1. If an element of the set $\{2,4,6,8,10,12,14,16\}$ is selected at random and substituted for x, find the probability that the value of $(x+3)$ represents a prime whole number. Express your answer as a common fraction reduced to lowest terms.

2. How many of the following three statements are true?
a) The reciprocal of 0.6 can be expressed as an improper fraction.
b) $\sqrt{11\frac{1}{9}}$ is a rational number.
c) No **real** number for x exists such that $x^2+x+1=0$.

3. **(Always, Sometimes, or Never)** For your answer, write the ***whole word*** **Always, Sometimes,** or **Never—**whichever is correct.

If the largest of four different positive integers is less than 19, then the sum of the four positive integers is more than 68.

4. If $(x+y)(x-y)=4.86$, find the exact value of $2.3x^2-2.3y^2$. Express your answer as a **decimal**.

5. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

An album contains x black-and-white photographs and y color photographs. If the album contains a total of 60 photographs, then all but one of the following can be true. Which one can **not** be true?

A) $x=y$ B) $x=2y$ C) $x=3y$ D) $x=4y$ E) $x=5y$ F) $x=6y$

Note: Be certain to write the correct capital letter as your answer.

6. Judy took a math test for which there were 25 problems. The test was scored as follows: 5 points for each problem answered correctly, -2 points for each problem answered incorrectly, and 0 points for a problem left blank. Judy's score was 53. Find the **maximum** number of problems that Judy could have had correct.

7. Let $B=\{3,6,9,12,14,15,16,18\}$. If one of the members of B is selected at random, find the probability that the member selected is **not** an integral multiple of 3. Express your answer as a common fraction reduced to lowest terms.

8. Judy is thinking of a whole number that is greater than 1 and less than 100. Bob is to make a "guess" and Judy will then truthfully tell Bob whether or not his "guess" is too large, too small, or is Judy's number. If Bob uses the best strategy, find the smallest number of "guesses" Bob will need to make to be **certain** that he has named Judy's number.

9. If the temperature is more than 1° Celsius but less than 74° Celsius, find the sum of all possible distinct **Fahrenheit** temperatures such that the Fahrenheit temperature is a **prime** integer and that the equivalent Celsius temperature is any integer.

10. How many distinct integers between 1000 and 2010, inclusive, are divisible by 3, 5, or 7?

11. A book store sells 3 types of books: math books for $5 each, physics books for $1.50 each and comic books for $.25 each. Derby buys 100 books and pays exactly $50.00. If Derby buys at least 1 book of each type, find the **ordered triple** of the form (# of math books, # of physics books, # of comic books) that Derby buys.

12. Find the product of all distinct values of x such that $2^{(x^2+2x)}=4^4$.

13. An original number is increased by $k\%$, and then the result is decreased by $w\%$. The final result is 85% of the original number. If k is a positive integer less than 173 and if w is a positive integer, find the sum of all possible distinct values of k.

14. Judy said to Bob: "I will give you 11 cents for each word you spell correctly if you will give me 6 cents for each word you spell incorrectly." Bob agreed. After 51 words, Bob and Judy were even—that is, neither owed the other anything. How many of the 51 words did Bob spell incorrectly?

15. How many distinct positive integers less than 1000 have prime factorizations of the form $p^a q^b r^c$ with p, q, and r distinct positive primes and with a, b, and c positive integers such that $p+q+r<13$?

16. If n is an integer such that $101<n<118$, for how many distinct values of n is the expression $11\left(16^n\right)-1$ a prime?

17. Let N be a three-digit number (with a non-zero hundreds digit) such that the sum of the digits of N is eight. If N^2 is divided by 9, the remainder is 1. How many distinct possibilities exist for the value of N?

18. The line whose equation is $ky+x=17$ contains the point $(17,289)$. Find the value of k.

19. Carol conducted a survey of the 2000 students that attended the 2008 ICTM State Math Contest Finals. It was discovered that 85% of the students responded to the survey and that 60% of those responding answered “no” to a certain yes-no question. If 63 students left that question blank, how many students answered “yes” to that question?

20. An ordinary clock with a minute hand and hour hand now indicates it is 12:00 P. M. Noon. The minute and hour hands are together at the usual dial positions, but the clock is actually running slow. When the minute and hour hands are first together after 3:00 pm, the clock has actually been running for 210 minutes. Find the number of true minutes lost by the clock during the time from 12:00 P. M. Noon on the clock to the first time after 3:00 P. M. that the minute and hour hands came together. Express your answer as an improper fraction reduced to lowest terms.

1. $\frac{3}{4}$ (Must be this reduced common fraction.)

2. 3 or "all are true"

3. Never (Must be the whole word.)

4. 11.178 (Must be this decimal.)

5. F (Must be this capital letter.)

6. 13 (Problems optional.)

7. $\frac{1}{4}$ (Must be this reduced common fraction.)

8. 7 (Guesses optional.)

9. 493

10. 550 ("Integers" optional.)

11. $(5,1,94)$ (Must be this ordered triple.)

12. -8

13. 245

14. 33 (Words optional.)

15. 32 ("Integers" optional.)

16. 0 or none

17. 36

18. 0 or zero

19. 617 (Students optional.)

20. $\frac{150}{11}$ (Must be this reduced improper fraction, minutes optional.)

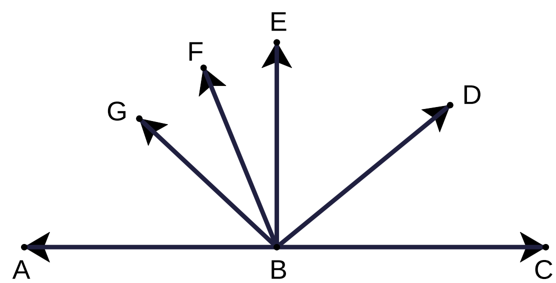
1. Given the following 3 statements:

   A. Every isosceles triangle has at least two angles that are congruent.
   B. Every right triangle has at least two angles that are congruent.
   C. Some scalene triangles have angles such that the sum of the degree measures of two of the three angles is more than the degree measure of the third angle.

   If one of these statements is selected at random, find the probability that the statement selected is a true statement. Express your answer as a common fraction reduced to lowest terms.

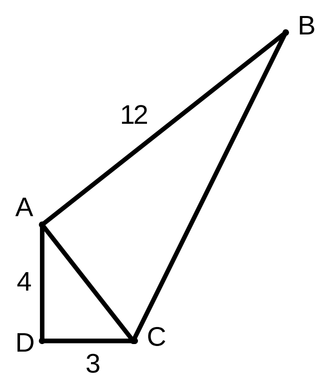
2. Rosa sliced a pizza into equal sixths. She then sliced each of these equal sixths into two equal tiny pieces. How many of these tiny pieces would Placido need to eat so that he had eaten half of the original pizza?

3. In the coplanar figure shown, A, B, and C are collinear, $\overline{EB} \perp \overline{AC}$, $\overrightarrow{BD}$ bisects $\angle EBC$, $\overrightarrow{BF}$ bisects $\angle GBE$, and $\angle FBD = 62°$. Find the degree measure of $\angle ABG$.

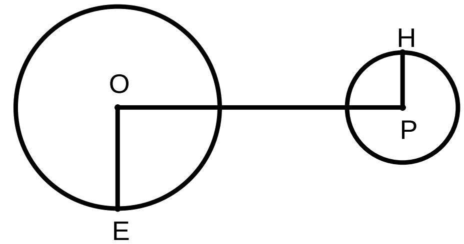


4. Two angles, $\angle A$ and $\angle B$, are congruent and complementary. Two angles, $\angle C$ and $\angle D$, are congruent and supplementary. By how many degrees does the measure of $\angle C$ exceed that of the measure of $\angle A$?

5. In the diagram, $\overline{AD} \perp \overline{CD}$ and $\overline{AB} \perp \overline{AC}$. $AB = 12$, $AD = 4$, and $CD = 3$. Find the area of quadrilateral $ABCD$.



6. In the diagram, E lies on the circle with center at O, and H lies on the circle with center at P. $EO = 8$, $HP = 3$, and $OP = 13$. Find the exact length of a **common internal** tangent segment of the two circles.

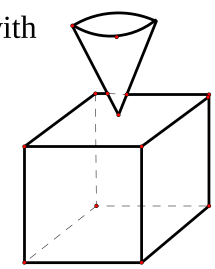


7. The degree measure of the supplement of an angle is k times the degree measure of the complement of the angle. If k is selected at random from the set $\{3,4,5,6\}$, find the probability that the supplement of the angle has a degree measure less than 118. Express your answer as a common fraction reduced to lowest terms.

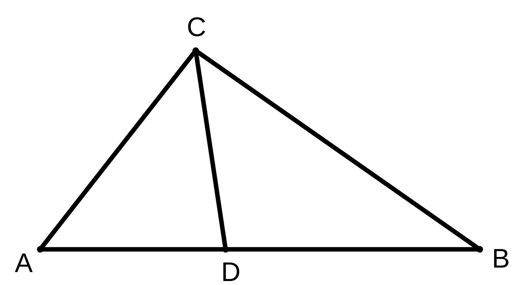
8. The lengths of the three sides of a triangle are 24, 35, and 53. Find the length of the altitude drawn to the shortest side.

9. A circular sheet of paper has a circumference of 72π. A sector whose central angle is $60°$ is removed from this circular sheet of paper, and the remaining portion of the paper is then folded to form a right circular cone. The volume of this right circular cone can be expressed in simplest radical form as $k\pi\sqrt{w}$ where k and w are positive integers. Find the value of $(k+w)$.

10. A right circular cone, height 8 and radius of its circular base 2, is resting with its apex (point) on the upper face of an empty cube with edge of length 4.396. A line from the center of the circular base of the cone through the apex of the cone is perpendicular to the upper face of the cube. The cone is filled with water and a small hole through the apex of the cone is opened to allow water to drip into the cube. (Assume the hole is small enough so that its radius can be disregarded for calculation purposes.) Let x represent the radius of the new circular base of water left in the cone. Find the value of x for the radius when x is also equal to the height of the water in the cube (height is measured from the lower face to the square base of the upper level of the water in the cube). Express your answer as a **decimal** rounded to the nearest hundredth.



11. In the diagram D lies on $\overline{AB}$. $\angle ACD \cong \angle BCD$. $AC=35$, $BC=42$, and $AB=49$. Find the **product** of AD and BD. Express your answer as an improper fraction reduced to lowest terms.



12. The equation of the line that is tangent to the circle whose equation is $x^2+y^2=53$ at the point $(7,2)$ can be written in the form $y=mx+b$. Find the value of $(m+b)$.

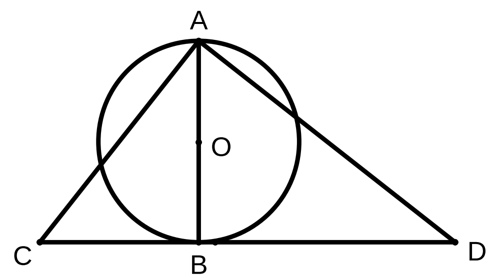
13. Let $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ be three parallel chords that are non-diameters of a circle on the same side of the center. The distance between $\overline{AB}$ and $\overline{CD}$ is equal to the distance between $\overline{CD}$ and $\overline{EF}$. If $AB = 24$, $CD = 20$, and $EF = 8\sqrt{3}$, find the distance from $\overline{AB}$ to the center of the circle.

14. **(Always, Sometimes, or Never)** For your answer, write the _whole word_ **Always, Sometimes,** or **Never**—whichever is correct.

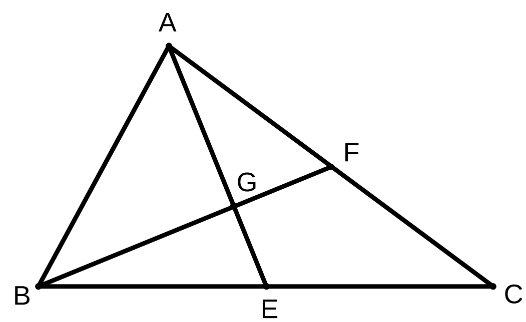
On the *x*-*y* plane, if a, b, and c represent real numbers with $c \neq 0$, then the graph of $(x-a)^2+(y-b)^2=c^2$ is a square.

15. A line segment has one end on the positive *x*-axis and the other end on the positive *y*-axis. The triangle formed by the segment and the axes has an area of 30. If the segment passes through the point $(3,2)$, then the **smallest** possible value of the slope of the line segment can be expressed in simplest radical form as $\dfrac{k+w\sqrt{p}}{-3}$ where k, w, and p are integers with $p>0$. Find the value of $(k+w+p)$.

16. In the diagram, $\overline{CD}$ is tangent to the circle at B, points A, O, and B are collinear, O is the center of the circle, and $\angle CAD = 90°$. If $AD = 13$ and $AB = 12$, find CD. Express your answer as a **decimal**.



17. In the diagram, E is the midpoint of $\overline{BC}$, F is the midpoint of $\overline{AC}$, $\overline{AE}$ and $\overline{BF}$ intersect at G, and $\angle BGE = 90°$. If $AC = 36$, and $BC = 42$, find AB.



18. A kite has diagonals whose lengths are 10.2 and 8.4. Find the area of the kite. Express your answer as a **decimal**.

19. A sphere is inscribed in a cube whose volume is 74088. The volume of this sphere is $k\pi$. Find the value of k.

20. Located inside equilateral triangle GHJ is a point M such that $GM = 13$, $HM = 18$, and $JM = 20$. Rounded to the nearest degree, find the degree measure of $\angle GMJ$.

2010 RAA
Geometry

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 6 (Pieces optional.)

3. 56 (Degrees optional.)

4. 45 (Degrees optional.)

5. 36

6. $4\sqrt{3}$ (Must be this exact answer.)

7. $\frac{1}{2}$ (Must be this reduced common fraction.)

8. 28

9. 1811

10. 1.28 (Must be this decimal.)

11. $\frac{72030}{121}$ (Must be this reduced improper fraction.)

12. 23

13. 10

14. NEVER (Must be the whole word.)

15. 25

16. 33.8 (Must be this decimal.)

17. $6\sqrt{17}$ (Must be this exact answer.)

18. 42.84 (Must be this decimal.)

19. 12348

20. 122 (Degrees optional.)

1. Set $A = \{1, 2, 3, 4\}$. If one of the members of Set A is selected at random and substituted for x, find the probability that $5^x > 27$. Express your answer as a common fraction reduced to lowest terms.

2. Solve for x in the determinant equation: $\begin{vmatrix} x & -2 \\ 4 & 3 \end{vmatrix} = 20$

3. Find the value of $\sum_{x=1}^{4} (\frac{1}{2x+1})$. Express your answer as a common fraction reduced to lowest terms.

4. If f is a real-valued function, find the sum of all distinct **positive integers** in the domain of f if $f(x) = \sqrt{6-x}$.

5. Let n be a positive integer. If $(a-b)^n$ is expanded and completely simplified, the numerical coefficients of the sixth and sixteenth terms are equal. If the expansion is written in terms of descending powers of a, then the third term of this expansion is $ka^w b^p$ where k, w, and p are positive integers. Find the value of $(k+w+p)$.

6. If $x^5 + 6x^4 + 2x^3 - 17x^2 - 9x + 5$ is factored with respect to the integers, then the third degree factor is $kx^3 + wx^2 + px + c$. Find the value of $(k+w+p+c)$.

7. From a standard deck of 52 cards, two cards are drawn, one at a time, without replacement. Find the probability that the first card drawn is a spade and that the second card drawn is a heart. Express your answer as a common fraction reduced to lowest terms.

8. An ellipse has foci at $(3,0)$ and $(-3,0)$ and has a major axis whose length is 12. Find the length of a latus rectum of this ellipse.

9. The sum of the squares of the roots for x of $7x^4 - kx^3 + 21x^2 - 986 = 0$ is 30. If $k > 0$, find the value of k.

10. Let a bag contain b blue marbles, g green marbles, and w white marbles such that $b:g:w = 5:4:3$. Let the bag contain no other marbles. If a marble is drawn at random from this bag, then the respective probabilities that a blue, a green, or a white marbles is drawn are the solutions for x of $ax^3 + cx^2 + dx + e = 0$. If a is a positive integer and c, d, and e are integers, find the smallest possible value of $(a + c + d + e)$.

11. Let $y = (2x - 3a)^2 + (x - 4b)^2$ where a and b are real numbered constants. The value of x that yields a minimum value for y can be expressed as $\dfrac{ka + wb}{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k + w + p)$.

12. $\begin{vmatrix} 1 & 3 & 1 \\ -1 & 2 & x \\ 0 & -1 & 5 \end{vmatrix} = 64$. Find the value of x.

13. For each real number x let $m(x)$ be the maximum of the numbers $\frac{1}{6}x + 5\frac{2}{3}$, $-\frac{2}{3}x + 7\frac{1}{3}$, and $x - 6$. Find the minimum value of $m(x)$.

14. Let $i=\sqrt{-1}$, and let x and y be real numbers. The reciprocal of $3+i$ can be expressed in the form $k+wi$. Find the value of $(60k+40w)$.

15. Find the sum of all distinct positive integral factors of 360.

16. Let $f(x)=3x-2$ and let $g(x)=\sqrt{x-3}+5$. Find the smallest number that is in the range of the composite function $f\circ g$.

17. Find the sum of all possible **distinct** integers for k such that the cubic equation $x^3-8x^2+kx-10=0$ will have at least one rational root for x.

18. Find the 50th term of the arithmetic sequence whose first four terms are respectively 7, 12, 17, and 22.

19. Find the vertex of the parabola whose equation is $y=-2x^2-4x+1$. Express your answer as an **ordered pair** of the form (x,y).

20. Let $x\in\{16,17,18,19,20\}$. If k and w are positive integers, find the sum of all distinct values of x such that, for some values of k and w, the remainder when x^{19} is divided by 13 is the same as the remainder when k^3+w^4 is divided by 13.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)

2. 4

3. $\frac{248}{315}$ (Must be this reduced common fraction.)

4. 21

5. 210

6. -1

7. $\frac{13}{204}$ (Must be this reduced common fraction.)

8. 9

9. 42

10. 42

11. 15

12. 38

13. 6

14. 14

15. 1170

16. 13

17. -275

18. 252

19. $(-1, 3)$ (Must be this ordered pair.)

20. 71

1. The base of a rectangle has a length of 8. One of the whole numbers greater than 5.2 and less than 7.3 is selected at random for the height of the rectangle. Find the probability that the area of this rectangle is greater than 50. Express your answer as a common fraction reduced to lowest terms.

2. If $f(x) = |3x - 12|$, then an equation of a line of symmetry is $x = k$. Find the value of k.

3. Give the smallest possible **positive** value of x for which $\cos(2x° - 60°) = 0$.

4. The cubic equation whose solution set is $\{1, 5, 11\}$ can be written as $x^3 + kx^2 + wx + p = 0$. Find the value of $(k + w + p)$.

5. The first term of a geometric sequence of real terms is 405. The fifth term of this sequence is 20480. Find the third term of this sequence.

6. A circle with center at $(-6, 3)$ contains the point $(2, -12)$. An arc of the circle has measure of $35°15'$. Find the area of the sector of the circle that is bounded by two radii and the arc. Express your answer as a **decimal** rounded to the nearest tenth.

7. One of the transformations necessary to produce the graph of $y = 2x^2 + 24x - 1$ from the graph of $y = x^2$ is a vertical shift k units downward. Find the value of $|k|$.

8. The function g satisfies the functional equation $g(x) + g(y) = g(x + y) + 2xy - 8$ for every pair (x, y) of real numbers. If $g(1) = 22$, find $g(-11)$.

9. When $y^2 + my + 14$ is divided by $(y-3)$, the quotient is $f(y)$ and the remainder is k. When $y^2 + my + 14$ is divided by $(y-7)$, the quotient is $g(y)$ and the remainder is w. If $w = 2k+1$, find the value of m.

10. All lengths of the sides of a triangle are integers. The cosine of the smallest angle of the triangle is $\frac{19}{21}$. The length of one of the adjacent sides of the triangle for that angle is 6. Find the smallest perimeter for such a triangle.

11. From the set $\{A, B, C, D, E, F\}$, two letters are selected at random without replacement. Find the probability that A or B or both A and B were selected. Express your answer as a common fraction reduced to lowest terms.

12. If ln is the symbol for natural logarithm, find the value of y such that $\ln(5+4y) - \ln(3+y) = \ln(3)$.

13. In Triangle ABC, $AB = 64$, $BC = 64$, and $AC = 48$. A, B, and C are the centers of three mutually externally tangent circles whose points of tangency are D, E, and F. Rounded to the nearest **whole number**, find the perimeter of Triangle DEF.

14. Last year UIC traced the job placement of their high school mathematics teacher candidates and found all took jobs teaching. The arithmetic mean salary of the candidates who took jobs in Chicago was \$48,600. The arithmetic mean of the rest of the candidates, who took jobs outside Chicago, was \$43,800. The overall arithmetic mean for all these candidates was \$45,000. Find the fraction of the candidates who took jobs outside Chicago. Express your answer as a common fraction reduced to lowest terms.

15. A farmer wishes to fence in a rectangular plot of ground bounded on one side by a river. He has 2200 feet of chain-link fence available. He will **not** fence the side along the river, but each of the two widths of the field perpendicular to the river will need a double layer (or "two-ply") of fence. If the area of the rectangular plot enclosed is to be a maximum, find the number of feet in the length of the field that is parallel to the river. (Consider only the lengths of fencing used in the problem. Ignore the thickness of the double fencing in your computations.)

16. Find the absolute value in the difference of the two values of x for which the three terms $x-2$, $x+1$, and $4x-8$ (in that order) will form a geometric progression?

17. (Multiple Choice) Write A if the conditions below determine a conic. Write B if the conditions below overdetermine a conic (that is, there is no conic meeting all the given conditions). Write C if the conditions below underdetermine a conic (that is, there is more than one conic that meets all the given conditions).

Ellipse with center at (0, 0) and eccentricity of $\frac{2}{3}$

Note: Be certain to write the correct capital letter as your answer.

18. If $\sin(3x°) = \frac{1}{2}$ and $133° < x < 178°$, find the value of x.

19. The length of the latus rectum of a hyperbola is 24.12, the eccentricity of the hyperbola is 3.62, the center of the hyperbola is at $(8.16, 19.02)$, and the principal axis of the hyperbola is parallel to the *y*-axis. One of the foci of the hyperbola is located at (x, y). Find the largest possible value of y. Express your answer as a **decimal** rounded to the nearest hundredth.

20. Find the **absolute** value of the distance from the point $(1,4,9)$ to the plane determined by the 3 points $(5,0,4)$, $(1,7,3)$, and $(2,8,0)$. Express your answer as a **decimal rounded to the nearest thousandth**.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)

2. 4

3. 75 (Degrees optional.)

4. -1

5. 2880

6. 88.9 (Must be this decimal.)

7. 73

8. -470

9. -16

10. 16

11. $\frac{3}{5}$ (Must be this reduced common fraction.)

12. 4

13. 84 (Must be this whole number.)

14. $\frac{3}{4}$ (Must be this reduced common fraction.)

15. 1100 (Feet optional.)

16. 4

17. C (Must be this capital letter.)

18. 170 (Degrees optional.)

19. 22.63 (Must be this decimal.)

20. 1.028 (Must be this decimal.)

NO CALCULATORS

1. The length of one of the altitudes of an equilateral triangle is $8\sqrt{15}$. From among the sides and the altitudes of this equilateral triangle, one segment is selected at random. Find the probability that the segment selected has a length that is more than $8\sqrt{15}$. Express your answer as a common fraction reduced to lowest terms.

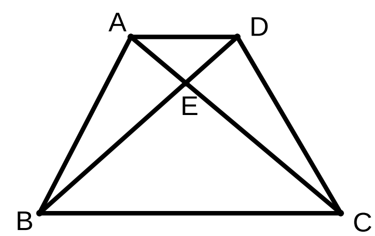
2. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   Which one of the following polygons can always be classified as a rectangle?

   A) square B) parallelogram C) non-isosceles trapezoid D) rhombus
   E) isosceles trapezoid

   Note: Be sure to write the correct capital letter as your answer.

3. In the diagram, $ABCD$ is an isosceles trapezoid with the segment from B to C as a base. The diagonals of the isosceles trapezoid intersect at E. $AB = 6x$, $EC = 2x + 18$, $DE = 4x + 4$, and $DC = 8x - 5$. Find BD.



4. One of the four conditions is selected at random. Find the probability that the figure referred to in the condition selected can be proved to be a parallelogram. Express your answer as a common fraction reduced to lowest terms.

   Condition A: Exactly one pair of sides of a given quadrilateral are congruent.
   Condition B: Both pairs of opposite sides of a given quadrilateral are congruent.
   Condition C: Both pairs of opposite angles of a given quadrilateral are congruent.
   Condition D: The diagonals of a given quadrilateral bisect each other.

5. In right triangle ABC, hypotenuse $\overline{AB}$ has a length of 40, and side $\overline{AC}$ has a length of 20. Points D and E lie on $\overline{AB}$ such that $\overrightarrow{CE}$ and $\overrightarrow{CD}$ trisect $\angle ACB$. Find the length of $\overline{DE}$.

6. A rectangular field has sides whose lengths are 9 and 12. A similar rectangular field has a diagonal whose length is 30. Find the perimeter of the second rectangle.

NO CALCULATORS

7. The lengths of the three sides of a triangle are 10, 10, and 12. Find the positive difference between the length of a radius of the triangle's circumcircle and the length of a radius of the triangle's incircle. Express your answer as an improper fraction reduced to lowest terms.

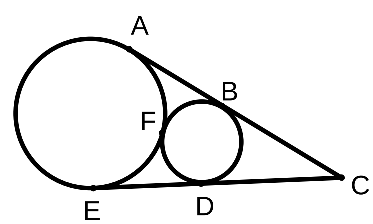
8. Cindy took six tests. Her first four scores were 78, 56, 89, and 43. Cindy's average score for the six tests was 72. Her score on the sixth test was 8 points higher than her score on the fifth test. Find Cindy's score on her sixth test.

9. The degree measures of the first two angles of a triangle are in the ratio of $2:3$. The number of degrees in the third angle of the triangle is 68 more than the number of degrees in one of the first two angles of the triangle. Find the sum of the degree measures of the two distinct possibilities for the degree measure of the third angle of the triangle.

10. The larger of two cubes has twice the volume of the smaller cube. A main diagonal of the smaller cube has a length of 1. The length of an edge of the larger cube can be expressed as $\frac{\sqrt[6]{k}}{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

11. Find the value of $\left|37-2\left|3-\left|14-15\right|\right|\right|$.

12. In the diagram, $\overleftrightarrow{AC}$ and $\overleftrightarrow{EC}$ are common external tangents of the two circles with points of tangency at A, B, D, and E. The circles are tangent at F and have radii of lengths 5 and 7. Find BC.



NO CALCULATORS

NO CALCULATORS

13. Find the perimeter of a rhombus with diagonals whose lengths are 16 and 30.

14. From the set $\{1, 2, 3\}$, two members are drawn—one at a time—with replacement. Find the probability that the two numbers drawn were the same or that the sum of the two numbers drawn was four. Express your answer as a common fraction reduced to lowest terms.

15. Let $x + y = 5$ and $x^3 + y^3 = 50$. Find the value of $\left(x^2 - xy + y^2\right)$.

16. A regular dodecagon (12-sided figure) is inscribed in a circle with a radius whose length is 16. Find the area of the regular dodecagon.

17. Let k be an integer such that $x + k$ is a factor of $x^2 + 6kx + 45$. Find the **product** of all possible distinct values of k.

18. If $1.3(c - 4) \le 2.6 + 0.7c$, then $c \le k$. Find the smallest possible value of k.

19. A line whose equation is $y = mx + 10$ intersects the line whose equation is $x = 1$ in Quadrant I and also intersects the line whose equation is $x = 16$ in Quadrant I. The area of the region bounded by $y = mx + 10$, $x = 1$, $x = 16$, and the *x*-axis is 73.5. Find the value of m. Express your answer as a **decimal**.

20. If k is the number of distinct positive integers that leave a remainder of 10 when divided into 1234, find the value of k.

NO CALCULATORS

NO CALCULATORS

ANSWERS:

1. $\frac{1}{2}$ (Must be this reduced common fraction.)

2. A (Must be this capital letter.)

3. 37

4. $\frac{3}{4}$ (Must be this reduced common fraction.)

5. 10

6. 84

7. $\frac{13}{4}$ (Must be this reduced improper fraction.)

8. 87 (Points optional.)

9. 210 (Degrees optional.)

10. 111

11. 33

12. $5\sqrt{35}$ (Must be this exact answer.)

13. 68

14. $\frac{5}{9}$ (Must be this reduced common fraction.)

15. 10

16. 768

17. -9

18. 13

19. -0.6 OR $-.6$ (Must be this decimal.)

20. 17

NO CALCULATORS

1. One of the six letters of ILLINI is selected at random. Find the probability that the letter selected is one of the letters of the state name INDIANA. Express your answer as a common fraction reduced to lowest terms.

2. It is known that x varies directly as y. If $x = 4$ when $y = 8$, find the value of x when $y = 20$.

3. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   For all values of x for which the trig functions in this problem are defined, $(1+\tan^2 x)(1+\cot^2 x) =$

   A) 1 B) $\tan^2 x$ C) $\cot^2 x$ D) $\sec^2 x\csc^2 x$ E) $\sin(2x)$

   Note: Be certain to write the correct capital letter as your answer.

4. Given the following recursive definition for a sequence: $t_1 = 29$ and for all positive integers n such that $n \ge 2$, $t_n = t_{(n-1)} - 3$. Find the sum of the first 30 terms of this sequence.

5. Let $P(A)$ represent the probability of A and $P(B)$ represent the probability of B. If $P(A) = 0.2$ and $P(B) = 0.3$, find $P(A \cup B)$ if A and B are independent events. Express your answer as a **decimal**.

6. The infinite geometric series $1 + 7x + 49x^2 + \cdots$ is a convergent geometric series for all values of x such that $k < x < w$. Find the value of $(w - k)$.

7. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   If $C \subseteq D$ and $E \subseteq D$, which one of the following statements must be true? (Note: In set notation, X' is one representation for the complement of X.)

   A) $C' \cup E' = (C \cup E)'$ B) $C' \cup E' = (C' \cap E)'$ C) $C = E$
   D) $C' \cap E' = (C \cup E)'$ E) $C' \cap E' = (C \cap E)'$

   Note: Be sure to write the correct capital letter as your answer.

NO CALCULATORS

8. The lengths of the sides of a **scalene acute** triangle are all integers. Find the smallest perimeter of any such triangle if the sine of one of the angles of the triangle is $\frac{4}{5}$.

9. Find the value of x such that $\log_6 x = 3.5$.

10. If n is a positive integer, find the value of n such that $\frac{1+2+3+\cdots+n}{1(2)+2(3)+3(4)+\cdots+n(n+1)} = \frac{1}{18}$.

11. Let vector $\vec{c} = (1,-4,k)$ and let vector $\vec{d} = (4,0,1)$. If the inner (dot) product $\vec{c} \bullet \vec{d}$ is 21, find the value of k.

12. The three real non-zero roots of $x^3 + (r-p)x^2 - qx + 3pq = 0$ are p, q, and r. Find the value of $(r-p)$.

13. The weights of fifteen students in a sixth-grade class were recorded as $\{62, 65, 69, 74, 75, 79, 80, 85, 86, 90, 92, 99, 104, 107, 113\}$. The mean weight and the median weight were calculated. Then a recording error was discovered: the 69 should have been 96. The mean and median were then recalculated. If $x = (\textit{new mean}) - (\textit{original mean})$ and $y = (\textit{new median}) - (\textit{original median})$, find $|x-y|$.

14. From the set of the 120 positive integers $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \cdots, 120\}$, an integer is selected at random. Find the probability that the left-most digit is 1. Express your answer as a common fraction reduced to lowest terms.

NO CALCULATORS

15. All ages in this problem are in whole numbers of years. A father has 3 sons whose present ages are in arithmetic progression and sum to the father's age now. Five years ago, the sum of the sons' ages then was half the father's present age, and the eldest son then was 4 times as old as the youngest son was then. Find the age of the oldest son at present.

16. Let $i = \sqrt{-1}$. If $\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$ is expressed in the form $x + yi$ where x and y are real numbers, then $(x+y)$ can be expressed in the form $\frac{\sqrt{k}+w}{p}$ where k, w, and p are integers. Find the smallest possible value of $(k+w+p)$.

17. The lengths of all sides of a triangle are integers, and two of these lengths are 7 and 10. From such possible triangles, two non-congruent triangles are selected without replacement at random. Find the probability that both triangles selected were acute triangles. Express your answer as a common fraction reduced to lowest terms.

18. Let $i = \sqrt{-1}$. Find the value of $\left|(6+8i)^3\right|$.

19. A parabola has its line of symmetry parallel to the *y*-axis and has its vertex at $(3,4)$. The point $(9,8)$ lies on the parabola. Find the ***y*-coordinate only** of the focus of this parabola. Express your answer as a **decimal**.

20. Let $S = \{a_1, a_2, a_3, \bullet\bullet\bullet, a_n\}$. In general, the harmonic mean of S is $\frac{n}{\sum_{i=1}^{n}\frac{1}{a_i}}$. Let $T = \{1, \frac{1}{3}, \frac{1}{6}, \frac{1}{10}, \bullet\bullet\bullet, \frac{1}{0.5n(n+1)}\}$. If the harmonic mean of T is $\frac{1}{100}$, find the value of n.

NO CALCULATORS

ANSWERS:

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 10

3. D (Must be this capital letter.)

4. -435

5. 0.44 OR .44 (Must be this decimal.)

6. $\frac{2}{7}$ OR $0.\overline{285714}$ OR $.\overline{285714}$ (No calculator allowed, but this is a common decimal.)

7. D (Must be this capital letter.)

8. 42

9. $216\sqrt{6}$ (Must be this exact answer.)

10. 25

11. 17

12. -7

13. $\frac{4}{5}$ OR 0.8 OR .8

14. $\frac{4}{15}$ (Must be this reduced common fraction.)

15. 13 (Years optional.)

16. 4

17. $\frac{5}{39}$ (Must be this reduced common fraction.)

18. 1000

19. 6.25 (Must be this decimal.)

20. 23

Answers should be expressed either in **scientific notation OR** as a **decimal,** and answers should be rounded to four **significant** digits. **However**, specific instructions in a given problem take precedence. For example, if instructions ask for the answer to be expressed as a **decimal** or as an **integer**, you may NOT use scientific notation for that answer.

1. Two angles are supplementary and the larger angle has a degree measure that is 2.536 times the degree measure of the smaller angle. Find the degree measure of the larger angle.

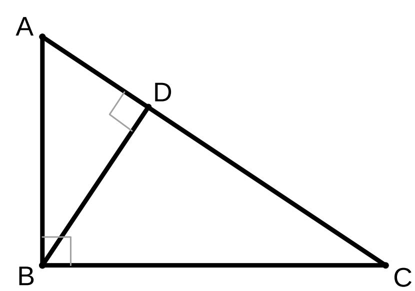
2. Two cars leave point A at the same time. One car travels due east at a constant speed of 40.00 mph. while the other car travels due north at a constant speed of 30.00 mph. How many **minutes** after they leave point A will the two cars be 5000 **feet** apart?

3. Find the value of x such that $\dfrac{x-2.003}{x-5.194} = \dfrac{x+3.112}{x+5.684}$.

4. Find the value of $\log_{8.023} 60.45$.

5. A car is being driven so that the wheels make 12.43 revolutions per second. In 2 minutes and 14 seconds, the car travels a distance of 10,513 feet. Assuming all wheels have equal radii, find the number of **inches** in a radius of one of the wheels of the car.

6. In the diagram, ΔABC is a right triangle with $\angle ABC$ being the right angle. $AB = 18.45$, and $BC = 21.04$. $\overline{BD}$ is the altitude drawn to the hypotenuse. Find the value of $(AD)^2 + (AD)(DC)$.

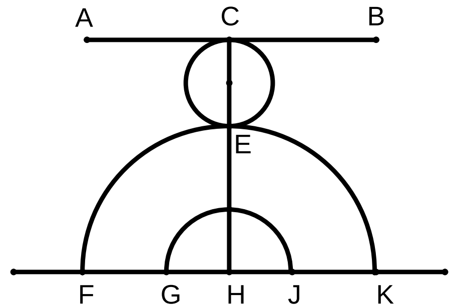


7. Assume for this problem that the ground is level and horizontal. A ladder leans against a vertical wall so that the angle of elevation from the foot of the ladder to the top of the ladder is $67.43°$. If the foot of the ladder is 12.23 feet from the base of the wall, find the number of feet in the length of the ladder.

8. Candy is in a lake and is 352.1 yards from the perfectly straight shoreline. She wishes to reach a point on the shore that is 867.3 yards down the shoreline. She can swim at the rate of 2.876 yards per second. Once she gets to the shoreline, she can walk along the shoreline at 3.524 yards per second. Find the smallest possible number of seconds for Candy to get from her present spot in the lake to the point that is 867.3 yards down the shoreline.

9. By how much does the area of a triangle with sides of lengths 89, 99, and 100 exceed the area of a triangle with sides of 48, 85, and 91?

10. In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{FK}$. Points A, C, and B are collinear, and points F, G, H, J, and K are collinear. Point H is the center of the semi-circular arc from F to K, and point H is also the center of the semi-circular arc from G to J. $\overline{CE}$ is a diameter of the circle. Point C is a point of tangency of $\overleftrightarrow{AB}$ and the circle. Point E is a point of tangency of the circle and the semi-circular arc from F to K. Points C, E, and H are collinear with $\overline{CH} \perp \overline{FK}$. If the radius of the semi-circular arc from G to J is 2, and if the distance between the parallel lines is 16, then the minimum sum of the areas of the circle and the semi-circular annulus (ring between the two semi-circular arcs) is $k\pi$. Find the value of k. Express your answer as an improper fraction reduced to lowest terms.



11. The perimeter of a rectangle is 876.8. The length of the rectangle is 1.103 times the width of the rectangle. Find the length of the rectangle.

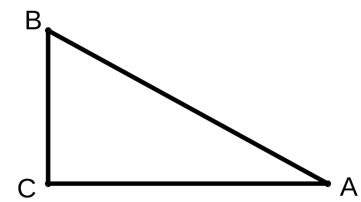
12. The nine positive integers from 1 through 9 are placed in a hat and drawn out at random one at a time without replacement. Find the probability that at least three of the odd integers were among the first 4 integers drawn. Express your answer as a common fraction reduced to lowest terms.

13. Let d be the symbol for deviation, let f be the symbol for frequency, and let s be the symbol for standard deviation for a frequency distribution that uses the "coding method" for calculating the standard deviation by the formula:

$s = \sqrt{\frac{\sum(fd^2)}{N} - \left(\frac{\sum(fd)}{N}\right)^2}$. Rounded to 5 decimal places, $s \approx 2.91295$ for the following chart. If k is a positive integer, find the value of k. Express your answer as an **integer**.

Class Mark	d	f
61	– 6	5
64	– 3	18
67	0	42
70	3	k
73	6	8

14. In the diagram, $\angle ACB = 90°$, $\angle BAC = 4°28'$, and $BC = 32,000$ **feet**. Find the number of **miles** in the length of the hypotenuse.



15. Let k be a prime integer such that $k < 40$. Find the sum of all possible distinct values of k such that k is the length of the hypotenuse of a right triangle in which all sides have lengths which are integers. Express your answer as an **integer**.

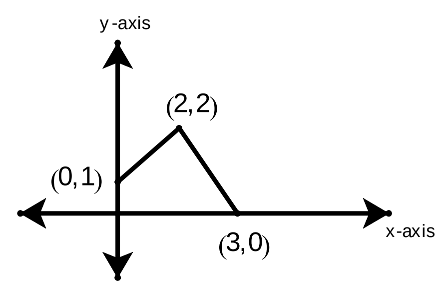
16. Find the real value of x such that $\dfrac{6^{(2x-4)}}{10^{(3x+1)}} = 1$.

17. A soda pop can in the shape of a right circular cylinder is to hold exactly 170 cubic units. Find the length of the radius that will yield a can of minimal ***total*** surface area.

18. $\dfrac{60}{3x - 18.45} = -5.431830527$. Find the value of x.

19. From a standard deck of 52 cards, three cards are drawn at random without replacement. Find the probability that two are of one denomination and the third is of a second denomination. Express your answer as a common fraction reduced to lowest terms.

20. The graph of f is shown to the right and consists of the union of a line segment from $(0,1)$ to $(2,2)$ and a line segment from $(2,2)$ to $(3,0)$. Find the absolute value of the shortest possible distance from $(5, -2)$ to any point on the graph of $y = -2f(x+1)+1$.



1. 129.1 or 1.291×10^2 (Degrees optional.)

2. 1.136 or 1.136×10^0 (Minutes optional.)

3. -0.8292 or $-.8292$ or -8.292×10^{-1}

4. 1.970 or 1.970×10^0 (trailing zero is necessary)

5. 12.05 or 1.205×10 or 1.205×10^1 (Inches optional.)

6. 340.4 or 3.404×10^2

7. 31.86 or 3.186×10 or 3.186×10^1 (Feet optional.)

8. 316.9 or 3.169×10^2 (Seconds optional.)

9. 1944 or 1.944×10^3

10. $\frac{122}{3}$ (Must be this reduced improper fraction.)

11. 229.9 or 2.299×10^2

12. $\frac{5}{14}$ (Must be this reduced common fraction.)

13. 29 (Must be this integer.)

14. 77.82 or 7.782×10 or 7.782×10^1 (Miles optional.)

15. 101 (Must be this integer.)

16. -2.849 or -2.849×10^0

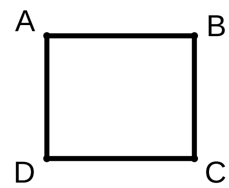
17. 3.002 or 3.002×10^0

18. 2.468 or 2.468×10^0

19. $\frac{72}{425}$ (Must be this reduced common fraction.)

20. 3.638 or 3.638×10^0

1. In the diagram, $\overline{AB} \cong \overline{DC}$ and $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$. $AD = 3x - 8$, $AB = 15$, and $BC = x + 4$. A horse that drinks 2**.**18 milliliters of water each second will drink k milliliters of water in $\frac{1}{12}$ hour. Find the value of $(x + k)$.



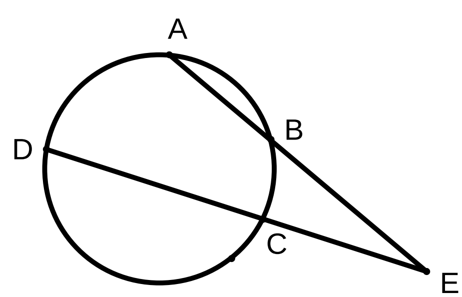
2. Find the value of $213_{eleven} + 4123_{eight}$. Express your answer in **base ten**.

3. Given the following system: $\begin{cases} 4x - 3y - 4z = 3 \\ x + y + z = 4 \\ 3x + 4y - 3z = 21 \end{cases}$ Find the value of $(2x + 3y - 4z)$.

4. If a chord whose length is 120 units is 25 units from the center of a circle, how many units from the center of this circle is a chord whose length is 112 units.

5. Rhombus $RHOM$ is inscribed in a circle whose center is point P. $MH = 80$. Find the area of ΔRPM.

6. Let k be the least possible sum of 4 consecutive positive integers if k is an integral multiple of 9. In the diagram, points A, B, C, and D lie on the circle. $AB = 7.8$, $BE = 5$, $EC = 4$, and $CD = w$. Points A, B, and E are collinear; points, and D, C, and E are collinear. Find the value of $(k + w)$.



7. The perimeter of ΔABC is 48 with $AB = 3x - 6$, $BC = 22 - x$, and $AC = 2x + 4$. Find the length of the shortest altitude of ΔABC.

8. Let k be the **perimeter** of a square whose diagonal has a length of $6\sqrt{3}$. Let w be the **perimeter** of an equilateral triangle whose altitude has a length of $\sqrt{18}$. Find the exact value of $(k+w)$.

9. Digit k is the units digit in the 8-digit number 2471465*k*, and the 8-digit number is an integral multiple of 11. Let r be the length of a radius of the circumcircle of a right triangle whose legs have lengths of 96 and 110. Find the value of $(k+r)$.

10. In ΔABC, $AB=36$, $BC=16$, and $AC=40$. $\overline{BD}$ and $\overline{BE}$ are respectively the altitude and median from B to $\overline{AC}$. Find the length of $\overline{DE}$.

ANSWERS

1. 660

2. 2387 OR 2387_{10} OR 2387_{ten}

3. 17

4. 33 (units optional)

5. 800

6. 30

7. 12

8. $18\sqrt{6}$ (Must be this exact answer.)

9. 80

10. 13

11. The coordinates of the vertices of a triangle are $(2,3)$, $(4,7)$, and $(5,6)$. Find the area of the triangle.

12. The sum of three numbers is $4x^2$, and the sum of four other numbers is $3x^3$. If $x=14$, find the average (arithmetic mean) of all seven numbers.

13. The lengths of all sides of a **scalene** triangle are whole numbers. If the largest side has a length of 17, find the smallest possible perimeter of the triangle.

14. If $\frac{3}{5}$ of a number is 6 more than $\frac{1}{2}$ of the number, find the number.

15. A woman was looking at a painting and stated: "That person's mother was my mother's daughter. I have no sisters, and I have no daughters." What was the relationship of the person in the painting to the woman? Express your answer in a whole word or words such as: **great-great-grandfather** or **niece**.

ANSWERS

11. 3

12. 1288

13. 35

14. 60

15. Son (Must be the whole word.)

1. A vector that has the same direction as the vector $(12,5)$ but has a magnitude of 39 is the vector (x,y). Find the ordered pair (x,y).

2. If $i=\sqrt{-1}$, one of the complex square roots of $-32+24i$ can be expressed as $x+yi$ where $x>0$ and $y>0$. Find the value of $(x+y)$.

3. The directrix of the parabola whose vertex is $(0,-2)$ and whose focus is $(0,6)$ is the line whose equation is $y=k$. Let p be the probability that in flipping a fair coin 7 times, there will be exactly 0 heads and 7 tails showing. Find the value of (kp). Express your answer as a common fraction reduced to lowest terms.

4. The arithmetic mean of two of the roots for x of $x^3-10x^2-wx-630=0$ is 8. The arithmetic mean of two of the roots for y of $y^3-14y^2+31y+k=0$ is 8. Find the value of $(k+w)$.

5. Given ΔABC with $A(0,-2)$, $B(17,1)$, and $C(0,k)$. If the area of ΔABC is 102, find the sum of all distinct possibilities for k.

6. Let $f(x)=3x+5$ and $g(x)=7x+k$. If $f(g(3))=62$, find the value of k.

7. Sherri invested k dollars at 4% annual percentage rate of interest. The interest was compounded annually. At the end of one year after the interest was credited, Sherri's investment was worth $260. Find the value of k.

8. Let $i=\sqrt{-1}$. The four roots of $x^4+kx^3+29x^2-24x+w=0$ are $2i$, $-2i$, $3+4i$ and $3-4i$. The first term of an arithmetic progression is 4. The sum of the first 18 terms of this arithmetic progression is 531. Let p be the third term of this arithmetic progression. Find the value of $(k+w+p)$.

9. If ln is the symbol for natural logarithm and if $f(x)=\ln\left(\dfrac{x+3}{x-1}\right)$, find all x such that $f(x)=f(2)+f(5)$. Express any and all answers as improper fractions reduced to lowest terms.

10. Let k be the number of distinct positive integers that leave a remainder of 47 when divided into 2372. Let $(6,-48)$ and $(x,36)$ be two perpendicular vectors. Find the value of $(k+x)$.

ANSWERS

1. $(36,15)$ (Must be this ordered pair.)

2. 8

3. $-\frac{5}{64}$ OR $\frac{-5}{64}$ OR $\frac{5}{-64}$ (Must be this reduced common fraction.)

4. 327

5. -4

6. -2

7. 250 (Dollars optional.)

8. 104

9. $\frac{13}{9}$ (must be this reduced improper fraction)

10. 294

11. The area of a rectangle is 168 square units, and a diagonal of the rectangle has a length of 25 units. Find the number of units in the perimeter of this rectangle.

12. The second term of an arithmetic sequence is l3 and the eighteenth term of this arithmetic sequence is 41. Find the twenty-ninth term. Express your answer as a **decimal**.

13. Solve the determinant equation for x: $\begin{vmatrix} 7 & -2 & -5 \\ -4 & 8 & x \\ 1 & 0 & 2 \end{vmatrix} = 106$.

14. If $y = -2x^2 + 44x - 11$ where x and y are real, find the largest possible value for y.

15. If x is a positive integer, find the smallest possible value of x such that $\cot(x°) < 0$ and such that $\cot(x°) = -\cot(-x°)$.

ANSWERS

11. 62 (Units optional.)

12. 60.25 (Must be this decimal.)

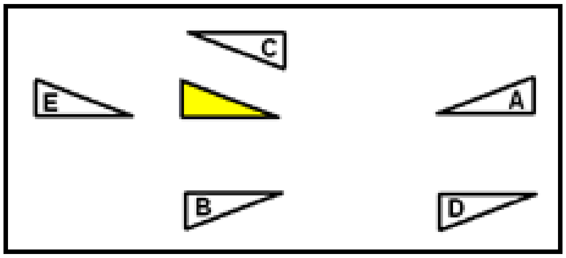
13. 15

14. 231

15. 91

Questions for the Oral Competition – Regional Level AA, 2010
Ch. 19, For All Practical Purposes, sixth edition

1) Identify the type of transformation for each figure below that has a letter in it. The initial figure has no letter in it.



2) For each of the following figures, explain which type of pattern it is and give the notation for the symmetry pattern.

(a)

(b)

(c)

3) The most famous Fibonacci problem deals with the sequence of pairs of rabbits. In this problem, a pair of rabbits is defined to be one male rabbit and one female rabbit. These rabbits procreate every month that they are adult rabbits, and always give birth to a pair of rabbits whose genders are one male and one female. For each of the following conditions, construct a table of the number of pairs of rabbits over the first ten months and describe the pattern.

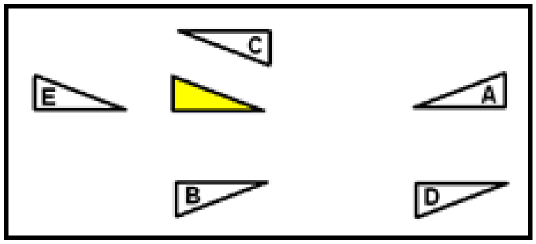
(a) The pairs of rabbits are born in the first month, are juvenile rabbits the second month, and are adults to give birth to a pair of rabbits the third month.

(b) The pairs of rabbits are born in the first month and are adults to give birth to a pair of rabbits the next month.

(c) The pairs of rabbits are born in the first month, are children rabbits the second month, are juvenile rabbits the third month, and are adults to give birth to a pair of rabbits the fourth month.

Answers for the Questions for the Oral Competition – Regional Level AA, 2010

1) Identify the type of transformation for each figure below that has a letter in it. The initial figure has no letter in it.

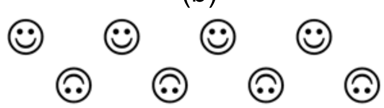


Answers:
A is a reflection along a vertical line, B is a reflection along a horizontal line, C is a rotation, D is a glide reflection, E is a translation.

2) For each of the following figures, explain which type of pattern it is and give the notation for the symmetry pattern.

(a)

(b)



(c)

Answers:
(a) rosette pattern because there are not infinitely many repetitions of a motif; it does not have reflection symmetry, but the motif coincides with itself under a rotation of 120°, so its notation is *c3*.
(b) strip pattern, because the motif is repeated horizontally, but not vertically; using the flowchart on p700 of the source, it does have vertical reflection lines, it does not have horizontal reflection lines, and there is a half-turn, so its notation is *pma2*.
(c) strip pattern, because the motif is repeated horizontally, but not vertically; using the flowchart on p700 of the source, it does have not vertical reflection lines, it does not have horizontal reflection lines or a glide reflection, and there is not a half-turn, so its notation is *p111*.

3) The most famous Fibonacci problem deals with the sequence of pairs of rabbits. In this problem, a pair of rabbits is defined to be one male rabbit and one female rabbit. These rabbits procreate every month that they are adult rabbits, and always give birth to a pair of rabbits whose genders are one male and one female. For each of the following conditions, construct a table of the number of pairs of rabbits over the first ten months and describe the pattern.

(a) The pairs of rabbits are born in the first month, are juvenile rabbits the second month, and are adults to give birth to a pair of rabbits the third month.

(b) The pairs of rabbits are born in the first month and are adults to give birth to a pair of rabbits the next month.

(c) The pairs of rabbits are born in the first month, are children rabbits the second month, are juvenile rabbits the third month, and are adults to give birth to a pair of rabbits the fourth month.

Answers:

Month	(a)	(b)	(c)
1	1	1	1
2	1	2	1
3	2	4	1
4	3	8	2
5	5	16	3
6	8	32	4
7	13	64	6
8	21	128	9
9	34	256	13
10	55	512	19

(a) Fibonacci numbers. (b) The rabbits double from how many there were the previous month. (c) The finite differences beginning with month four (month 4 – month 3) are the same as the sequence of the number of rabbits beginning in month one. Alternately, $f(1) = f(2) = f(3) = 1$ and $f(n) = f(n-1) + f(n-3)$ for $n \geq 4$.

Questions for the Oral Competition – Regional Level AA, 2010
Extra Questions

1) Assume that the capital letters of the alphabet are written as A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. Which of these letters have all three types of the following symmetries: a horizontal line of reflection symmetry, a vertical line of reflection symmetry, and a rotational symmetry?

2) Give an example of a geometric figure in which the ratio of the lengths of two segments is ϕ.

3) All of the rigid motions can be described in terms of reflections alone.
(a) Describe how to achieve a translation using one or more reflections.
(b) Describe how to achieve a glide reflection using one or more reflections.

Answers for the Extra Questions for the Oral Competition
Regional Level AA, 2010

1) Assume that the capital letters of the alphabet are written as A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. Which of these letters have all three types of the following symmetries, a horizontal line of reflection symmetry, a vertical line of reflection symmetry, and a rotational symmetry? Answers: H, I, O, X

2) Give an example of a geometric figure in which the ratio of the lengths of two segments is ϕ.

Answers: Answers will vary. Possibilities include the golden rectangle (ratio of the length of the longer side to the length of the shorter side) and a pentagon with at least one diagonal (ratio of the length of the diagonal to the side length). The question does not ask for a particularly creative answer, so any figure with the appropriate ratio should receive credit.

3) All of the rigid motions can be described in terms of reflections alone.

(a) Describe how to achieve a translation using one or more reflections.

(b) Describe how to achieve a glide reflection using one or more reflections.

Answers: (note that answers may vary; students need not provide more specificity than is given here)

(a) Use two reflections over lines perpendicular to the direction of translation.

(b) Use three reflections—two to achieve a translation (the “glide” part), then one to achieve the “reflection” part.