1. Set $A = \{1, 2, 3, 4\}$. If one member of Set A is selected at random and substituted for x in the expression $\frac{2x+1}{3}$, find the probability that this expression will then have a value that is an integer. Express your answer as a common fraction reduced to lowest terms.

2. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

Assuming non-zero denominators: $\frac{ax-cx}{4a-4c} =$:

A) $\frac{x}{2}$ B) $\frac{x}{4}$ C) 0 D) $-\frac{x}{2}$ E) 1 F) $-\frac{x}{4}$

Note: Be sure to write the correct capital letter as your answer.

3. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

If the largest of four different positive integers is less than 8, then the average of the four positive integers is more than 6.

4. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

If x, y, z are real numbers and $x^2y^3z < 0$, then which of the following **must** be true?

A) $x^2 < 0$ B) $y^2 < 0$ C) $y^3 < 0$ D) $y < 0$ E) $z < 0$ F) $yz < 0$

Note: Be certain to write the correct capital letter as your answer.

5. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

Let $A = \{1, 4, 9, \cdots, n^2, \cdots, 14161\}$. The sum of all distinct members of A that are exactly three more than an integral multiple of four is k. Then:

A) $k = 0$ B) $0 < k < 1000$ C) $1000 \le k < 10,000$
D) $10,000 \le k < 100,000$ E) $k \ge 100,000$

Note: Be sure to write the correct capital letter as your answer.

6. Find the value of k such that the two roots for x of the quadratic equation $x^2 + 5x - (2k + 4) = 0$ are equal. Express your answer as a **decimal**.

7. John has 4 pennies and 8 nickels. If one of these coins is selected at random, find the probability that the coin is a nickel. Express your answer as a common fraction reduced to lowest terms.

8. Find the maximum possible value of $4y$ if x and y satisfy the system:

$$\begin{cases} x \geq 2 \\ y \geq 1 \\ x + y \leq 7 \end{cases}$$

9. Today, there are k dollars in a bowl. Every 5 days from today, \$6 will be added to the bowl. Every 8 days from today, \$9 will be removed from the bowl. After the addition and/or subtraction is made from the bowl on the day which is 365 days from today, there will be \$74 in the bowl. Find the value of k.

10. It is rumored that Illinois is considering a law that would require all students in Grade 9 to pay three-fourths as much income tax (percentage-wise) as they make dollars per month. [For example, if a Grade 9 student earned \$12 per month, (s)he would pay three-fourths of 12% (that is, 9%) of \$12 or \$1.08 in income tax and leave \$10.92 in after-tax pay.] If such a law passes, how many dollars per month should a Grade 9 student earn in order to have maximum after-tax pay? If necessary, round your answer to the nearest dollar. Assume that after-tax pay is **not** rounded to the nearest cent but is left as an exact fractional portion of a cent.

11. In the following number base problem, k represents a number base that is a positive integer. Find the value of k.

$$31004_{five} = 2666_k$$

12. Let $A = \left\{ p, 2p, 3p, 4p, p+2, p+3, p+4, p+24, \frac{p}{2}, \frac{p}{3} \right\}$. If p represents some positive integer that is divisible by 8 without remainder, how many of the distinct ten members of A must be **integers** that are divisible by 8 without remainder?

13. Let $f(x)$ be defined for all integral $x \geq 0$ such that $f(a+b) = f(a) + 2f(b) - 2f(ab)$ where *a* and *b* are positive integers. If $f(10) = 1000$, find $f(2)$.

14. At Fairfield High School 5% of the female students and 4% of the male students are on the Math Team. If $\frac{25}{41}$ of the students at Fairfield High School are male and there are 27 students on the Math Team at Fairfield High School, find the total number of students at Fairfield High School.

15. Alice brought one more can of soda than Bob. Bob brought one more can of soda than Carol. Dick brought nothing, but all 4 drank the same number of cans, and no soda was left. A fair distribution would be for Dick to give Alice $3k$ cents, to pay Bob w cents, and to pay Carol k cents. If each can of soda was identical and cost 36 cents, find the number of cans that Carol brought.

16. Mowing at constant rates, Cindy can mow the whole lawn in 6 hours: Lee, in 4 hours: and Jeffrey, in 3 hours. Assuming no loss of efficiency, find the number of **minutes** it will take before the lawn is mowed if all three start mowing the lawn at the same time.

17. Find the smallest 3-digit positive integer that meets all 3 of the following conditions:
   a) If two is added to the number formed by any two of its digits (in either order), the result is a prime.
   b) If two is added to the number formed by its three digits (in any order), the result is a prime.
   c) Not all 3 of the digits can be alike.

18. Penny has three times as many quarters as she has dimes. If the total value of her quarters and dimes is $4.25, find the number of quarters that Penny has.

19. How many of the following five statements are true for all real numbers x and y?

   A) $|x| = -|x|$
   B) $|3-y| = |y-3|$
   C) $|y-5x| = |5y-x|$
   D) $|x+y| \le |x| + |y|$
   E) $|x-y| \le |x| + |y|$

20. The original price of an item was $100. The original price was increased by x% to a new price of p. This new price of p was reduced by y% to a second new price of q. This second new price of q was increased by 50% to a final selling price of $135. If x and y are both positive integers such that $1 < x < 103$ and $1 < y < 103$, find the sum of ***<u>all</u>*** distinct possibilities for x.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)

2. B (Must be this capital letter.)

3. Never (Must be the whole word.)

4. F (Must be this capital letter.)

5. A (Must be this capital letter.)

6. -5.125 (Must be this decimal.)

7. $\frac{2}{3}$ (Must be this reduced common fraction.)

8. 20

9. 41 ($ optional.)

10. 67 ($ optional.)

11. 9 or nine (Base optional.)

12. 5

13. 1000

14. 615 (Students optional.)

15. 7 (Cans optional.)

16. 80 (Minutes optional.)

17. 177

18. 15 (Quarters optional.)

19. 3

20. 275

1. Two of the three angles of a triangle have measures of $55°$ and $60°$ respectively. One of the three angles of this triangle is selected at random. Find the probability that the angle selected has a measure greater than $58°$. Express your answer as a common fraction reduced to lowest terms.

2. A line segment has endpoints at $(12,5)$ and $(4,27)$. Find the **ordered pair** that represents the midpoint of that line segment.

3. **(Yes or No)** Does $(3,7)$ lie on the circle whose equation is $x^2+(y-1)^2=39$? Be sure to write the whole word **YES** or **NO** as your answer.

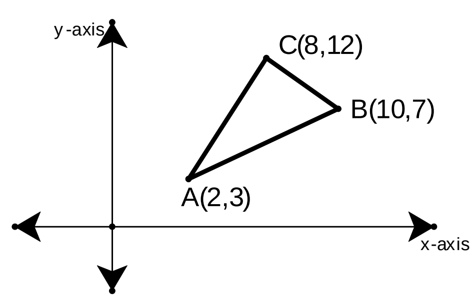
4. A rectangle has an area of 42 square units, and the length of the base of the rectangle is 6. Find the exact length of a diagonal of the rectangle.

5. A trapezoid has sides with respective lengths: 2, 41, 20, 41. Find the length of an altitude of this trapezoid.

6. An original rectangular solid with dimensions of lengths 3, 36, and k has the same volume as a cube whose edge has a length of 12. Find the total surface area of the original rectangular solid.

7. The lengths of the legs of right triangle ABC are 3 and 4. The lengths of the legs of right triangle DEF are 5 and 12. The lengths of the legs of right triangle GHJ are 16 and 63. One of these triangles is selected at random. Find the probability that the length of the hypotenuse of the triangle selected is an integral multiple of 5. Express your answer as a common fraction reduced to lowest terms.

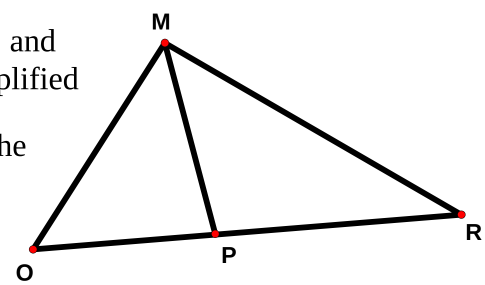
8. Using the diagram with coordinates as shown, find the exact length of the line segment that joins the midpoints of $\overline{AC}$ and $\overline{BC}$.



9. Rounded to the nearest centimeter, find the circumference of a circle in which a chord whose length is 80 centimeters is 9 centimeters from the center of the circle.

10. The sum of the degree measure of the complement of an angle and the degree measure of the supplement of the angle is more than 200, but five times the degree measure of that same angle exceeds the degree measure of the supplement of the angle by at least 6. The set of all possible degree measures y of the angle is $\{y : k \le y < w\}$. Find the value of $(k+w)$.

11. In the diagram, $\angle OMP \cong \angle RMP$. $MO = 12$, $RP = 15$, and $MR = 18$. The area of ΔMOR can be expressed in simplified form as $\frac{w\sqrt{k}}{4}$. If w and k are positive integers, find the smallest possible value of $(k+w)$. Note: O, P, and R are collinear.



12. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

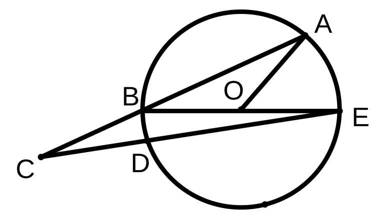
The midpoint of the hypotenuse of a right triangle is equidistant from the 3 vertices of the right triangle.

13. In rhombus $ABCD$, $\angle DAB = 60°$. A circle passes through vertices A, B, and D, and intersects diagonal $\overline{AC}$ at E. If $AE = 9$, find the exact perimeter of the rhombus.

14. The length of the hypotenuse of a right triangle is 73, and the difference of the lengths of the two legs of this right triangle is 7. Find the length of the smaller leg.

15. The center of the circle that passes through the points $(5,3)$, $(7,1)$, and $(8,2)$ can be expressed as an ordered pair of the form (x,y). Find the value of $(x+y)$.

16. Points A, B, D, and E lie on the circle O. Center O lies on diameter $\overline{BE}$. Point B lies on $\overline{AC}$, and point D lies on $\overline{CE}$. If $\angle AOB = 114°$ and the degree measure of $\angle BEC$ is 2 times the degree measure of $\angle ACE$, find the degree measure of minor arc $\overset{\frown}{DE}$.



17. The points $(7,k)$ and $(11,w)$ lie on a circle whose equation is $x^2 + y^2 = p$. If k and w are both positive integers, and if $k + w < 132$, find the sum of all possible distinct values of p.

18. A classroom in the shape of a rectangular solid has a volume of 8400 cubic feet. If the width and length of the classroom are respectively 20 feet and 30 feet, find the number of feet in the height of the classroom.

19. Find the exact area of an equilateral triangle if the perimeter of the triangle is 6.

20. A scalene triangle has sides of lengths 16, 20, and 24. A point H is located in the interior of the triangle such that the distance of H from one side of the scalene triangle is 4, such that the distance of H from a second side of the scalene triangle is 8, and such that the distance of H from the third side of the scalene triangle is x. Find the largest possible value of x. Express your answer as a **decimal** rounded to the nearest thousandth.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. $(8,16)$ (Must be this ordered pair.)

3. No (Must be the whole word.)

4. $\sqrt{85}$ (Must be this exact answer, units optional.)

5. 40

6. 1464 (Square units optional.)

7. $\frac{2}{3}$ (Must be this reduced common fraction.)

8. $2\sqrt{5}$ (Must be this exact answer.)

9. 258 (Centimeters optional.)

10. 66

11. 6484

12. Always (Must be the whole word.)

13. $18\sqrt{3}$ (Must be this exact answer.)

14. 48

15. 9

16. 136 (Degrees optional.)

17. 710

18. 14 (Feet optional.)

19. $\sqrt{3}$ (Must be this exact answer, square units optional.)

20. 4.675 (Must be this decimal.)

1. In a drawer are 8 white socks, 6 black socks, and 6 blue socks. Stacy reaches into this drawer and picks one sock at random. Find the probability that the sock selected was white. Express your answer as a common fraction reduced to lowest terms.

2. Two boys work at the same constant rate when mowing a certain lawn. If it takes the two boys a total of 3 hours to mow the lawn when they work together, how many hours would it take one of these boys to mow the lawn by himself?

3. Let $i = \sqrt{-1}$. Then $i^7 - i^{13} = ki$ where k is a real number. Find the value of k.

4. If $(2x+3y)^8$ is expanded and completely simplified, then kx^3y^5 would be one of the terms. Find the value of k.

5. $\begin{bmatrix} 7 & 3 & -2 \\ 8 & 6 & 12 \end{bmatrix} + \begin{bmatrix} 11 & -3 & 3 \\ 1 & 7 & -5 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. Find the value of $acd - 2cf$.

6. Find the sum of the first thirty terms of the arithmetic sequence: $1.23, 2.37, 3.51, \cdots$. Express your answer as a decimal.

7. A triangle has vertices at $(0,0)$, $(5,6)$, and $(5,-2)$. A point is selected at random in the interior of the triangle. Find the probability that the point selected lies in Quadrant I. Express your answer as a common fraction reduced to lowest terms.

8. Solve the determinant equation for k: $\begin{vmatrix} 1 & -2 & -6 \\ 4 & 1 & 0 \\ 5 & -3 & k \end{vmatrix} = 165$.

9. The first three terms of an arithmetic progression are respectively: $3x-7$, $8x-58$, and $2x+78$. Find the value of x.

10. Let the equation of a parabola be $y = x^2 - kx + w$. The sum of the squares of the x-intercepts of the parabola is 4210. If 18 times the sum of the x-intercepts of the parabola is subtracted from the product of the x-intercepts of the parabola, the result is 183. Find the least possible value of y.

11. Find the vertex of the parabola whose equation is $y = x^2 + 4x + 11$. Express your answer as an **ordered pair** of the form (x, y).

12. When $2.2\overline{3}$ (where only the 3 repeats) is written as an improper fraction reduced to lowest terms, what is the value?

13. Let $i = \sqrt{-1}$ and let x represent an integer. Find the sum of the smallest possible value of x and the largest possible value of x such that $\left((x+i)^3 + (x+i)^4 + x^4 i - 11x^3 i - 22x^2 i + 167xi - 209i\right)$ represents an integer.

14. If p, r, and s are the three distinct zeroes of $f(x) = x^3 + 174x^2 - 696x + w$, find the value of $(p + r + s)$.

15. Let $f(x) = 2x^2 + 3$ and let $g(x) = kx + w$ where k and w are positive integers. If $f(g(3)) = 5411$, find the largest possible value of $(k + w)$.

16. If there are 31 members in the Math Club at University High School, in how many different ways can the offices of President, Secretary, and Treasurer of this club be filled if a different member of the club must occupy each office?

17. Let $A = \{1, 2, 3, 7, 8\}$. Let B be the set of all four digit integers that can be formed using four different elements of A. How many of the elements of B are integral multiples of 11?

18. If $k(\log(9)) = \log(27)$, find the value of k. Express your answer as a **decimal**.

19. If $x^4 - 5x^2 - 36$ is factored over the complex numbers into four linear factors, one of those linear factors is $x - ki$ where $i = \sqrt{-1}$ and k is a positive integer. Find the value of k.

20. Let $P(x) = 0$ be a third degree polynomial equation with **integer** coefficients. Let the polynomial equation have at least one root that is an **integer**. If $P(5) = 6$, $P(4) = -2$, and $P(-2) = -92$, find $P(13)$.

1. $\frac{2}{5}$ (Must be this reduced common fraction.)

2. 6 (Hours optional.)

3. -2

4. 108,864

5. 148

6. 532.8 (Must be this decimal.)

7. $\frac{3}{4}$ (Must be this reduced common fraction.)

8. 7

9. 17

10. -1429

11. $(-2, 7)$ (Must be this ordered pair.)

12. $\frac{67}{30}$ (Must be this reduced improper fraction.)

13. 2

14. -174

15. 50

16. 26970

17. 24

18. 1.5 (Must be this decimal.)

19. 2

20. 1078

ANSWERS **ANSWERS** **ANSWERS** **ANSWERS**

1. In a sack are 4 slips of paper. Each slip of paper has one number written on the slip of paper. The first slip has a 3; the second slip has a 7, the third slip has a 13, and the fourth slip has a 48. If one of the slips is selected at random, find the probability that the slip selected has a prime number written on it. Express your answer as a common fraction reduced to lowest terms.

2. ΔBOA has its vertices at $B(0,22)$, $O(0,0)$, and $A(16,0)$. The median from point O intersects $\overline{AB}$ at D. Give the **ordered pair** representing the coordinates of D.

3. Does the circle $(x-3)^2+(y+2)^2=25$ contain the point with coordinates of $(6,6)$ in its interior? For your answer, write the *whole word* **Yes** or **No**, whichever is correct.

4. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   Which of the following is (are) true?

   I. $\sin 60° = \cos 30°$
   II. $\cos 87° = -\cos 267°$
   III. $\tan\left(\frac{\pi}{5}\right) = \tan\left(\frac{6\pi}{5}\right)$

   A) I only B) II only C) III only D) I and II only E) I and III only
   F) II and III only G) I, II, and III

   Note: Be certain to write the correct capital letter as your answer.

5. Find the sum of a geometric progression of 9 terms if the first term is 2, and the common ratio is 3.

6. Let x be an integer such that $0 < x < 150$. Find the sum of all possible distinct values of x such that $\cos(2x+8)° > 0$ and $\sin(5x-12)° < 0$.

7. Find the amplitude of the graph of the equation: $16y = 80\sin(64(4x° - 32°))$.

8. A sine function has amplitude of 5, a radian period of $\frac{\pi}{2}$, and a radian phase shift of $\frac{\pi}{6}$. The point $(\frac{x\pi}{12},5)$ is on the graph of this sine function. If $2.9 < x < 4.4$, find the value of x. Express your answer as a **decimal**.

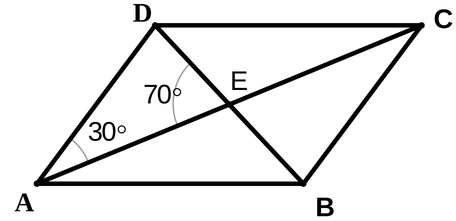
9. In a right isosceles triangle, find the value of the **sum** of the tangent of one **acute** angle and the cotangent of the other **acute** angle.

10. Let $f(x) = x^3 - 21x^2 + 72x + 2327$. The set of values for the real number k such that $f(x) - k$ will have three distinct real zeroes is $\{k : w < k < p\}$. Find the value of $(w + p)$.

11. If one of the six points $(-2,5)$, $(1,4)$, $(3,-5)$, $(3,4)$, $(-7,1)$, $(5,5)$ is selected at random, find the probability that the point selected is in the interior of the circle whose equation is $x^2 + y^2 = 36$. Express your answer as a common fraction reduced to lowest terms.

12. An apartment rental company has a total of 400 apartments available of which 250 are presently rented at $550 per month. The company has decided it will only raise or lower the rent per month, for all rented apartments, by integral increments of $10. A survey has shown that, so long as there are apartments that have not been rented, for each $10 drop in rent per month, there will be 8 new tenants. Find the number of dollars in the monthly rent that will maximize total income.

13. In the diagram, $ABCD$ is a parallelogram in which the diagonals intersect at E. If $\angle DAC = 30°$ and $\angle DEA = 70°$, find the degree measure of $\angle CAB$. Express your answer as a **decimal** rounded to the nearest hundredth of a degree.



14. A light metal rod of uniform density is 1 meter and 4 centimeters long. It is suspended in the middle and balances at the 52 cm mark. There are one gram weights hung on the rod at the following distances from the left end: 4 cm, 19 cm, 36 cm, 47 cm, 51 cm, 74 cm, 77 cm, and 87 cm. There is a three gram weight hung on the rod at k cm from the left end. Find the value of k.

15. A right circular cylinder has a ***total*** surface area of 132π. Find the maximum volume of such a right circular cylinder. Express your answer as a **decimal** rounded to the nearest hundredth.

16. When $(2x-y)^{10}$ is expanded and completely simplified, find the numerical coefficient of the term containing x^4y^6.

17. For what value of k will the graph of $f(x)=17\cos\left(\frac{1}{k}x\right)^\circ$ have a **degree** period of 3? Express your answer as a common fraction reduced to lowest terms.

18. If the magnitude of the three dimensional vector $(2,3,p)$ is $\sqrt{38}$, find the smallest possible value of p.

19. In convex pentagon $ABCDE$, the degree measure of $\angle BAE$ is twice the degree measure of $\angle ABC$. If $\angle AED=176^\circ$, $\angle EDC=141^\circ$, and $\angle DCB=40^\circ$, find the degree measure of $\angle BAE$.

20. In Triangle ABD, $\angle BAD=90^\circ$. Point C lies on $\overline{BD}$ such that $\angle BAC\cong\angle DAC$, $BC=8$, and $CD=9$. A second triangle, Triangle EFG, exists such that $\overline{AB}\cong\overline{EF}$, $\overline{FG}\cong\overline{BC}$, and $\angle BAC\cong\angle FEG$. However, the area of Triangle BAC is **not** the same as the area of Triangle FEG. Find the area of Triangle FEG. Express your answer as a **decimal** rounded to the nearest hundredth.

1. $\frac{3}{4}$ (Must be this reduced common fraction.)

2. $(8,11)$ (Must be this ordered pair.)

3. No (Must be the whole word.)

4. G (Must be this capital letter.)

5. 19682

6. 2167

7. 5

8. 3.5 (Must be this decimal.)

9. 2

10. 4290

11. $\frac{2}{3}$ (Must be this reduced common fraction.)

12. 430 ($ optional)

13. 22.12 (Must be this decimal, degrees optional.)

14. 59 (Cm. optional.)

15. 648.36 (Must be this decimal, cubic units optional.)

16. 3360

17. $\frac{1}{120}$ (Must be this reduced common fraction.)

18. -5

19. 122 (Degrees optional.)

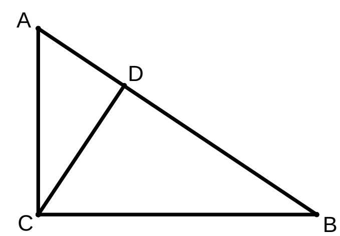
20. 30.01 (Must be this decimal, sq. units optional.)

NO CALCULATORS

1. From the first ten letters in the alphabet, one letter is selected at random. Find the probability that the letter selected was a vowel. Express your answer as a common fraction reduced to lowest terms.

2. A straight line contains the points $(1,4)$ and $(-5,0)$. Through which quadrant does the line **not** pass? Express your answer as a **Roman numeral**.

3. The diagram shows a right triangle with $\overline{CD}$ as the altitude to the hypotenuse. If $AD=16$ and $BD=25$, find CD.



4. From the set $\{44,46,48,49,50,51,52,53,54\}$ one member is selected at random. Find the probability that the number selected could not possibly be the perimeter of a square if it is required that this square must have sides whose lengths are integers. Express your answer as a common fraction reduced to lowest terms.

5. If x is an **integer**, find the sum of all distinct values of x such that $|x-1| \le 7$.

6. If x is an **integer**, find the sum of all distinct values of x such that $1<|6-x|<11$.

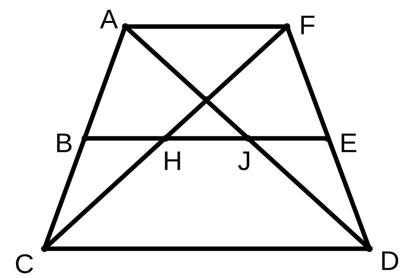
7. $\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{6}}=\dfrac{a+b\sqrt{2}+c\sqrt{3}+d\sqrt{6}}{e}$. If a, b, c, d, and e are positive integers, find the minimum possible value of $(a+b+c+d+e)$.

8. One side of a triangle has a length of 100. A line segment of length k is parallel to that side of the triangle and has its endpoints on the other two sides of the triangle. This line segment divides the original triangular region into a smaller triangular region and a quadrilateral region. The ratio of the area of the smaller triangular region to the area of the quadrilateral region is 4:21. Find the value of k.

NO CALCULATORS

9. Let y vary inversely as the square root of x. If $y = 18$ when $x = 4$, find the value of y when $x = 9$.

10. In the diagram, $ACDF$ is a trapezoid with $\overline{AF} \parallel \overline{CD}$. B is the midpoint of $\overline{AC}$. H is the midpoint of $\overline{CF}$. E lies on $\overline{FD}$, points B, H, J, and E are collinear. J lies on $\overline{AD}$. If $AF = 16$ and $CD = 48$, find EH.



11. In a math contest, Team Orange received 5 points for each question answered correctly during the first minute, 4 points for each question answered correctly during the second minute, and 3 points for each question answered correctly during the third minute. Team Orange had a point total of 11 points, but did not answer any question correctly during the first minute. How many questions did Team Orange answer correctly during the third minute?

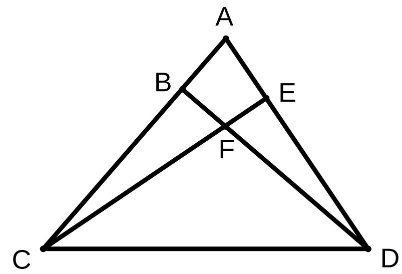
12. The altitudes of a triangle have respective lengths of 5, 6, and 10. Find the length of a radius of the circle that is inscribed in the triangle. Express your answer as an improper fraction reduced to lowest terms.

13. Find the sum of all distinct real values of x for which the fractions $\frac{x^3 - x}{x^3 + 5x^2 - 6x}$ and $\frac{x+1}{x+6}$ are **not** equivalent real numbers.

14. From an equiangular pentagon, an equiangular hexagon, and an equiangular heptagon, one polygon is selected at random. Find the probability that an interior angle of the polygon selected has a degree measure that is an integer. Express your answer as a common fraction reduced to lowest terms.

NO CALCULATORS

15. In the diagram, A, B, and C are collinear, A, E, and D are collinear, and altitudes $\overline{BD}$ and $\overline{CE}$ of ΔACD intersect at F. If $\angle CAD = 76°$, and $\angle ACD = 62°$, find the degree measure of $\angle CFD$.



16. If x is an integer, find the sum of all distinct values of x such that $x^2 + 5x < 24$.

17. $\frac{8}{11} = \frac{1}{k} + \frac{1}{w} + \frac{1}{f} + \frac{1}{4070}$ where k, w, and f are positive integers. Find the smallest possible value of $(k + w + f)$.

18. Each of the four digits 3, 7, 8, 9 is used exactly once as digits to form two numbers for the variables in the equation $\sqrt{k - w} = 27$. Find the value of the product (kw).

19. In ΔJCM, H lies on $\overline{JC}$, A lies on $\overline{JM}$, and T lies on $\overline{CM}$. $\overline{JT}$, $\overline{HM}$, and $\overline{CA}$ are concurrent at P. $AP : PC = 1 : 7$, and $HP : PM = 3 : 8$. Then $JP : PT = k : w$ where k and w are positive integers. Find the smallest possible value of $(2k + 3w)$.

20. Let k and w be two positive integers with $k < w$, $k < 4$, and $w < 6$. If $k = 2$, then $w \neq 4$. Let x and y be positive integers such that $\frac{2x + 3y}{7x - 2y} = \frac{k}{w}$. If $x < 34$, find the sum of all distinct possible values of x.

NO CALCULATORS

ANSWERS:

1. $\frac{3}{10}$ (Must be this reduced common fraction.)

2. IV (Must be this Roman numeral.)

3. 20

4. $\frac{2}{3}$ (Must be this reduced common fraction.)

5. 15

6. 108

7. 48

8. 40

9. 12

10. 24

11. 1 (Question optional.)

12. $\frac{15}{7}$ (Must be this reduced improper fraction.)

13. 1

14. $\frac{2}{3}$ (Must be this reduced common fraction.)

15. 104 (Degrees optional.)

16. -25

17. 44

18. 6642

19. 229

20. 243

NO CALCULATORS

1. Let $x = -3$. From the four expressions $-2x^2$, $(-2x)^2$, $-2(x^2)$, $(-2)(x)^2$, one expression is selected at random. Find the probability that the value of the expression selected is -18. Express your answer as a common fraction reduced to lowest terms.

2. Find the value of $\log_{27}\left(\frac{1}{9}\right)$. Express your answer as a common fraction reduced to lowest terms.

3. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

Let a and b be real numbers, let $i = \sqrt{-1}$, and let $\overline{a+bi}$ be the complex conjugate of $a+bi$. If P is a polynomial with real coefficients such that $P(a+bi) = 17-23i$, then $P(\overline{a+bi}) =$

A) 17 B) -23 C) 23 D) -17 E) $17+23i$ F) $-17-23i$

Note: Be certain to write the correct capital letter as your answer.

4. $\begin{bmatrix} k & 3 & -2 \\ 4 & 6 & 10 \end{bmatrix} - \begin{bmatrix} 9 & -3 & 5 \\ w & 7 & -2 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. If $ae - 2df = 382$ and if $d - 3a = -22$, find the value of $(k+w)$.

5. On a math exam there are 3 easy questions and 1 difficult question. If a student works 2 of these questions at random, find the probability that both questions are easy. Express your answer as a common fraction reduced to lowest terms.

6. As $x \to 2$, $\dfrac{x^3 + 11x^2 + 10x - 72}{x^2 - x - 2} \to k$. Find the value of k.

NO CALCULATORS

7. In the *x*-*y* plane, the three points represented by $(-2,20)$, $(10,-7)$, and $(18,5)$ lie on a parabola whose axis of symmetry is parallel to the *y*-axis. Find the **ordered pair** (x, y) that represents a point on that parabola such that $(x+y)$ is a minimum under the condition that both x and y are positive integers.

8. If the sum of the infinite geometric series whose first three terms are respectively 3, $5x$, and $\frac{25x^2}{3}$ is 15, find the value of x. Express your answer as a common fraction reduced to lowest terms.

9. If n represents an odd positive integer, find the value of $\sum_{k=1}^{n} \cos(k\pi)$.

10. $ABCD$ is a convex quadrilateral with $AB = 52$, $BC = 50$, and $CD = 86$. $\angle ABC$ is obtuse, $\sin(\angle ABC) = \frac{7}{25}$, and $\cos(\angle BCD) = -\frac{7}{25}$. Expressed in simplest radical form, $AD = k\sqrt{w}$ where k and w are positive integers. Find the value of $(k+w)$.

11. Is the infinite geometric series $6 - 3 + 1\frac{1}{2} - \frac{3}{4} + \frac{3}{8} + \cdots$ a convergent series or a divergent series? For your answer, write the *whole word* **convergent** or **divergent**, whichever is correct.

12. Let a, b, and c each represent a single non-zero digit. Find the number of distinct **ordered triples** of the form (a,b,c) for which $\frac{abc}{a+b+c} < 1$.

13. If $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$, find the value of $(a+2b+3c+d)$.

NO CALCULATORS

14. If one of the first one hundred positive integers is selected at random and substituted for m, find the probability that the expression: $(4m+1)^2$ has a units digit of 1. Express your answer as a common fraction reduced to lowest terms.

15. The graph of $y=\frac{3x^2+5x-2}{x-5}$ has an oblique or slant asymptote of $y=mx+b$. Find the value of $(m+b)$.

16. How many ounces of pure silver must be added to 180 ounces that is 40% pure in order to obtain a mixture that is 50% pure?

17. Assume that whenever Arturo runs in a marathon race, his probability of finishing first (fastest time) is $\frac{1}{4}$, and his probability of finishing second (second fastest time) is $\frac{1}{3}$. Arturo plans to run in 15 marathon races next year. Find the probability that his third finish of second place will come in the eighth of those 15 marathon races. Express your answer as a common fraction reduced to lowest terms.

18. Let $x^3+28x^2-185x-4524=(x-13)(x+k)(x+w)$ where k and w are integers. Find the value of $|k-w|$.

19. Let the parametric equations $x=3t$ and $y=t^2+5$ represent a parabola. If the parametric equations are converted to a Cartesian equation, find the ***y*-coordinate only** of the focus of the parabola. Express your answer as an improper fraction reduced to lowest terms.

20. $\sin(10°)+\cos(24°)=k=\sin(10x°)+\cos(24x°)$. If x is a positive integer greater than 1, find the smallest possible value of x.

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

NO CALCULATORS

ANSWERS:

1. $\frac{3}{4}$ (Must be this reduced common fraction.)

2. $-\frac{2}{3}$ OR $\frac{-2}{3}$ OR $\frac{2}{-3}$ (Must be this reduced common fraction.)

3. E (Must be this capital letter.)

4. 31

5. $\frac{1}{2}$ (Must be this reduced common fraction.)

6. 22

7. $(2,5)$ (Must be this ordered pair.)

8. $\frac{12}{25}$ (Must be this reduced common fraction.)

9. -1

10. 102

11. Convergent (Must be the whole word.)

12. 28 ("Ordered triples" optional.)

13. 3

14. $\frac{2}{5}$ (Must be this reduced common fraction.)

15. 23

16. 36 (Ounces optional)

17. $\frac{224}{2187}$ (Must be this reduced common fraction.)

18. 17

19. $\frac{29}{4}$ (Must be this reduced improper fraction.)

20. 89

Answers should be expressed either in **scientific notation OR** as a **decimal,** and answers should be rounded to four **significant** digits. **However**, specific instructions in a given problem take precedence. For example, if instructions ask for the answer to be expressed as a **decimal** or as an **integer**, you may NOT use scientific notation for that answer.

1. Ace scored a total of 680 points in 32 games. Find his average number of points scored per game.

2. The lengths of the two legs of four right triangles are respectively 6 and 8, 12 and 35, 28 and 45, and 20 and 48. If one of these right triangles is selected at random, find the probability that the length of the hypotenuse is an odd integer. Express your answer as a common fraction reduced to lowest terms.

3. Find the value of $\left((2.023^5)(2.023^{1.671})^3\right)^{1.116}$.

4. For the following system, find the value of $(x+y)$: $\begin{cases} 2.4x+3.5y=1063.81 \\ 7.12x-2.15y=450.023 \end{cases}$

5. Find the volume of a right circular cylinder whose base has a radius of length 9.685 and whose height has a length of 16.26.

6. A square is inscribed in a circle. The length of a side of the square is 4.321. Find the area of the region that is in the interior of the circle but is in the exterior of the square.

7. In ΔABC, $AB=18.00$, $BC=10.00$, and $\angle ABC=96°42'$. Find the area of ΔABC.

8. Find the **exact value** of the 43rd term of the Fibonacci sequence whose first 7 terms are: $1, 1, 2, 3, 5, 8, 13$. Express your answer as an **integer**.

9. Find the largest root of $x^3 - 3.538x^2 - 41.398949x + 107.346376 = 0$.

10. An airplane is flying at a height of 37,000 feet above level ground. The angle of depression to the airport is $4°32'$. If the plane descends directly along the line of sight to the airport, how far will the airplane travel before touch down? Round your answer to the nearest mile and express your answer as that **integer**.

11. The area of a circle and the area of a square are equal. The length of a side of the square is k times the length of a radius of the circle. Find the value of k.

12. From a standard deck of 52 cards, four cards are drawn at random without replacement. Find the probability that none of the four cards drawn is a spade. Express your answer as a common fraction reduced to lowest terms.

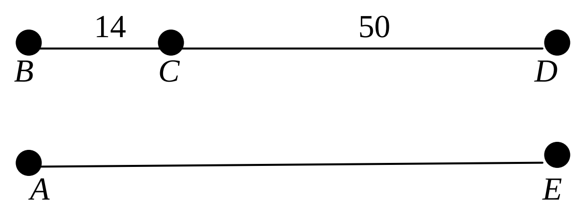
13. From the sample set $\{25, 33, 34, 39, 46, 48, 51, 65, 69, 81\}$, one number is selected at random. Find the probability that the number selected is within one standard deviation from the arithmetic mean of the set. Express your answer as a common fraction reduced to lowest terms.

14. Find $\sec(Arc\tan(2.678))$ where $Arc\tan$ means principal value.

15. The wheels of a car have a 14–inch radius. When the car is being driven so that the wheels make 11 revolutions per second, find the number of **feet** the car will travel in 1 minute.

16. A regular polygon with 20 sides is inscribed in a circle. If the area of the regular polygon is 8764, find the length of a radius of the circle.

17. In the diagram, Bob is at point *A* on the straight center line $\overline{AE}$ of a straight road and wishes to reach point *D* on the curb $\overline{BD}$. The road is ***entirely*** in sun as is the portion of the curb from *B* to *C*(14 units), while the portion from *C* to *D*(50 units) is in the shade. Bob uses 18% less energy walking in the shade than when walking in the sun. Assuming $\overline{BD} \parallel \overline{AE}$ and that the perpendicular distance from *A* to *B* is 12 units, Bob should aim at a point on $\overline{BD}$ that is x units from *D* in order to minimize the energy used for his walk. Find the value of x.



18. The lengths of two of the sides of a right triangle are 77 and 85. The length of the third side is also an integer. Find that integer. Express your answer as an **exact integer**.

19. $t_n = (\frac{t_{(n-1)}}{4})^2 + 1.1$ and $t_1 = 1.3$. Find t_{14}.

20. Find the length of the radius of the inscribed circle of a right triangle with hypotenuse of length 16 and acute angles whose measures are 20° and 70°.

1. 21.25 or 2.125×10 or 2.125×10^1 (Points optional.)

2. $\frac{1}{2}$ (must be this reduced common fraction)

3. 2626 or 2.626×10^3

4. 344.3 or 3.443×10^2

5. 4791 or 4.791×10^3

6. 10.66 or 1.066×10 or 1.066×10^1

7. 89.39 or 8.939×10 or 8.939×10^1

8. 433494437 (Must be this integer.)

9. 7.214 or 7.214×10^0

10. 89 (Must be this integer.)

11. 1.772 or 1.772×10^0

12. $\frac{6327}{20825}$ (Must be this reduced common fraction.)

13. $\frac{7}{10}$ (Must be this reduced common fraction.)

14. 2.859 or 2.859×10^0

15. 4838 or 4.838×10^3 (Feet optional.)

16. 53.25 or 5.325×10 or 5.325×10^1

17. 46.81 or 4.681×10 or 4.681×10^1

18. 36 (Must be this integer.)

19. 1.188 or 1.188×10^0

20. 2.254 or 2.254×10^0

1. From the four numbers 6, 15, 42, 51, one number is selected at random. Find the probability that the number selected has exactly two prime positive factors. Express your answer as a common fraction reduced to lowest terms.

2. An odd integer is represented by x, and the next larger odd integer is represented by $8x-33$. The perimeter of a rectangle is 64, and one of the sides of the rectangle has a length that is 4 more than the length of a second side. Find the sum of x and the length of one of the shorter sides of the rectangle.

3. A clerk gives you 35 cents in American coins, and you discover that you were given at least one nickel. Find the absolute value of the difference between the maximum number of coins you could have been given and the minimum number of coins you could have been given.

4. A straight line passes through the point $(2,7)$ and is perpendicular to the line whose equation is $x-5y-2=0$. The equation of that straight line can be expressed as $y=mx+b$. Find the value of $(m+b)$.

5. For all real numbers a and b, $a \oplus b=(a-5)(b+3)$. Let k be the number of sides of a convex polygon whose sum of the degree measures of its interior angles is 2160. Find the value of $7 \oplus 20 + k$.

6. Let k and w be positive integers such that $k \neq 1$, $w \neq 1$ and $k > w$. If $k\sqrt{w}=432$, and the sum of k and w is an odd integer, find the largest possible value of $(k+w)$.

7. The lengths of all sides of a **scalene** triangle are integers. Two of the sides have lengths of 8 and 11. Let k be the possible number of non-congruent **scalene** triangles. Let $A=\{25,68,91,73,62,w\}$, and let the arithmetic mean (average) of A be 79. Find the value of $(k+w)$.

8. One of the interior angles of a regular polygon has the same degree measure as the obtuse angle formed by the minute hand and the hour hand of a clock at 11:20. Find the number of sides of the regular polygon.

9. If the same value for x satisfies both systems, find the value of k.

$$\begin{cases} 7x+4y=79 \\ x-2y=-17 \end{cases} \qquad \begin{cases} 3x+y=8 \\ kx-5y=95 \end{cases}$$

10. Parallelogram $ABCD$ is inscribed in a circle. $AB=8$, $BD=16$, and $AD=k$. A multiple dwelling has 46 letter boxes; 115 pieces of mail were correctly delivered to those boxes. No box received more than 3 pieces of mail. Let w be the least number of boxes that could have received 3 pieces of mail. Find the value of $(k+w)$.

ANSWERS

1. $\frac{3}{4}$ (Must be this reduced common fraction.)

2. 19

3. 28

4. 12

5. 60

6. 153

7. 168

8. 9 (Sides optional.)

9. 12

10. $23+8\sqrt{3}$ or $8\sqrt{3}+23$

11. If $\frac{3}{5}$ of a number is 6 more than $\frac{1}{2}$ of the number, find the number.

12. The lengths of all sides of a **scalene** triangle are whole numbers. If the largest side has a length of 17, find the smallest possible perimeter of the triangle.

13. The sum of three numbers is $4x^2$, and the sum of four other numbers is $3x^3$. If $x = 14$, find the average (arithmetic mean) of all seven numbers.

14. The coordinates of the vertices of a triangle are $(2,3)$, $(4,7)$, and $(5,6)$. Find the area of the triangle.

15. A woman was looking at a painting and stated: "That person's mother was my mother's daughter. I have no sisters, and I have no daughters." What was the relationship of the person in the painting to the woman? Express your answer in a ***whole word or words*** such as: great-great-grandfather or niece.

ANSWERS

11. 60

12. 35

13. 1288

14. 3

15. Son (Must be the whole word.)

1. From the three lines $y = 5x + 8$, $3y + 5x = 300$, $2y = 7 - 8x$, one line is selected at random. Find the probability that the line selected has a slope that is negative. Express your answer as a common fraction reduced to lowest terms.

2. If 5 "*finals*" $= 2\pi$ *radians*, then $120° = k$ "*finals*". Find the value of k. Express your answer as an improper fraction reduced to lowest terms.

3. Find the value of $\sin(22°) + \cos(68°) + \sin(158°) + \cos(248°) + \sin(202°) + \sin(338°)$

4. Find the value of: $\log_3(99) - \log_3(11)$.

5. Let k be the common ratio of an infinite geometric progression with 1st term of 15 and sum of 25. Let w be the common ratio of an infinite geometric progression with 1st term of 63 and sum of 81. Find the value of $(k + w)$. Express your answer as a common fraction reduced to lowest terms.

6. Let k be the probability of drawing 5 hearts when 5 cards are selected at random (without replacement) from 13 hearts and 3 spades. Let S be the sum of all distinct positive integral multiples of 3 that are less than 56. Find the smallest integer that is greater than (kS).

7. Let a, b, and c represent integers. The solution set of $(x-a)(x-2b)(x-3c) = 0$ is $\{18, 30, 36\}$. Let k be the largest possible value of $(a+b+c)$. The cubic equation $y^3 - 14y^2 + wy - 90 = 0$ has 3 distinct positive integral solutions for y. Find the value of $(k + w)$.

8. The equation of a line whose *x*-intercept is 4 and which passes through $(10,12)$ can be expressed in the form $y = mx + b$. The inverse of $y = 2x + 3$ can be expressed in the form $y = kx + w$. Find the value of $(m + b + k + w)$.

9. Let $f(x) = \frac{5x+2}{x+6}$ and $g(x) = \frac{3x+8}{4x+7}$. Find $f(g(1)) + f(g(-2)) + g(f(1)) + g(f(-2))$.

10. Let S be the sum of all distinct integral values of x such that $\left|\sqrt{x} - \sqrt{3}\right| < 2$. The sum of two numbers is 13, and the cube of their sum exceeds the sum of their cubes by 858. Let p be the product of those two numbers. Find the value of $(S + p)$.

ANSWERS

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. $\frac{5}{3}$ (Must be this reduced improper fraction, finals optional.)

3. 0 or zero

4. 2

5. $\frac{28}{45}$ (Must be this reduced common fraction.)

6. 152

7. 120

8. -7

9. -2

10. 113

11. If x is a positive integer, find the smallest possible value of x such that $\cot(x°) < 0$ and such that $\cot(x°) = -\cot(-x°)$.

12. If $y = -2x^2 + 44x - 11$ where x and y are real, find the largest possible value for y.

13. Solve the determinant equation for x: $\begin{vmatrix} 7 & -2 & -5 \\ -4 & 8 & x \\ 1 & 0 & 2 \end{vmatrix} = 106$.

14. The second term of an arithmetic sequence is l3 and the eighteenth term of this arithmetic sequence is 41. Find the twenty-ninth term. Express your answer as a **decimal**.

15. The area of a rectangle is 168 square units, and a diagonal of the rectangle has a length of 25 units. Find the number of units in the perimeter of this rectangle.

ANSWERS

11. 91

12. 231

13. 15

14. 60**.**25 (Must be this decimal.)

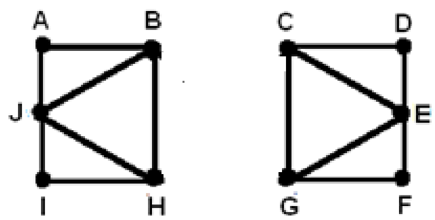
15. 62 (Units optional.)

Questions for the Oral Competition – Regional Level A, 2010

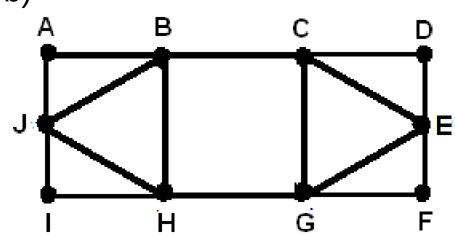
1) a) Use a diagram of your own to show and define a circuit.
   b) Use a diagram of your own to show and define an Euler circuit.
   c) Explain the difference between a circuit and an Euler circuit.
   d) For the diagram in part (b), give the valence of each vertex and explain how the valence numbers can help determine if a graph does or doesn't have an Euler circuit.

2) Which graphs have Euler circuits? For the one(s) that do, find the Euler circuits by numbering the edges in the order the Euler circuit uses them starting at point A. For the one(s) that doesn't (don't) explain why no Euler circuit is possible.

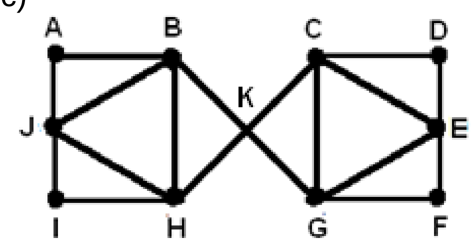
a)



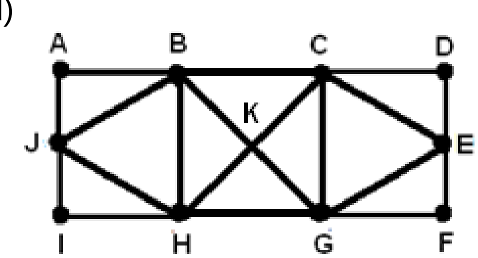
b)



c)

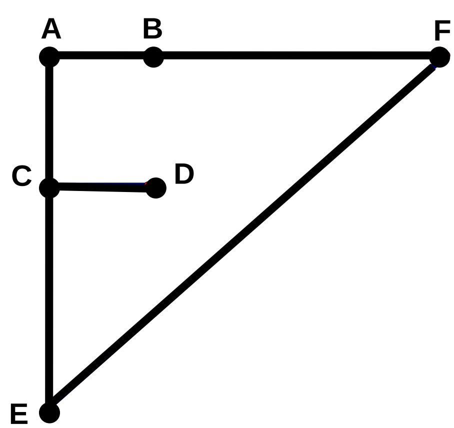


d)

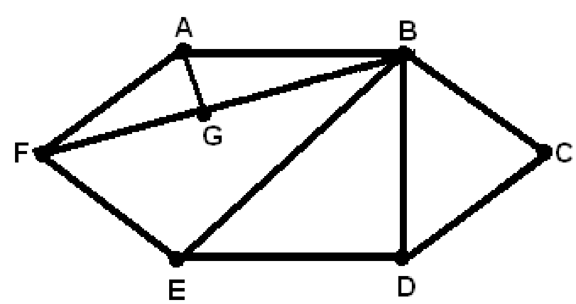


3) Eulerize these graphs using the minimum number of new segments.

a)



b)



Solutions for the Questions for the Oral Competition
Regional Level A, 2010

1.

a) Any diagram that shows a circuit as a path that starts and ends at the same vertex.

b) Any diagram that shows an Euler circuit as a circuit that traverses each edge of a graph exactly once.

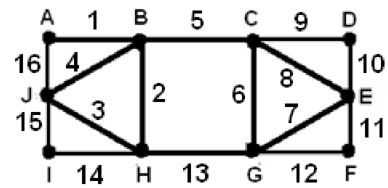
c) In a circuit, an edge may be traversed more than once or not all edges may be traversed; in an Euler circuit every edge is traversed exactly once.

d) The valence of a vertex in a graph is the number of edges meeting at the vertex. In an Euler circuit, all of the valences must be even numbers greater than 1.

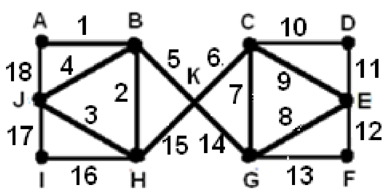
2. Which graphs have Euler circuits? For the one(s) that do, find the Euler circuits by numbering the edges in the order the Euler circuit uses them starting at point A. For the one(s) that doesn't (don't) explain why no Euler circuit is possible.

a) It is not an Euler graph because it is not connected.

b) Answers may vary. Below is one possible solution.



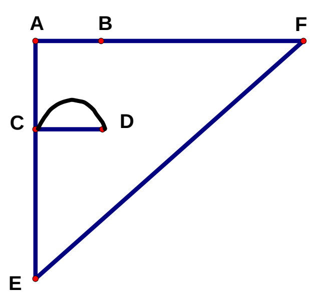
c) Answers may vary. Below is one possible solution.



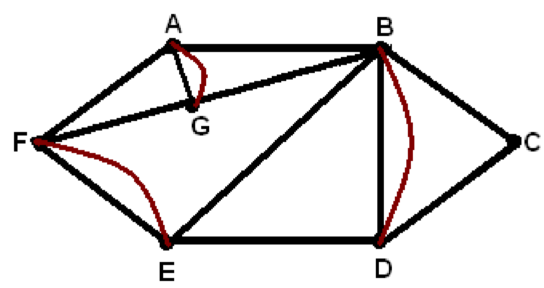
d) It is not an Euler circuit because the valences of vertices B, C, G, and H are odd.

3.

a) One segment is needed. Add segment CD



b) Three segments are needed. One solution is to add segments AG, BD and FE.

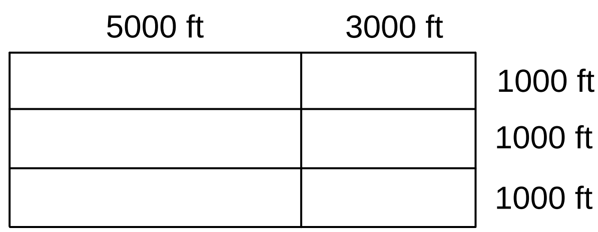


Extra Questions for Judges

4) a) Draw a graph with more than one vertex and one vertex with a valence of zero. Explain what a valence of zero means in terms of a graph.

b) Can this graph contain an Euler circuit? Why or why not?

5) The figure below contains rectangular blocks with distances as marked.



a) Determine the minimum distance that needs to be added to Eulerize the graph. Show the edges added.

b) What is the minimum total length of the Euler circuit?

Solutions for Extra Questions

4) a) Draw a graph with more than one vertex and one vertex with a valence of zero. Explain what a valence of zero means in terms of a graph

Answer:

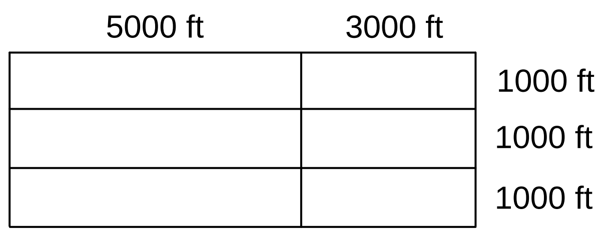
Any graph with at least one non-connected vertex should be accepted. A valence of zero means that there is no path connecting the vertex to any other part of the graph.

b) Can this graph contain an Euler circuit? Why or why not?

Answer:

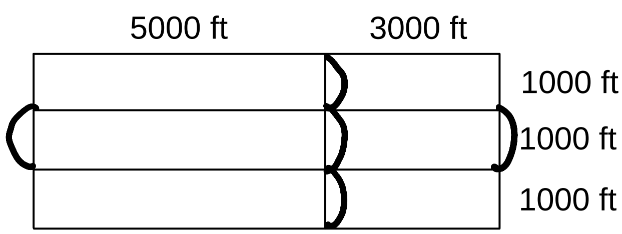
No, because it is not a connected graph.

5) The figure below contains rectangular blocks with distances as marked.



a) Determine the minimum distance that needs to be added to Eulerize the graph. Show the edges added.

Answer:



Adding the paths shown, for a total length of 5(1000) = 5000 is the minimum added distance.

b) What is the minimum total length of the Euler circuit?

Answer:

The total length is 4(5000) + 4(3000) + 14(1000) = 46000.

**Note: Some credit should be given for students who correctly Eulerize the graph, even if they do not find an Eulerization that minimizes the distance added.