1. If 4 less than a number is equal to 8, find the number.

2. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   From which of the following statements can it be deduced that $x > y$?

   A) $x - y > 0$
   B) $x + y > 0$
   C) $x + 2y > 0$
   D) $x + y \geq 0$
   E) $x + 1 = y$

   Note: Be certain to write the correct capital letter as your answer.

3. Find the maximum possible value of $4y$ given x and y satisfy the system: $\begin{cases} x \geq 2 \\ y \geq 1 \\ x + y \leq 7 \end{cases}$

4. A rectangular sheet of cardboard is 24 inches long by 9 inches wide. At each of the 4 corners, a two-inch by two-inch square is removed. The remaining cardboard is then folded to form an open rectangular box. Find the number of cubic inches in the volume of this open rectangular box.

5. The width of a lake is 880 feet at the point where it is spanned by a bridge. If $37\frac{1}{2}\%$ of the bridge is over land on one side of the lake and $16\frac{2}{3}\%$ of the bridge is over land on the other side of the lake, find the number of feet in the total length of the bridge.

6. If $3x + 4 < 0$, then $|x|$ equals:

   A) x
   B) $-x$
   C) x or $-x$
   D) $\frac{4}{3}$
   E) $-\frac{4}{3}$

   Note: Be certain to write the correct capital letter as your answer.

7. Let x be a positive integer such that $9 < x < 100$. Find the sum of all distinct values of x such that the product of the two digits of x is an odd number.

8. Start with any three distinct positive integers, with each integer being less than 9. Multiply the first integer by 2. Add 5 to your product. Multiply this sum by 5. Now add the second integer to your result. Multiply your answer by 10. Now add the third integer to this last result. Subtract 250 from that result to obtain your final answer. By how much does the sum of the digits of your final answer exceed the sum of the original three integers?

9. Given the system: $\begin{cases} 3x + 8y = 23778 \\ 17x + ky = 14952 \end{cases}$ Let k and x both be **positive integers** and let y be an **integer**. Then if $k < 76$, there are two consecutive positive **odd** integers for k that satisfy the given conditions. Find the sum of those two consecutive positive **odd** integers.

10. A bag contains exactly k marbles. If the marbles were counted 3 at a time, there would be 2 left over; if the marbles were counted 5 at a time, there would be 2 left over. If $150 < k < 200$, find the sum of all distinct possible values of k.

11. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

    If x is 25% of y and y is 50% of z, then z is what percent of x?

    A) 75%
    B) 175%
    C) 275%
    D) 375%
    E) 475%
    F) 800%

    Note: Be certain to write the correct capital letter as your answer.

12. Christie said to Katie: "Give me one of your pennies, and I will then have as many pennies as you will then have." Katie said: "I won't do it, but if you give me one of your pennies, I will then have three times as many pennies as you will then have." How many pennies did Katie have at the start of this problem?

13. Let *a*, *b*, and *c* represent single digit positive integers such that $f(x) = ax^2 + bx + c$. If $f(6) = 94$ and $f(10) = 234$, find the value of $f(13)$.

14. For all positive integers x and y such that $x < 100$ and $y < 100$, let $x \oplus y = (x+7)(y+4) - 6k$. If $x \geq y$, then $k = x$. If $x < y$, then $k = 2y$. If $x \oplus y = 170$, find the sum of all distinct possible values of y.

15. For how many distinct integers x is $-5 \leq 2x \leq 6$?

16. All ages in this problem are in whole numbers of years and both parents are less than 100 years of age. An Illinois man is two years older than his wife. The product of their ages is 13 times the product of the ages of their son and daughter. Their son is one year older than their daughter. Find the number of years in the daughter's age.

17. Let k represent a positive integer. Let k be divided into 5 parts such that each part is a positive integer and such that the sum of the 5 parts is k. If the first part were increased by 6, the second part decreased by 6, the third part divided by 6, the fourth part multiplied by 6, and the sum of the fifth part and 2 were multiplied by 1.5, then the 5 results would be equal. Find the smallest possible value of k.

18. Wally ran x miles in y hours to the park and then immediately turned around and walked the same number of miles in z hours. Find the number of miles per hour in Wally's average rate for the round trip. Express your answer as a simplified single fraction in terms of x, y, and z.

19. In the math headquarters room, there are only men and women. Four women leave, and there are now twice as many men as women left in the room. Then eighteen men leave. Now there are twice as many women as men left in the room. Find the total number of people there were in the room at the start of this problem.

20. The sum of the cubes of the roots for the equation $x^2 - 80x + k = 0$ is 244,160. Find the larger of the two roots for this equation.

2009 RAA
Algebra I

Name ANSWERS

School ________________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

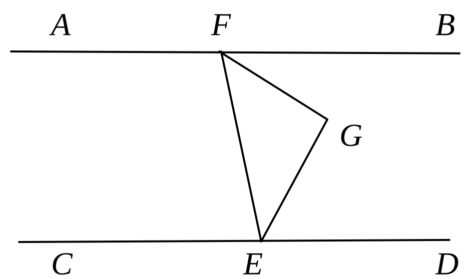
Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 12
2. A (Must be this capital letter.)
3. 20
4. 200 ("cubic inches" optional.)
5. 1920 ("feet" optional.)
6. B (Must be this capital letter.)
7. 1375
8. 0
9. 96
10. 698
11. F (Must be this capital letter.)
12. 5 ("pennies" optional.)
13. 381
14. 67
15. 6
16. 7 ("years" optional.)
17. 104
18. $\frac{2x}{y+z}$ or $\frac{2x}{z+y}$ (Must be single, reduced fraction, mph optional.)
19. 40
20. 62

1. If a square has the same area as a rectangle with sides whose lengths are 4.16 and 9.36, find the perimeter of the square. Express your answer as a **decimal**.

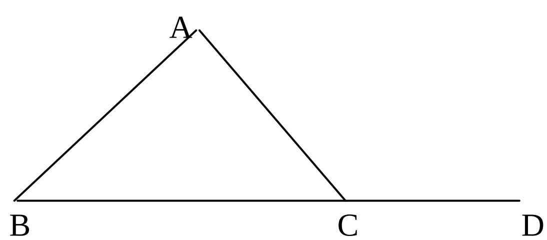
2. **(Always, Sometimes, or Never)** For your answer, write the _whole word_ **Always, Sometimes,** or **Never**—whichever is correct.

   In the diagram, the line containing points A, F, and B is parallel to the line containing points C, E, and D, $\overrightarrow{FG}$ bisects $\angle BFE$, and $\overrightarrow{EG}$ bisects $\angle FED$. Thus, $\angle GFE$ is complementary to $\angle GEF$.



3. A right triangle has one leg with a length of 6 and has an area of 18. Find the absolute value of the difference in the degree measures of the two acute angles of the triangle.

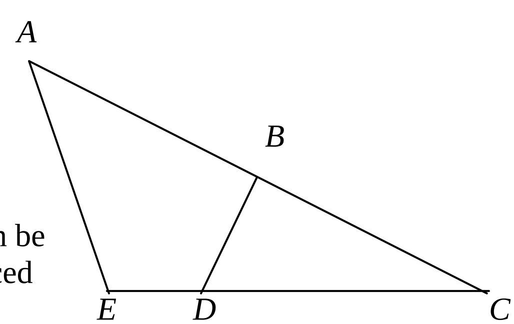
4. In the diagram, points B, C, and D are collinear. $\angle BAC = (2x+8)°$, $\angle ABC = (3x)°$, and $\angle ACD = (8x-52)°$. Find the degree measure of $\angle ACB$.



5. Two angles are supplementary, and the degree measure of one of the angles is 16 more than the degree measure of the other angle. Find the degree measure of the larger angle.

6. The equation of a circle is $x^2 + y^2 - 8y = 10$. The area of this circle is $k\pi$. Find the value of k.

7. In the diagram, B lies on $\overline{AC}$, and D lies on $\overline{EC}$. If $\frac{AB}{BC} = \frac{4}{5}$ and $\frac{ED}{DC} = \frac{1}{7}$, then the ratio of the area of ΔBDC to the area of quadrilateral $ABDE$ can be expressed as a common fraction reduced to lowest terms. Find that common fraction.

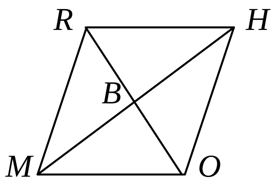


8. The length of the hypotenuse of a right triangle is 97, and the sum of the lengths of the two legs of this right triangle is 137. Find the length of the longer leg.

9. Located inside equilateral triangle TVW is a point Y such that $TY = 38$, $VY = 42$, and $WY = 44$. Rounded to the nearest integer, find the radius of the circumscribed circle of equilateral triangle TVW.

10. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

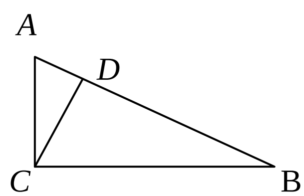
$RHOM$ is a rhombus as shown. Which of the following **cannot** be deduced?



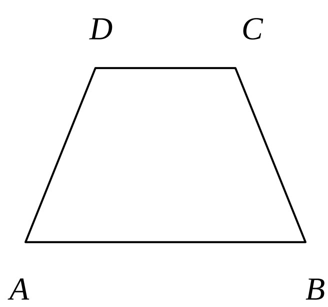
A) the perimeter of $RHOM$ is more than the sum of MH and RO
B) $RM = MO$
C) $\angle RBM$ is a right angle
D) $\angle MRO \cong \angle MOR$
E) $\angle MRB \cong \angle RMB$

Note: Be certain to write the correct capital letter as your answer.

11. The diagram shows a right triangle with $\overline{CD}$ as the altitude to the hypotenuse. If $CD = 20$ and $BD = 25$, find AD.

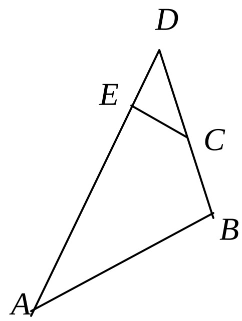


12. In the diagram, $ABCD$ is a trapezoid with $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$. $AD = 6$, $DC = 5$, $CB = 9$, and $AB = 12$. The area of trapezoid $ABCD$ can be expressed as $\frac{k\sqrt{w}}{f}$ where k, w, and f are positive integers. Find the smallest possible value of $(k + w + f)$.



13. One of the angles of an isosceles trapezoid has a degree measure of 42. Find the largest possible degree measure sum of two of the other angles of the trapezoid.

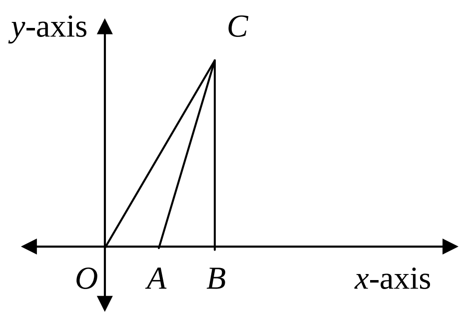
14. In the diagram, $AD = 24$, $BD = 12$, and $\angle ADB = 30°$. $\angle DAB \cong \angle DCE$, E lies on segment $\overline{AD}$, and C lies on segment $\overline{BD}$. If the lengths of $\overline{ED}$ and $\overline{CD}$ are integers, find the sum of all distinct possibilities for the area of quadrilateral $ABCE$. Express your answer as an exact decimal.



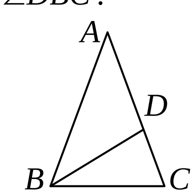
15. In a rectangle that is a square, the length of one of the diagonals is $\sqrt{578}$. In a rectangle that is **not** a square, the length of one of the diagonals is 26, and the length of at least one of the sides is an integral multiple of 5 with all such possible lengths being equally likely. From among the diagonals and the sides of these two rectangles, one segment is selected at random. Find the probability that the length of the segment selected is an integer. Express your answer as a common fraction reduced to lowest terms.

16. The measure of an interior angle of an equiangular polygon is 176.4°. Find the number of sides of the equilateral polygon.

17. In the diagram, $\overline{CB} \perp \overline{OB}$, O is the origin, and points A and B lie on the x-axis. The equation of $\overleftrightarrow{OC}$ is $y = 3x$. $\angle OCA \cong \angle ACB$. $\dfrac{OA}{AB} = \dfrac{\sqrt{k}}{w}$ where k and w are positive integers. Find the smallest possible value of $(k + w)$



18. In the diagram, $\overline{AB} \cong \overline{AC}$, and $\overrightarrow{BD}$ bisects $\angle ABC$. If the degree measure of $\angle BAC$ is 68, find the degree measure of $\angle DBC$.



19. A right triangle is inscribed in a circle. The side-lengths of this right triangle are all positive integers. If the area of the circle is 1806.25π, find the **sum** of all possible distinct areas of the right triangle.

20. In rhombus $ABCD$, $\angle DAB = 60°$. A circle passes through vertices A, B, and D, and intersects diagonal $\overline{AC}$ at E. If $EC = 12$, find the exact area of the circle **that is inscribed in the rhombus.**

2009 RAA
Geometry

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 24.96 **(Must be decimal answer.)**
2. Always **(Must be the whole word.)**
3. 0 **(Degrees optional.)**
4. 72 **(Degrees optional.)**
5. 98 **(Degrees optional.)**
6. 26
7. $\frac{35}{37}$ **(Must be this reduced common fraction.)**
8. 72
9. 41
10. E **(Must be this capital letter.)**
11. 16
12. 151
13. 276 **(Degrees optional.)**
14. 332.5 **(Must be decimal answer.)**
15. $\frac{7}{10}$ **(Must be this reduced common fraction.)**
16. 100
17. 13
18. 28 **(Degrees optional.)**
19. 5166
20. 81π **(Must be this exact answer.)**

1. Determine the value of $(b+c)$ so that the following two polynomials are identical:
$x^3 + (5b-7)x^2 + 17 + 3c + 5x$ and $163x^2 + x^3 + 5x + 50$.

2. Given the set of 5 **distinct** integers: $\{2, 4, 6, 8, k\}$. One number is selected at random from the given set and the probability that the number selected is the square of an integer is 0.2. If k represents a positive integer greater than 10, find the smallest possible value of k.

3. Find the **sum** of all distinct members of the **union** $A \cup B$ of the two sets $A = \{1, 2, 4, 5, 7\}$ and $B = \{3, 4, 6, 7, 8\}$.

4. If $f(x) - x\left(f(-x)\right) = x^2$, find $f(5)$. Express your answer as an improper fraction reduced to lowest terms.

5. Find the value of $(x+y)$ for the ordered pairs as shown: $(x-2y, 3x+4y) = (3, 29)$.

6. If $x^4 + 3x^3 - x^2 + 11x - 4$ is divided by $x+4$, the quotient is $x^3 + kx^2 + wx + p$. Find the value of $(k+w+p)$.

7. Find the smallest positive integer that has exactly 32 distinct positive integral divisors, but has only 3 distinct prime positive factors.

8. Find the **sum** of all negative integers that are members of the solution set for x of: $|x| < 5$.

9. Claire gave a test consisting of 5 problems: A, B, C, D, and E. Claire noticed that the percentage of students who had problem A correct was 65%; B, 75%; C, 60%; D, 55%; and E, 45%. Pairwise, the percentage correct was A,B, 50%; A,C, 40%; A,D, 30%; A,E, 30%; B,C, 40%; B,D, 45%; B,E, 40%; C,E, 30%; and D,E, 30%. Three at a time, the percentage correct was A,B,C, 25%; A,B,D, 25%; A,B,E,25%; A,C,D, 15%; A,C,E, 20%; A,D,E, 20%; B,C,D, 25%; B,C,E, 25%; B,D,E, 30%; and C,D,E, 20%. By fours, A,B,C,D, 10%; A,B,C,E, 15%; A,B,D,E, 20%; A,C,D,E, 10%; and B,C,D,E, 20%. Finally, the percentage having all five correct A,B,C,D,E was 10%. Five per cent of the students missed every problem. There were $k\%$ of the students who had C and D correct. Find the value of k. Do **not** attach % to your answer. **Hint:** the value of k is an integer.

10. Let $5+\sqrt[3]{7}$ be a root for x of a polynomial equation in terms of x of the smallest possible degree in which all coefficients are integers. If the coefficient of the term of the highest degree is 1, find the sum of the numerical coefficient of the x^2 term and the numerical coefficient of the x term.

11. Let $i=\sqrt{-1}$. If $x+2i=y$, find the value of $|x-y|+|y-x|$.

12. Find the length of the minor axis of an ellipse whose equation is $27x^2+144y^2=3888$.

13. The reciprocal of each root of $36x^3+228x^2+73x+6=0$ is also a root for x for the cubic equation $x^3+kx^2+wx+6=0$. Find the value of $(k+w)$. Express your answer as a proper or improper fraction as appropriate, reduced to lowest terms.

14. If n is a positive integer, find the value of n such that

$$\frac{1(2)+3(4)+5(6)+\cdots+(2n-1)(2n)}{1(2)(3)+2(3)(4)+3(4)(5)+\cdots+n(n+1)(n+2)}=\frac{29}{150}.$$

15. Find the degree of the following polynomial: $\left(3x^2\right)\left(x^3\right)+2x\left(x\right)^3-13x^2+5x-6$.

16. Find the remainder when 11^{27^6} is divided by 7.

17. The terms of an arithmetic sequence are: $50,75,100,\cdots$. The terms of the triangular sequence whose n^{th} term is $\frac{n^2+n}{2}$ are: $1,3,6,10,\cdots$. Find the value of the first term of the triangular sequence that is greater than the corresponding term of the arithmetic sequence.

18. If Lee selects 1 coin at random from k coins, the probability the coin will be a nickel is $\frac{1}{3}$. If Cindy selects 2 coins without replacement at random from the same k coins, the probability both coins will be nickels is $\frac{1}{12}$. Find the least value of k.

19. If $\log_a 6=4$ and $\frac{1}{\log_a\left(r\right)}=3$, find the value of $\log_a\left(\sqrt{6r^3}\right)$. Express your answer as a **decimal**.

20. Two friends have agreed to meet for lunch this Saturday. They have agreed that each will arrive at a random time between 11:00 A. M. and 12:00 noon and that each will wait for the other for 8 minutes or until 12:00 noon. Otherwise, the first to arrive will leave. Find the probability the two will actually have lunch together this Saturday. Express your answer as a common fraction reduced to lowest terms.

2009 RAA
Algebra II

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 45
2. 11
3. 36
4. $\frac{75}{13}$ (Must be this reduced improper fraction.)
5. 9
6. 1
7. 1080
8. –10
9. 35
10. 60
11. 4
12. $6\sqrt{3}$ (Must be this exact answer.)
13. $\frac{301}{6}$ (Must be his reduced improper fraction.)
14. 22
15. 5
16. 1
17. 1326
18. 9
19. 2.5 (Must be decimal answer.)
20. $\frac{56}{225}$ (Must be this reduced common fraction.)

1. $\begin{bmatrix} 3 & 2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} k \\ w \end{bmatrix}$. Find the value of $(2k+3w)$.

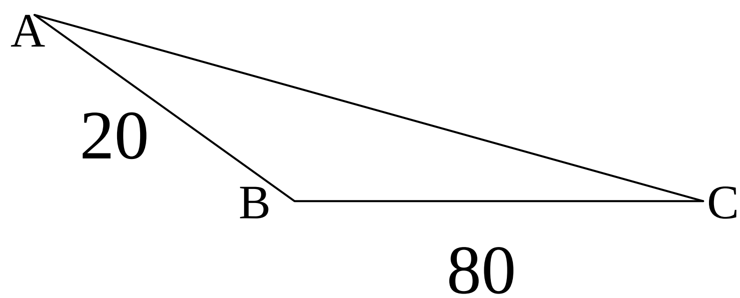
2. Given the set: $\{\log_2(16),\ \log_3(16),\ \log_4(16),\ \log_5(16)\}$. If one of the 4 members of the set is drawn at random, find the probability that the member drawn could represent a positive integer. Express your answer as a common fraction reduced to lowest terms.

3. Find the ordered pair that represents the sum of the following two vectors: $(-5,6)$ and $(17,7)$.

4. Let n be a positive integer. For $n \geq 1$, $x_{(n+2)} = x_{(n)} + x_{(n+1)}$. If $x_1 = 3$ and $x_2 = 1$, find the value of x_{10}.

5. Matrix $[A]$ contains 23 rows and 13 columns. Matrix $[B]$ contains 13 rows and 92 columns. Find the number of rows in the product matrix $[A][B]$.

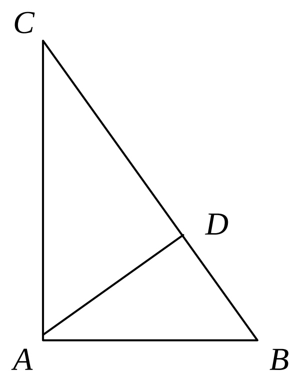
6. In ΔABC as with side-lengths as shown, $\angle ABC = 120°$. Find the exact length of $\overline{AC}$.



7. A woman is sitting in the stands behind one end zone of a football field. She observes, in the same vertical plane with her position, two players standing on their own goal lines, exactly 100 yards apart. Looking at the spots on the ground where they are standing, a player at one goal line is at an angle of depression of 15° and a player at the other goal line is at an angle of depression of 8°. Assuming the football field is horizontal, find the **number of feet** in the vertical height of the woman's eye above the horizontal plane of the football field. Express your answer as a **decimal** rounded to the nearest hundredth of a foot.

8. If three real geometric means are inserted between 15 and $\frac{5}{27}$, find the value of the middle term. Express your answer as an improper fraction reduced to lowest terms.

9. In the diagram ΔCAB is a right triangle with $\angle CAB = 90°$. Point D is on $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$. Let the lengths of ΔCAB be a, b, and c with these lengths respectively opposite the angles with vertices at A, B¸ and C. If $b\sin C\left(b\cos C + c\cos B\right) = 41$, and if the length of $\overline{AD}$ is an integer, find the largest possible value of AD.



10. The roots of $x^3 + kx^2 - 59x + 420 = 0$ are 5, -7, and 12. Find the value of k.

11. When $(2x-3)^8$ is expanded and completely simplified, find the numerical coefficient of x^5.

12. The first term of an infinite geometric sequence of real terms is 225, and the fourth term of this geometric sequence is 14.4. Find the sum of the terms of this infinite geometric sequence.

13. The sum of the terms of an arithmetic sequence with 13 terms and common difference of 3 is found to be 286. Find the thirteenth term of this sequence.

14. Let there be k objects, each of weight w. When these objects are weighed in pairs, the sum of the weights of all possible pairs is 770. When these objects are weighed in groups of four, the sum of the weights of all possible groups of four is 9240. Find the value of $\left(k+w\right)$.

15. Let $i=\sqrt{-1}$. Find the exact value of $\left|\sqrt{6}+i\sqrt{8}\right|$. Write your answer in simplified radical form.

16. Tom is a gambler. He has a pair of fair, standard cubical dice. He selects two different numbers in advance from the face numbers 1, 2, 3, 4, 5, and 6. He then rolls the pair of dice. If both numbers showing match his two selections, he wins \$108. If exactly one number showing matches one of his two selections, he neither wins nor loses. If neither number showing matches either of his two selections, he loses \$54. Find Tom's mathematical expectation each time he rolls the pair of dice. Express your answer by including the word "lose" or the word "win." For example, your answer might be "lose \$2" or might be "win \$7."

17. Let D be the point that is $\frac{1}{3}$ of the way from B to A for equilateral triangle ΔABC. Let P be a point on $\overline{BC}$ such that $AP+PD$ is the smallest possible value. If the perimeter of ΔABC is 63, find the exact value of $\left(AP+PD\right)$.

18. Let $i=\sqrt{-1}$ and let k and w represent real numbers. If $(k+wi)(4+i)=33-13i$ then find the value of $\left(k+w\right)$.

19. Find the acute angle between the normal vectors of the following two planes:

$$3x+2y+5z+4=0 \text{ and } -2x-3y+6z-2=0$$

Round your answer to the nearest minute and then express your answer in degrees and minutes in the form: $k°w'$.

20. Points A, B, C, and D are the vertices of a rectangle. Point E is in the interior of this rectangle. $AE=8$, $EC=21$. The lengths of $\overline{DE}$, $\overline{BE}$, and $\overline{BC}$ are integers. If $DE<BE$, find the largest possible value for the length of $\overline{BC}$.

2009 RAA

Pre-Calculus

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 68
2. $\frac{1}{2}$ (Must be this reduced common fraction.)
3. $(12,13)$ (Must be this ordered pair.)
4. 97
5. 23
6. $20\sqrt{21}$ (Must be this exact simplified radical.)
7. 88.67 (Must be this decimal, feet optional.)
8. $\frac{5}{3}$ (Must be this reduced improper fraction.)
9. 4
10. -10
11. -48384
12. 375
13. 40
14. 18
15. $\sqrt{14}$ (Must be this exact simplified radical.)
16. Lose 18 (Must have "lose", not -18, \$ or dollars optional.)
17. $7\sqrt{13}$ (Must be this exact simplified radical.)
18. 2
19. $65°21'$ (Must be in the form Degrees and minutes.)
20. 19

NO CALCULATORS

1. A school purchases resin-coated variable contrast photographic paper in boxes of 250 sheets for $70 per box. They cut each of the 250 sheets in half. Find the number of **cents** in the cost of each half-sheet.

2. A certain brand of perfume costs 88 cents per $\frac{1}{4}$ ounce. At that rate, find the number of **cents** that one gallon of that perfume would cost.

3. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   If the expression $5x+3$ represents a whole number that is an integral multiple of 7, which of the following expressions **must** also be a whole number that is an integral multiple of 7?

   A) $7x+3$
   B) $5x+7$
   C) $7x+7$
   D) $15x+9$
   E) $12x+10$

   Note: Be certain to write the correct capital letter as your answer.

4. From the set $\{\pi,\ \frac{4}{\frac{\pi}{2}},\ 4\sqrt{\pi},\ \frac{6}{\sqrt{\pi}},\ 8\}$, one member of the set is selected at random, and the member selected is the number of units in the radius of a circle. Find the probability that the number of square units in the area of that circle is a whole number. Express your answer as a common fraction reduced to lowest terms.

5. Let $f(x)=\frac{x^2+kx+w}{x+p}$ have zeroes for x of 7 and 17 and be undefined when $x=12$. Find the value of $(2k+3w+4p)$.

6. Two vertices of an isosceles right triangle are $A(0,0)$ and $B(12,0)$. If $\overline{AB}$ is the hypotenuse, and the remaining vertex of the triangle lies in Quadrant IV, find the **ordered pair** that represents this remaining vertex.

7. Find the area of an equilateral triangle if a side of the equilateral triangle has a length that is equal to the units digit of 12345678^{2009}.

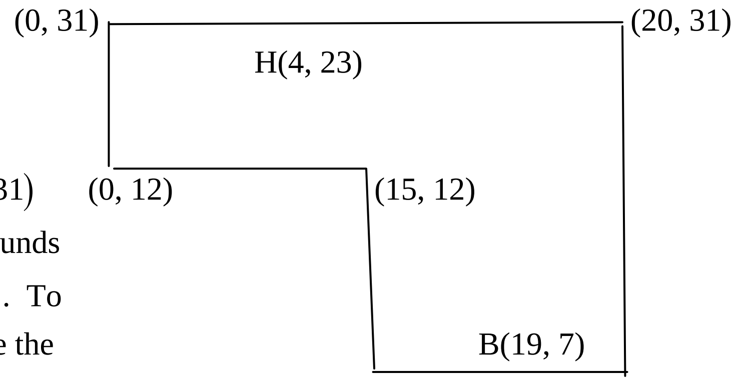
NO CALCULATORS

NO CALCULATORS

8. If k is the number of distinct positive integers that leave a remainder of 12 when divided into 3587, find the value of k.

9. All side-lengths of a triangle are whole numbers. If two of the sides have lengths of 15 and 19, find the largest possible perimeter of the triangle.

10. On a miniature golf course as shown, the ball is at $B(19,7)$. A golfer wishes to strike the ball so that it hits the barrier running from $(0,31)$ to $(20,31)$ and then rebounds into the hole at $H(4,23)$. To do so, the ball must strike the barrier at $(x,31)$. Find the value of x. Express your answer as an improper fraction reduced to lowest terms.



11. Given the system: $\begin{cases} 3x = k+9 \\ 3y = 5k+6 \end{cases}$ For all real numbers x and y, $k = \dfrac{x+y-p}{m}$. Find the value of $(3p+2m)$.

12. The point $A(5,17)$ is rotated 270° clockwise about the point $(10,5)$ to point B. Find the distance from point B to the line represented by $8x-15y+135=0$.

13. The equation of the line containing all points in the plane that are equidistant from the points $(4,2)$ and $(-2,-3)$ can be expressed in the form: $y = mx+b$. Find the value of $(300m+120b)$.

NO CALCULATORS

14. For how many different ordered pairs (a,b) where a is a number selected from List A and b is a number selected from List B, is $a-2b>0$?

List A	List B
1	1
2	2
3	3
6	4
9	5

15. Shoeboot Company manufactures only brown boots and black boots, both of which are available as either waterproof or plain. On the basis of the information on the table below, how many black boots did Shoeboot Company manufacture in March, 2008?

Shoeboot Company's March 2008 Production

	Waterproof	Plain	Total
Brown	4,200		
Black		1,600	
Total		4,300	12,000

16. Quadrilateral $ABCD$ is inscribed in a circle such that $AB=6$, $BC=7$, $CD=2$, and $DA=9$. Sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively, are extended past B, C, D, and A, respectively, to points E, F, G, and H, respectively, creating congruent segments. (Thus $\overline{AB}\cong\overline{BE}$, $\overline{BC}\cong\overline{CF}$, and so forth.) Find the area of quadrilateral $EFGH$.

17. If the line passing through the points $(5,5)$ and $(-3,1)$ is reflected through the *y*-axis, find the *x*-intercept of this reflected line. **Note:** the *x*-intercept is the ***x*-coordinate only**, and is **not** an ordered pair.

18. Find the midpoint of the line segment that joins point $A(1,2,5)$ to point $B(7,-6,1)$. **Express your answer as an ordered triple.**

19. Let x, y, and z be digits and let the three-digit number N be represented as xyz_{ten}. **Note:** xyz **is a 3-digit number and <u>not</u> the product** $(x)(y)(z)$**.** If k, w, and p are positive integers, find the least values of k, w, and p such that N **must** be divisible by 8 if $kx+wy+pz$ is divisible by 8. Express your answer as an **ordered triple** of the form (k,w,p).

20. Choosing from the set $\{1,2,3,4,5,6,7,8,9\}$, how many distinct sets of 5 different integers exist where the 5 different integers are represented by a, b, c, d, and e in some order where $a<b<c$ and where a, b, and c are the one and only triplet, in any particular 5 member set, in the respective ratio $1:2:3$?

NO CALCULATORS

2009 RAA

Fr/So 8 Person

School ___ANSWERS___

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 14 **(Cents optional.)**
2. 45056
3. D **(Must be this capital letter.)**
4. $\frac{1}{5}$ **(Must be this reduced common fraction.)**
5. 261
6. $(6,-6)$ **(Must be this ordered pair.)**
7. $16\sqrt{3}$ **(Must be this exact answer.)**
8. 9
9. 67
10. $\frac{31}{4}$ **(Must be this reduced improper fraction.)**
11. 19
12. 7
13. -276
14. 7
15. 5100
16. 150
17. 5
18. $(4,-2,3)$ **(Must be this ordered triple.)**
19. $(4,2,1)$ **(Must be this ordered triple.)**
20. 39

NO CALCULATORS

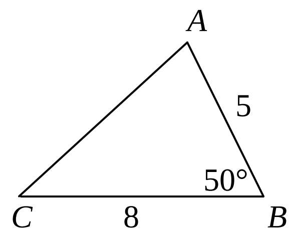
1. Find the value of $\left(16^{0.43}\right)\left(16^{2.07}\right)\left(16^{\left(\frac{1}{4}\right)}\right)$. Express your answer as an **integer**.

2. Find the maximum **sum** of the distinct members of the largest **proper** subset of $\{1,2,3,4,5\}$.

3. A line passes through the points represented by $\left(-3,7\right)$ and $\left(k,32\right)$. If this line has a direction vector of $\left(2,5\right)$, find the value of k.

4. There are two values of x for which the three terms: 3, $2x+1$, and $7x-1$, taken in that order, will form a geometric sequence. One of those values for x is an integer. Find the **other** value for x. Express your answer as a **decimal**.

5. In ΔABC with measures as shown, $8^2+5^2-(AC)^2=k\cos(50°)$. Find the value of k.



6. An ellipse has the equation of $\dfrac{4(x-2)^2}{169}+\dfrac{(y+3)^2}{16}=1$. Find the length of the major axis of this ellipse.

7. Let $i=\sqrt{-1}$. Find the exact product of the two distinct roots of the equation $x^2=9i$.

NO CALCULATORS

NO CALCULATORS

8. Bob and Judy are playing a game with four dice whose faces are numbered as follows:
   Orange Die: 4, 4, 4, 4, 4, 4
   Blue Die: 8, 8, 2, 2, 2, 2
   Scarlet Die: 7, 7, 7, 1, 1, 1
   Gray Die: 6, 6, 6, 6, 0, 0

   Bob must select a die first. After Bob selects his die, he rolls that die once. Then Judy must select her die. Judy then rolls her die once. The winner is the one who rolled the higher number. Bob selects the scarlet die and rolls. Judy knows Bob selected the scarlet die, but not the result of his roll. If Judy applies her best strategy, find the probability that Judy will win the game. Express your answer as a common fraction reduced to lowest terms.

9. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   Which of the following is **not** an arithmetic series?

   A) $\sum_{n=1}^{18} 2n+1$
   B) $\sum_{a=1}^{6} \frac{1}{6}a$
   C) The sum of the first 10 integral perfect squares.
   D) The sum of the first 10 integral positive multiples of 3.
   E) The sum of the first 10 positive integers.

   Note: Be certain to write the correct capital letter as your answer.

10. Let $i=\sqrt{-1}$. If $|6+ki|=6\sqrt{5}$, find the smallest possible real value of k.

11. The graph of $y=\dfrac{x^3+11x^2+10x-72}{x^2-x-2}$ has a slant or oblique asymptote of $y=mx+b$. Find the value of $(m+b)$.

12. One of the angle bisectors of the angles formed by the graphs of $5x-12y+80=0$ and $12x+5y-2=0$ is represented by the equation $kx-7y+w=0$ where k and w are integers. Find the value of $(k+w)$.

13. How many distinct numbers are in the **range** of the relation: $\{(5,2),(4,0),(6,2),(854,2)\}$?

NO CALCULATORS

NO CALCULATORS

14. If $0° < x < 45°$, find the sum of all distinct values of x such that $\sin(24x)° + \cos(6x)° = 0$.

15. The linear regression line (also known as the least squares regression line) for the three points $(5,23)$, $(6,41)$, and $(7,95)$ is $y = 36x - w$. Find the value of w.

16. $ABCD$ is a convex quadrilateral with $AB = 4$, $BC = 30$, and $CD = 56$. $\angle BCD$ is obtuse, $\cos(\angle ABC) = -\frac{3}{5}$, and $\sin(\angle BCD) = \frac{3}{5}$. Find the length of $\overline{BD}$.

17. All ages in this problem are in whole numbers of years. A father has three sons whose ages form an arithmetic progression. The father's age is now equal to the sum of the sons' ages. Five years ago, the sum of the sons' ages at that time was half of the father's present age, and the oldest son then was 4 times as old as the youngest son then. Find the number of years in the present age of the oldest son.

18. Let k be selected at random from the set: $\{2,3,5,7,8\}$. Find the probability that, for all radian values of θ, that $\cos(\frac{5\pi}{k} - \theta) = \sin(\theta)$. Express your answer as a common fraction reduced to lowest terms.

19. If x and y are real, and $x^2 + y^2 = 1$, find the maximum value of $(x+y)^8$.

20. Find the exact value of the diameter of a circle that is circumscribed about a triangle with sides of lengths 11, 31, and 35.

2009 AA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 2048

2. 14

3. 7

4. 0.25 or .25 (Must be decimal answer.)

5. 80

6. 13

7. $-9i$ or $0-9i$

8. $\frac{2}{3}$ (Must be this reduced common fraction.)

9. C (Must be this capital letter.)

10. -12

11. 13

12. 95

13. 2

14. 113 (Degrees optional.)

15. 163

16. 82

17. 13 (Years optional.)

18. $\frac{1}{5}$ (Must be this reduced common fraction.)

19. 16

20. $\frac{62\sqrt{3}}{3}$ or $20.\overline{6}\sqrt{3}$ (Must be this exact answer.)

1. Dee Fender, the soccer goalie, had a save average of 97% (she stopped 0.97 of all shots taken against her). If the opponents averaged 15 shots on goal per game and Dee played in 3186 entire soccer games, find the total number of goals that Dee allowed. Round your answer to the nearest integer and express your answer as that **integer**. Do **not** use scientific notation.

2. An observer in a building that rises vertically notes that two objects on a horizontal road below have respective angles of depression of 27° and 18° respectively. If the distance from the base of the building to the far object is 1458 feet and if the horizontal road runs directly away from the observer, find the number of feet that the eye of the observer is above the horizontal road.

3. Let x represent the length of the third side of a right triangle whose other two sides have lengths of 4.000 and 5.000 in some order. Find the absolute value of the difference between the smallest possible value of x and the largest possible value of x.

4. Find $\cot\left(\sec^{-1}(2.678)\right)$ where $\sec^{-1} x$ (also written $Arc\sec(x)$) means principal value.

5. Find the length of the radius of the inscribed circle of a triangle whose longest side has a length of 18 and whose acute angles have measures of 28° and 47°.

6. Assume that 74.56% of all mathletes at the ICTM State Math Contest chew gum. From a random group of mathletes at the ICTM State Math Contest, 100 students are selected at random. Calculate the standard deviation for the number of these mathletes who chew gum. Express your answer as a **decimal** rounded to 4 significant digits.

7. A parallelogram has sides whose lengths are 12.21 and 15.37. One of the angles of the parallelogram is 29.13°. Find the area of the parallelogram.

8. Find the **exact value** of the 50th term of the Fibonacci sequence whose first 7 terms are: 1, 1, 2, 3, 5, 8, 13. Express your answer as an **integer**.

9. When $x = 9.888$, $y = 4.12$. If y varies directly as x, find the value of x when $y = 2.9616$.

10. Beulah wants to know at what annual percentage rate of interest to invest her money if it is to double in 6.134 years when compounded continuously. Find the annual percentage rate of interest needed. Express your answer as a per cent.

11. Find the value of $\csc(70°57')$.

12. The first four terms of a sequence that is either geometric, arithmetic, or harmonic are: 0.244140625, 0.1953125, 0.15625, and 0.125. By finding what type of sequence is involved, find the sum of the first 13 terms of this sequence.

13. Find the minimum value of the expression $2\log(x+2) - \log(2x)$.

14. In ΔABC, $BC = 24.68$, $\angle ABC = 27.00°$, and $\angle ACB = 84.00°$. Find AC.

15. Find the value of x such that $\log_{3.115}(2.111x - 5.678) - \log_{3.115} 2.444 + \log_{3.115} 1.116 = 4.197$

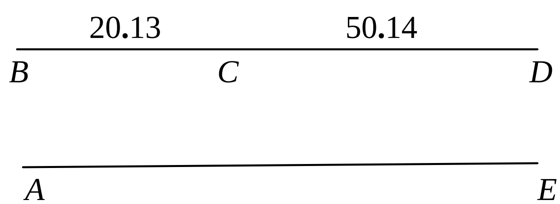
16. In ΔABC, a, b, and c are the sides opposite the respective angles $\angle A$, $\angle B$, and $\angle C$. $a = 6$, $b = 8$, and $m\angle A = 35°$. Find the sum of all possible length(s) for side c.

17. Roberta is at $(-3,-6)$ and walks to a point on the graph of the ellipse whose foci are $(4,0)$ and $(-4,0)$ and whose major axis has a length of 10. Then she ends her walk by walking from that point on the graph of the ellipse to $(-1,-4)$. Find the shortest possible length of Roberta's total walk.

18. Find the value of x such that $\sqrt{x+2}+\sqrt{x+5}=x^2+\sqrt{\frac{x}{3}}$.

19. A cube with a volume of 216 is truncated by removing congruent regular pyramids from each vertex so that the remaining solid has faces that are regular octagons and equilateral triangles. Find the total surface area of this truncated cube.

20. In the diagram, Bob is at point A on the straight center line $\overline{AE}$ of a straight road and wishes to reach point D on the curb $\overline{BD}$. The road is entirely in sun as is the portion of the curb from B to C (20.13 units), while the portion from C to D (50.14 units) is in the shade. Bob uses 18% less energy walking in the shade than when walking in the sun. Assuming $\overline{BD} \parallel \overline{AE}$ and that the perpendicular distance from A to B is 12 units, Bob should aim at a point on $\overline{BD}$ that is x units from D in order to minimize the energy used for his walk. Find the value of x.



2009 RAA

Calculator Team

School **ANSWERS**

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 1434 **(Must be this integer.)**
2. 473.7 or 4.737×10^{2}
3. 3.403 or 3.043×10^{0}
4. 0.4025 or $.4025$ or 4.025×10^{-1}
5. 2.852 or 2.852×10^{0}
6. 4.355 or 4.355×10^{0}
7. 91.36 or 9.136×10 or 9.136×10^{1}
8. 12,586,269,025 **(Must be this integer.)**
9. 7.108 or 7.108×10^{0}
10. 11.30 or 1.130×10 or 1.130×10^{1} **(Trailing zero necessary, % optional.)**
11. 1.058 or 1.058×10^{0}
12. 1.154 or 1.154×10^{0}
13. 0.6021 or $.6021$ or 6.021×10^{-1}
14. 12.00 or 1.200×10 or 1.200×10^{1} **(Trailing zeroes necessary.)**
15. 124.9 or 1.249×10^{2}
16. 13.11 or 1.311×10 or 1.311×10^{1}
17. 4.679 or 4.679×10^{0}
18. 1.954 or 1.954×10^{0}
19. 200.3 or 2.003×10^{2}
20. 50.14 or 5.014×10 or 5.014×10^{1}

1. Let $-1 < x < 4$. If y is an integer such that $y = \left| x - 3\frac{1}{2} \right|$, find the sum of all distinct possible values for y.

2. If $x - 18 = 2y$, then $|2y - x| = k$. The area of an equilateral triangle is $45\sqrt{3}$. Let p be the perimeter of this equilateral triangle. Find the value of $(k + p)$.

3. One number is positive, and a second number is negative. The difference of their reciprocals is $\frac{1}{4}$. Find the greatest integral value for the negative number.

4. Start by choosing a number that is a positive improper fraction (and may differ from every other team's number.) A is computed by multiplying the original number by 2, adding 16 to the result, then dividing that sum by 2, and finally subtracting the original number. B is computed by dividing the original number by 2, then subtracting 16 from the result, then multiplying that difference by 2, and finally subtracting the original number. Find $(A - B)$.

5. Find the value of $10110_{two} + 2102_{three} + 3112_{four} + 534_{six}$. Express your answer in **base five.**

6. Let p be the maximum integral value such that the roots of $2x^2 - 7x + 3p = 0$ are real. Let q be the minimum integral value such that $2x^2 + 7x + 3q = 0$ has no real solutions. Let $f(x) = x^3 - 6x^2 + 12x - 8$. Find the value of $(f(p) - f(q))$.

7. Point A lies on the line represented by $3x - 4y + 24 = 0$, and point C lies on the line represented by $3x - 4y + 12 = 0$ such that $\overline{AC}$ is perpendicular to the line represented by $3x - 4y + 24 = 0$. Point $B(4, k)$ lies between A and C such that $AB : BC = 2 : 3$. Find the value of k. Express your answer as an improper fraction reduced to lowest terms.

8. Let k be the value of the expression: $x^2 + xy + y^2$ if $x^3 - y^3 = 84$ and $\frac{x - y}{1.125} = \frac{32}{9}$. Let $w\pi$ be the area of the circle circumscribed about a rectangle with side-lengths of 32 and 60. Find the value of $(k + w)$.

9. The lengths of all sides of a triangle are integers and one side has length 10. The perimeter of the triangle is 23. Find the number of distinct triangles possible.

10. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ k & 2 & 7 \\ 3 & k & 8 \end{vmatrix} = 48$, where k is an integer.. The perimeter of the rhombus, whose diagonals have lengths of 12 and 18, in simplified form, is $w\sqrt{p}$. Find the value of $(k + w + p)$.

2009 RAA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		Score (to be filled in by proctor)
1.	10	
2.	$18+18\sqrt{5}$ **(or exact, simplified equivalent.)**	
3.	-5	
4.	40	
5.	4003 or 4003_5 or 4003_{five}	
6.	-1	
7.	$\frac{39}{5}$ **(Must be this reduced improper fraction.)**	
8.	1177	
9.	5	
10.	33	

TOTAL SCORE: ________

(*enter in box above)

Extra Questions:

11. 31

12. 28

13. 51.34 **(Must be this decimal.)**

14. 2637

15. 41

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let $i=\sqrt{-1}$. If two of the following seven numbers are selected at random, what is the probability both are equivalent to i? i^7 i^8 i^9 i^{10} i^{361} i^{761} i^{951}

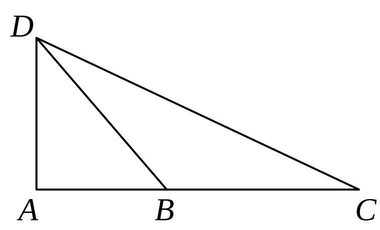
2. The distance from $(k,5)$ to $(-4,20)$ is 17. If $k>0$, find the value of k. Find w, the sum of the distinct real solutions of $|x-3| + |x+5| = 10$. Report as your answer the sum $(k+w)$.

3. Let x be a positive integer such that $x<2009$. Find the sum of all distinct values of x such that $3>\frac{6}{x-4}$.

4. A ring of 6 circular disks covers the boundary of a unit circle exactly with each disk tangent to the disk on either side. The sum of the areas of the six disks is $k\pi$. Find k.

5. Let k be the average rate in mph. of a person who drives from A to B at a constant rate of 21 mph. and immediately returns from B to A at a constant rate of 28 mph. Let w be the average rate in mph. of a person who drives 2 hours at a constant rate of 21 mph. and immediately drives 2 more hours at a constant rate of 28 mph. Find the value of $(k+w)$.

6. Let $R=\{2.567, 3.889, 4.997, 18.25\}$. Let k be the total population standard deviation (σ_X) of R. Points A, B, and C are collinear. $\angle DAB = 88.13°$ and $\angle DCA = 11.97°$, and $DA = 124.3$. Find the value of $(k+AC)$. Express your answer as a **decimal** rounded to the nearest tenth.



7. Every constant a_i in a given polynomial: $a_n x^n + a_{(n-1)} x^{(n-1)} + \cdots + a_1 x + a_0$ is either equal to a constant, c, or is equal to one. It is given that $c \neq 1$, and that the choosing of c is equally as likely as choosing one for any a_i. Find the probability that the absolute value of the product of the zeroes of the polynomial is one. Express your answer as a common fraction reduced to lowest terms.

8. Let $\sum_{a=1}^{50}(3a)=S$ and $\sum_{k=5}^{M}(k+9)=S$ and $M>0$. Find the value of M

9. When the long division is carried out, the quotients $\dfrac{x^2+2x-8}{x^2+3}$ and $\dfrac{x^4+2x^2+Ax+(B+1)}{x^2+1}$ have the same remainder. For some real constants C and D, $x^4-32x^2+Cx+D=0$ has zeroes 3 and $(2+i)$. Find $(A+B+C+D)$.

10. Find the sum of all distinct positive integral factors of 3024.

2009 RAA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. $\frac{1}{7}$ **(Must be this reduced common fraction.)**	
2. 2	
3. 2,017,021	
4. 2	
5. 48.5 or $48\frac{1}{2}$ or $\frac{97}{2}$	
6. 596.3 **(Must be this decimal.)**	
7. $\frac{1}{2}$ **(Must be this reduced common fraction.)**	
8. 79	
9. –10	
10. 9920	

TOTAL SCORE: ____________

(*enter in box above)

Extra Questions:

11. 6

12. 57340

13. –368

14. 23

15. 23

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

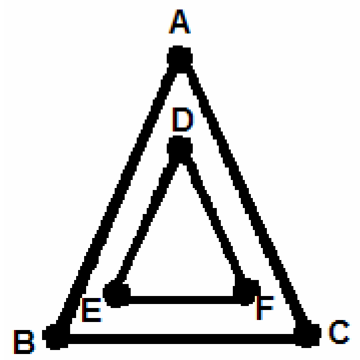
GRAPHS AND NETWORKS

Ch. 1, For All Practical Purposes, sixth edition

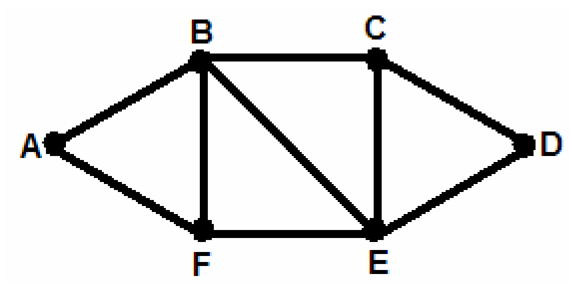
1) Explain the following, in your own words:
   a) What is an Euler circuit?
   b) What does it mean to Eulerize a graph? Give an example.
   c) What is the valence number for a vertex? Give an example.
   d) Describe the valence numbers in a graph which has an Euler circuit.

2) Which graphs have Euler circuits? For the one(s) that do, find the Euler circuits by numbering the edges in the order the Euler circuit uses them starting at point A. For the one(s) that doesn't (don't) explain why no Euler circuit is possible.

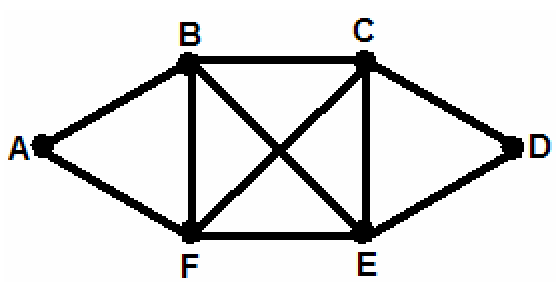
a)



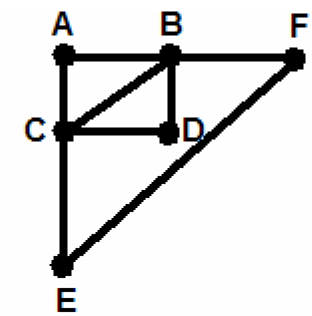
b)



c)



d)



3) If a rectangular street is *r* blocks by *s* blocks, find a formula for the minimum number of edges that must be added to Eulerize a graph representing the network in terms of *r* and *s* when both *r* and *s* are > 1*.* Determine a formula when
   a) both *r* and *s* are odd . Test your formula with the case: 5 blocks by 5 blocks using the edge walker technique.
   b) both *r* and *s* are even. Test your formula with the case: 6 blocks by 6 blocks using the edge walker technique.
   c) *r* is odd and *s* is even. Test your formula with the case:
      5 blocks by 4 blocks using the edge walker technique.

Solutions for the Questions for the Oral Competition

ICTM Regional, Division AA 2009

GRAPHS AND NETWORKS

Ch. 1, For All Practical Purposes, sixth edition

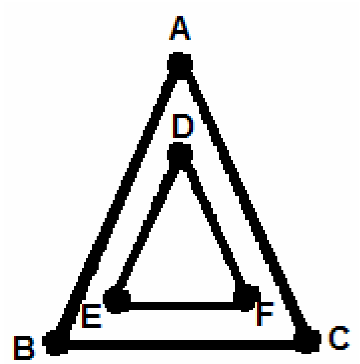
1) a) An Euler circuit is a circuit that traverses each edge of a graph exactly once.

b) Eulerizing a graph is adding new edges to a graph so as to make it a graph that possesses an Euler circuit. The new edges added must be duplicates of existing edges.

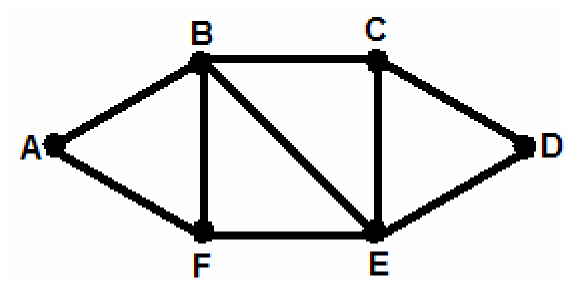
c) The valence of a vertex is the number of edges that touch the vertex.

d) All of the valences need to be even numbers greater than 1.

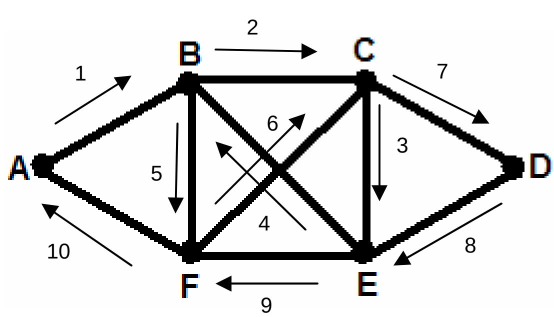
2) a) Does not have an Euler circuit.
It is not a connected graph.



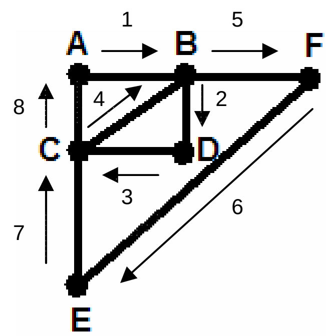
b) Does not have an Euler circuit.
Vertices C and F do not have an even valence.



c) Graph has an Euler circuit.
There are several possibilities.
One is ABCEBFCDEFA



d) Graph has an Euler circuit.
There are several possibilities.
One is ABDCBFECA.



3) If a rectangular street is *r* blocks by *s* blocks, find a formula for the minimum number of edges that must be added to Eulerize a graph representing the network in terms of *r* and *s* when both *r* and *s* are >1. Determine a formula when

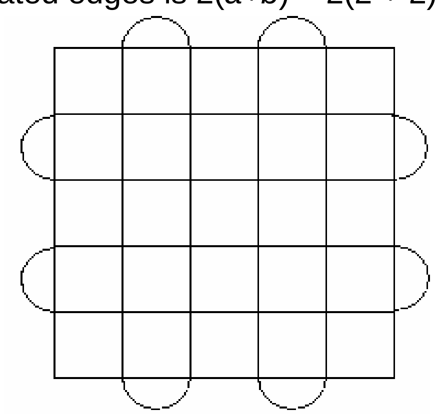
a) both *r* and *s* are odd . Test your formula with the case: 5 blocks by 5 blocks using the edge walker technique.

b) both *r* and *s* are even. Test your formula with the case: 6 blocks by 6 blocks using the edge walker technique..

c) either *r* or *s* is even and the other one is odd. Test your formula with the case: 5 blocks by 4 blocks using the edge walker technique..

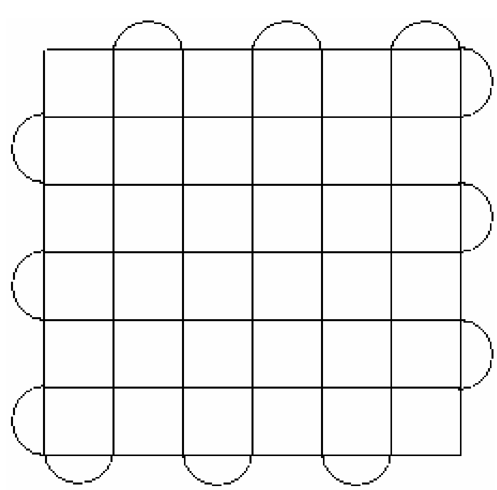
a) If *r* and *s* are odd, where $r = 2a+1$ and $s = 2b+1$ (*a* and *b* positive integers which are at least 1) then a formula for the number of repeated edges is $2(a + b)$.

For 5 blocks by 5 blocks, $r = 2a+1$ so $a = 2$ and $s = 2b+1$ so $b = 2$; therefore, the minimum number of repeated edges is $2(a+b) = 2(2 + 2)= 8$.



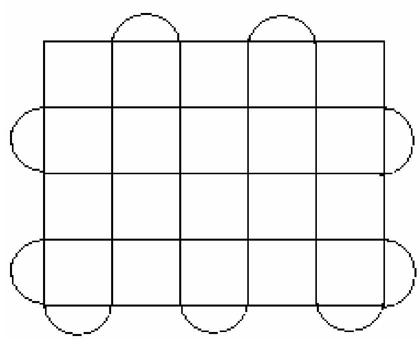
b) If *r* and *s* are even, where $r = 2a$ and $s = 2b$ (*a* and *b* positive integers which are at least 1) then a formula for the number of repeated edges is $2(a + b)$.

For 6 blocks by 6 blocks, $r = 2a$ so $a = 3$ and $s = 2b$ so $b = 3$; therefore, the minimum number of repeated edges is $2(a + b) = 2(3 + 3) = 12$.



c) If r is odd and s is even, where $r = 2a+1$ and $s = 2b$ (a and b positive integers which are at least 1) then a formula for the number of repeated edges is $2(a + b) + 1$.

For 5 blocks by 4 blocks, $r = 2a + 1$ so $a = 2$ and $s = 2b$ so $b = 2$; therefore, the minimum number of repeated edges is $2(a + b) + 1 = 2(2 + 2) +1 = 9$.



Extemporaneous Questions – Division AA Oral - ICTM Regionals 2009

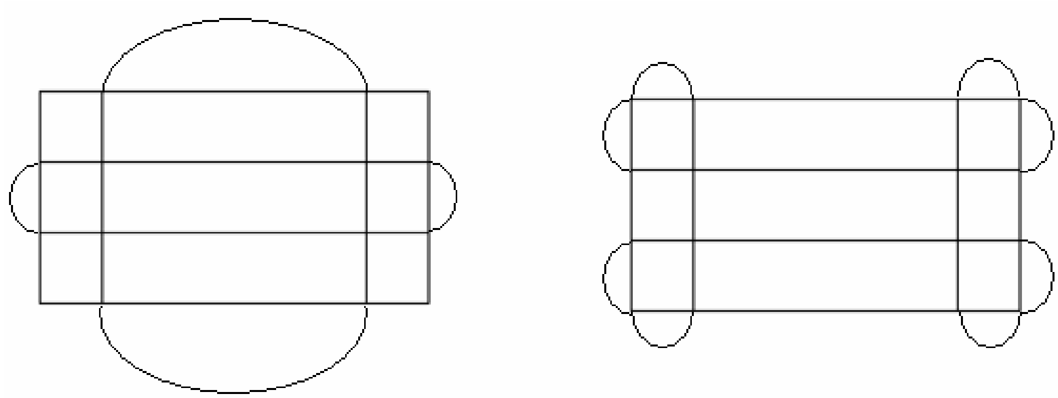
1) In the figure below, all blocks are 1000 by 1000 feet, except for the middle column of blocks, which are 1000 by 4000 feet.

a) Draw an Euler circuit on this graph, or Eulerize if needed.
b) Determine the minimum distance that needs to be added to the graph so that it has an Euler circuit.
c) What is the minimum total distance travelled if someone were to travel an Euler circuit on this graph?

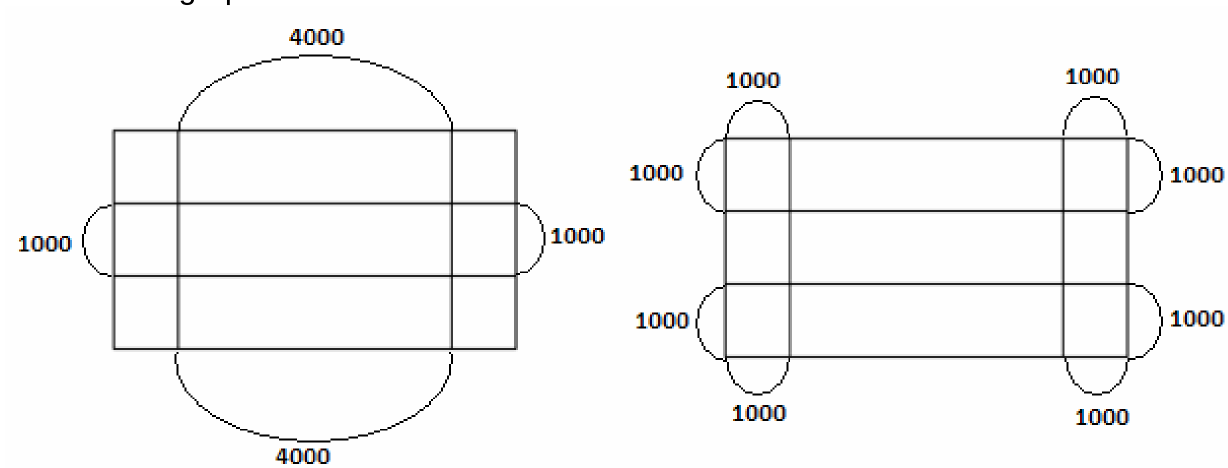
2) The text mentions practical uses for developing Euler circuits of checking parking meters, snow plowing, collecting garbage, and delivering mail. What other real world applications of Euler circuits come to your mind?

Solutions for Extemporaneous Questions
Division AA Oral - ICTM Regional 2009

1) a) There are two basic ways to Eulerize this circuit.



b) T he graph on the left needs to have 10,000 ft added to it in order to Eulerize the graph; whereas, the graph on the right needs to have 8,000 ft added it to it in order to Eulerize the graph..



d) If one were to travel the circuit, the least total distance is 44,000 ft.. The distance of the original graph is 36,000 ft.

Total distance = original distance + distances for added edges
For the circuit on the left, the total distance is 36,000 + 10,000 = 46,000 ft.
For the circuit on the right, the total distance is 36,000 + 8,000 = 44,000 ft.

2) There will be a variety of responses. Consider those that are non-traditional to be worth more than the obvious ones.