 2009 ICTM Regional Competition
Voting Theory – Extemporaneous Questions

1. Explain what is meant by the "Pareto condition". Name a method of picking a winner which can violate the Pareto condition.

2. True or false? If a candidate receives a majority of the first place votes, that candidate will be a Condorcet winner. Explain.

3. Suppose that in a Borda count election there are n candidates and v voters, with each first place vote worth $n-1$ Borda points. What is the total number of Borda points to be distributed among the candidates?

Oral Competition
Division A - 2009 ICTM Regional Competition
Voting Theory – Extemporaneous SOLUTIONS

1. The Pareto condition requires that if every voter prefers alternative A to alternative B, then B should not be the winner of the election. The method of sequential pairwise comparisons can violate the Pareto condition.

2. True. If candidate A receives a majority of first place votes, then more than 50% of the voters rank candidate A higher than any other candidate. Thus, in each pairwise comparison, candidate A will be preferred by at least 50% of the voters and hence will win that particular pairwise comparison. Winning every pairwise comparison makes candidate A a Condorcet winner.

3. Each voter has $(n-1)+(n-2)+\cdots+3+2+1+0 = \frac{n(n-1)}{2}$ Borda points to allocate among the candidates. Thus, the total number of Borda points to be distributed is $\frac{v \cdot n \cdot (n-1)}{2}$.