1. Find the value of x that satisfies $\frac{x-3}{x-7}=\frac{x+4}{x+7}$.

2. If k is a positive integral multiple of four and $k<60$, find the largest possible value of k.

3. If $2x-3(x-4)<-5$, then the solution set is $\{x : x>k\}$. Find the value of k.

4. When $x=-9$, x^2 exceeds $-x^2$ by kx. Find the value of k

5. The largest number which can be obtained as the product of two positive integers whose sum is 104 can be expressed as $a^b c^d$ where a, b, c, and d are positive integers and where a and c are prime numbers such that $a<c$. Find the value of $(a+2b+3c+4d)$.

6. In a survey of 100 persons, it was found that 64 persons read “Seventeen” and that 75 persons read “Sports Illustrated.” It was also found that 47 persons read both magazines. If one of these 100 persons is selected at random, find the probability that the person selected read **neither** magazine. Express your answer as a **decimal**.

7. If $\frac{y}{8.25}=16$, find the value of y.

8. The wholesale cost of an item to a dealer was $k\%$ of the price for which he sold the item. The difference between the wholesale cost and the selling price was \$420. The profit was 10% of the wholesale cost and overhead expenses were $\frac{57}{13}$ of the profit. Write your answer as the value of k from the $k\%$ of the price above. (Note: The selling price is the sum of the wholesale cost of an item, the profit made, and the expenses paid.)

9. In the addition alphametic shown to the right, each letter stands for the same digit throughout the puzzle. No digit is represented by more than one letter. For your answer write the 4 digits (from left to right in order) that represent MOOD.

$$\begin{array}{cccc} & & I & S \\ + & D & I & M \\ \hline M & O & O & D \end{array}$$

10. Find the sum of all the distinct positive integral factors of 2010.

11. The roots for x of the quadratic equation $x^2 - kx + w = 0$ are unequal, and both of the roots for x are positive even integers. Find the smallest possible value of w.

12. Bubbles can completely fill her empty spa bathtub in 40 minutes with the hot water faucet alone while the drain takes half as long to empty a completely full bathtub. The cold water faucet alone would take only 30 minutes to completely fill the empty tub. Bubbles turned on both water faucets to fill her empty tub, but forgot to close the drain. Five minutes later, she discovered her error and closed the drain. Ten minutes after closing the drain, she shut off the cold water faucet so the water would be warm. In total, how many minutes did it take Bubbles to completely fill her bathtub?

13. For all ordered triples (x, y, z) that satisfy $\frac{15}{x} - \frac{9}{y} = \frac{6}{z}$, solving for x yields $x = \frac{kyz}{wz + py}$ where k, w, and p are positive integers. Find the least value of $(k + w + p)$.

14. If k, w, and p are positive integers, find the smallest possible value of k such that $kx = wy$ **and** $kx = pz$ for all ordered triples (x, y, z) that satisfy $3x + 2y = 7z$ and $11x + z = 5y$.

15. Find the value of $\left(4^{\left(\frac{5}{6}\right)}\right)\left(1024^{\left(\frac{2}{15}\right)}\right)$

16. Let $x + y = 55$ and let $y = 33 + x$. Find the value of $\frac{y}{x}$.

17. For all real numbers, $a \otimes b = \frac{3a + b^2}{4b}$. If $17 \otimes b = 5$, find the sum of all possible value(s) of b.

18. The hypotenuses of two right triangles have equal lengths. All the legs have lengths which are integers. One right triangle has congruent legs while in the second right triangle, the length of one leg is 14 more than the length of the other leg. Find the area of the second right triangle. Express your answer as a **decimal**.

19. If $2x - 3(x + 4y) = 5(5y - 3) - x$, find the value of y. Express your answer as a common or improper fraction reduced to lowest terms.

20. Solve for x if $\frac{2}{x^2 + 9x + 20} + \frac{2}{x^2 + x - 12} = \frac{5}{x^2 + 2x - 15}$.

2011 RA
Algebra I

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

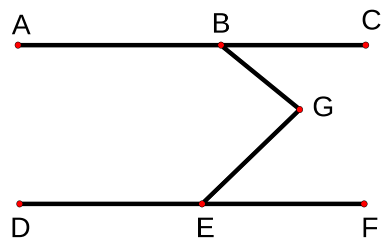
1. -1
2. 56
3. 17
4. -18
5. 57
6. 0.08 or .08 (Must be this decimal.)
7. 132
8. 65
9. 1009
10. 4896
11. 8
12. 30 (Minutes optional.)
13. 10
14. 2960
15. 8
16. 4
17. 20
18. 263.5 (Must be this decimal.)
19. $\frac{15}{37}$ (Must be this reduced, common fraction.)
20. -16

1. A circle with center $(-4,-6)$ is tangent to the $x-axis$ and has area $k\pi$. Find the value of k

2. The lengths of all sides of an isosceles triangle are integers, and one side has a length of 10. The perimeter of this isosceles triangle is 23. Find the length of the shortest side of this isosceles triangle.

3. Find the average degree measure of an interior angle of a convex polygon with 24 sides.

4. In the diagram, the line containing A, B, and C is parallel to the line containing D, E, and F. If $\angle BGE = 104°$ and $\angle FEG \cong \angle CBG$, find the degree measure of $\angle ABG$.



5. Find the exact perimeter of a square inscribed in a circle with area 25π.

6. One circle has an equation of $(x-8)^2+(y+2)^2=25$. A second circle has an equation of $2x^2+2y^2+12y=1$. Find the slope of a line that is perpendicular to the line that joins the centers of the two given circles.

7. A regular polygon has k sides. Half the measure of an interior angle is 6 more than three times the measure of an exterior angle. Find the value of k.

8. Let the perimeter of a regular hexagon be $3x+12y+3$. The sum of the lengths of any 4 sides of this regular hexagon is $5x+3y$. If x and y represent two-digit prime positive integers, find the largest possible area of the regular hexagon. Round your answer to the nearest integer and express your answer as that **integer**.

9. Let $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ be three parallel chords that are non-diameters of a circle on the same side of the center. Let $\overleftrightarrow{GJ}$ be tangent to the circle at H such that $\overleftrightarrow{GJ} \parallel \overline{AB}$. The distance between $\overline{AB}$ and $\overline{CD}$ is equal to the distance between $\overline{CD}$ and $\overline{EF}$ and is also equal to the distance between $\overline{EF}$ and $\overleftrightarrow{GJ}$. If $AB = 24$, $CD = 20$, then the distance from the center of the circle to $\overline{AB}$ can be expressed as $\frac{k\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.

10. Quadrilateral $QUAD$ is inscribed in a circle. $m\angle UQD = 110^\circ$ and $m\angle QUA = 80^\circ$. Find the degree measure of $\angle UAD$.

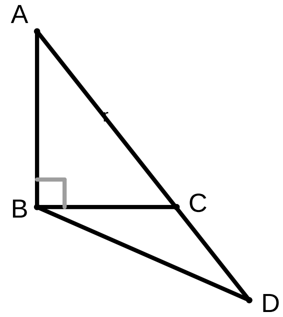
11. An isosceles trapezoid with legs of length 10 has bases of lengths 24 and 40. Find the length of an altitude to a base of this trapezoid.

12. The apothem of a regular hexagon has length $6\sqrt{2}$. Find the exact area of this hexagon.

13. The perimeter of a rectangle is 72.6 units. A second similar rectangle has an area that is 125% of the area of the original rectangle. Find the perimeter of the second rectangle, rounded to the nearest tenth of a unit.

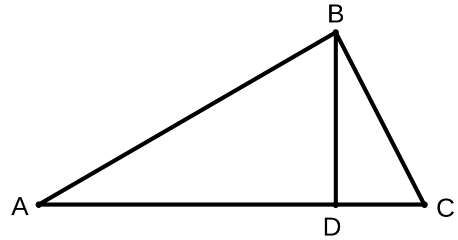
14. The sum of the radii of two concentric circles is 50. $\overline{CD}$ is a chord of the larger circle and has length 40. C, A, M, B, and D lie on $\overline{CD}$ in that order and divide $\overline{CD}$ into equal lengths. The smaller circle passes through A and B. The area of the smaller circle is $k\pi$. Find the exact value of k.

15. In the diagram (not necessarily drawn to scale), points A, C, and D are collinear. $AB = 12$, $BC = 5$, $\angle CBD = 45°$, and $\overline{AB} \perp \overline{CB}$. Find the length of $\overline{CD}$. Express your answer as an improper fraction reduced to lowest terms.



16. Think of the hands of a clock as lying along two radii of a circle. Find the degree measure of the minor arc intercepted by the hands of a clock at 11:15 AM. Express your answer as a decimal.

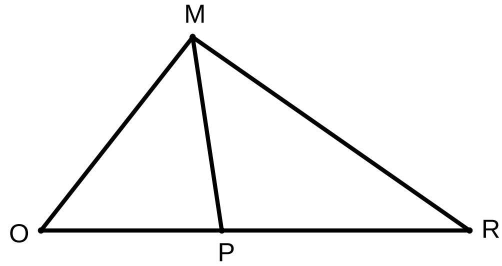
17. In the diagram (not necessarily drawn to scale), $\angle ABC = 90°$, and D lies on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$. $AC = \frac{17}{15}$ and $AB = 1$. Then $\frac{AD}{DC} = \frac{k}{w}$ where k and w are relatively prime positive integers. Find the smallest possible value of $(2k + 3w)$.



18. The sides of ΔABC have lengths 1, 2, and $\sqrt{3}$ while those of ΔDEF have lengths 2, 2, and $\sqrt{8}$. If one of the six angles A, B, C, D, E, or F is selected at random, find the probability the degree measure of the angle is a multiple of $30°$. Express your answer as a common fraction reduced to lowest terms.

19. Coplanar circles with centers at A, D, and E are mutually tangent externally so that each circle is tangent to each of the other two. Points B and C lie on the circle with center at A and are points of tangency with the other two circles. All radii of the circles have lengths that are positive integers. If $AB = 3$ and $BC = 3\sqrt{2}$, find the sum of all possible distinct perimeters of ΔADE.

20. In the diagram, $\angle OMP \cong \angle RMP$. $MO = 40$, $RM = 60$, $OP = k$, and $PR = w$, where k and w are integers. The area of ΔMOR is $75\sqrt{255}$. If O, P, and R are collinear, find PR.



2011 RA

Geometry

Name **ANSWERS**

School ____________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 36

2. 3

3. 165 (Degrees optional.)

4. 128 (Degrees optional.)

5. $20\sqrt{2}$ (Must be this exact answer, units optional)

6. -8

7. 15

8. 59239 (Must be this integer.)

9. 25

10. 70 (Degrees optional.)

11. 6

12. $144\sqrt{3}$ (Must be this exact answer.)

13. 81.2 (Must be this decimal.)

14. 484

15. $\frac{65}{7}$ (Must be this reduced improper fraction.)

16. 112.5 (Must be this decimal, degrees optional.)

17. 642

18. $\frac{2}{3}$ (Must be this reduced common fraction.)

19. 132

20. 42

1. Find the ninth term of the sequence: $1, \frac{1}{2!}, \frac{1}{3!}, \cdots$. Express your answer as a common fraction reduced to lowest terms.

2. If $x^4 - x^2 - 20 = 0$, how many of the distinct roots are non-real?

3. Let $i = \sqrt{-1}$. If $(3i)(ki) = -12$, find the value of k.

4. Sally wishes to produce an 8 digit code (sequence of digits) using only the digits 1, 2, 3, and 4 for each code. How many distinct codes are possible? Give your answer as the integer number of distinct codes.

5. The simplified value of $\frac{(n+2)!}{n!} - n$ can be expressed as $kn^2 + wn + p$ where n, k, w, and p represent positive integers. Find the value of $(k + w + p)$.

6. Find the smallest positive integer that is an integral multiple of 18 distinct positive integers and is 4 less than the square of a positive integer.

7. Let $i = \sqrt{-1}$. If $ik - 8i = 4$, then $k = w + pi$ where w and p are real numbers. Find the value of $(w + p)$.

8. A drawer contains 3 black socks, two brown socks, and two blue socks. Two socks are drawn at random from this drawer without replacement. Find the probability that the two socks drawn were of the same color. Express your answer as a common fraction reduced to lowest terms.

9. Let k be an integer such that $30 < k < 45$. Find the sum of all distinct values of k such that k is **not** the sum of 2 or more consecutive **odd** natural numbers.

10. The equation of the hyperbola whose vertices are at $(0,4)$ and $(0,-4)$ and whose eccentricity is 2.5 can be expressed in the form $\frac{y^2}{k} - \frac{x^2}{w} = 1$. Find the value of $(2k+w)$.

11. Give the mode for the data shown in the stem and leaf graph at the right.

Stem	*Leaf*									
5	6	6	6	8	9	2	2	4		
6	8	1	3	4	1	7	4			
7	4	4	2	1	2	3	4	6	8	4
8	0	1	2	3	2	1	0	4		

12. Let k and w be positive integers. If k is reduced by the sum of $k's$ digits, the result is w. If w is increased by the sum of $w's$ digits, the result is $k+2$. Find the sum of the seven largest distinct possible three-digit values of k.

13. Find the sum of all the distinct positive integral multiples of 7 if each multiple of 7 is less than 2010.

14. An athlete needs to combine a swim and then a walk for a total trip from a point A on one bank of a 1.5 km wide river, to a point B down river on the opposite bank. B is 30 km from the point, on the same side of the river as B, which is nearest A. The athlete can average 2 km per hour swimming and 6 km per hour walking. Assuming the sides of the river are parallel, find the minimum possible time for the trip. Express your answer as a **decimal** rounded to the nearest hundredth of an hour.

15. If $x < 0$, then find the value of x such that $|2 - 3x| = 23$.

16. The solutions for x of the quadratic equation $2x^2 - 10x + c = 0$ are k and $k + 2$. Find the value of c.

17. For how many distinct integers x greater than 100,000 and less than 200,000 will x be an integral multiple of all three of the following numbers: 7, 11, and 13?

18. Let a, b, and c be integers such that $a < 0$, $c > 0$, and $f(x) = ax^2 + bx + c$. If $f(7) = -37$ and $f(9) = -85$, find the smallest possible value of c.

19. For all real values of x, $x^2 - 6x + 14 = (x - k)^2 + w$. Find the value of w.

20. A bag contains exactly 6 marbles—3 red, 2 white, and 1 blue. A girl draws a marble at random, replaces the marble, and continues to draw in this fashion. Find the probability that after 7 draws she has drawn at least 2 marbles of each color. Express your answer as a common fraction reduced to lowest terms.

2011 RA

Algebra II

Name ANSWERS

School ________________

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{362880}$ (Must be this reduced common fraction.)
2. 2
3. 4
4. 65536
5. 5
6. 252
7. 4
8. $\frac{5}{21}$ (Must be this reduced common fraction.)
9. 266
10. 116
11. 74
12. 6592
13. 289296
14. 5.71 (Must be this decimal.)
15. -7
16. $\frac{21}{2}$ or $10\frac{1}{2}$ or 10.5
17. 100
18. 5
19. 5
20. $\frac{35}{216}$ (Must be this reduced common fraction.)

1. A sequence is defined recursively by $a_{(n+1)} = 2a_n + 3$ for all integers $n \geq 1$. If $a_{10} = 17$, find the value of a_{11}.

2. Given that the polynomial equation $x^5 + kx^4 + wx^3 + px^2 + fx + g = 0$ has 5 distinct real roots for x, how many times does the graph of $y = x^5 + kx^4 + wx^3 + px^2 + fx + g$ intersect the $x-axis$?

3. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   The quadrants which contain points of the graph of $y = -5(x+1)^2 - 12$ are:

   A) I and III B) I and II C) II and III D) II and IV E) III and IV

   Note: Be certain to write the correct capital letter as your answer.

4. Find the value of the indicated sum: $\sum_{1}^{4}(2.613k + 43.43)$. Express your answer as an **exact decimal.**

5. The eighth term of a geometric sequence of real terms is 32, while the thirteenth term is 1024. Find the third term of this geometric sequence of real terms.

6. Given that f is an **odd** function and that $f(2) = 4$, find the value of $f(-2)$.

7. All lengths of the sides of a triangle are integers. The sine of the middle-sized angle of the triangle is $\frac{\sqrt{3}}{2}$. The length of one of the adjacent sides of the triangle for that angle is 15. Find the smallest perimeter for such a triangle.

8. If $\sin(x) = \frac{4}{5}$ and $\cot(y) = \frac{12}{5}$, then there are two possible answers for $\tan(x-y)$. Find the sum of these two possible distinct answers. Express your answer as an improper fraction reduced to lowest terms.

9. Let k, w, and p be the roots of $x^3 - 16x^2 + 76x - 96 = 0$, and let $(k+1)$, $(w+2)$, and $(p+3)$ be the roots for y of $y^3 + ay^2 + by - 308 = 0$. Find the value of $(a+b)$.

10. $(a+b)^n$ is expanded for a particular positive integer n and written in decreasing powers of a. Specific positive integers are then substituted for a and b in each term. The string of terms produced includes the three consecutive numbers (separated by + signs) 1620, 4320, and 5760. Find the next number after 5760 in this string of terms.

11. In the real-numbered function, find the sum of all distinct negative integers for x such that $\log_{\left(\frac{1}{2}\right)}(x+15) \le 0$.

12. Find the smallest positive integer k such that all the zeros for x of the polynomial $x^3 - kx^2 + 216x$ are integers and such that all the zeros for y of the polynomial $3y^2 - 2ky + 216$ are integers.

13. If $A = 2\begin{bmatrix} 2 & 3 \\ 1 & -3 \end{bmatrix} - 3\begin{bmatrix} 5 & 4 \\ 1 & 7 \end{bmatrix}$, find the value of $\det(A)$.

14. It is known that 475 logs are piled in such a way that each row has one more log than the row above. If the top row has as few logs as necessary to stack all 475 logs, find the number of logs in the bottom row.

15. Let e be the base for natural logarithms and let $\ln$ be the symbol for natural logarithms. Find, written as a single fraction in terms of e, the value of x such that $\ln(x)-\ln(x-1)=1$.

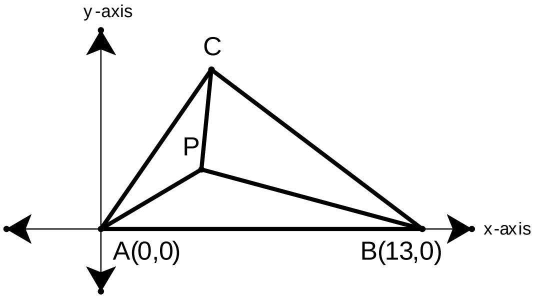
16. Given that $\sin(A)=\frac{2}{5}$ and $0<A<\frac{\pi}{2}$ (where A is measured in radians). Then $\sin(2A)$ can be expressed in simplest radical form as $\frac{k\sqrt{w}}{p}$ where k and p are relatively prime integers and $p>0$. Find the value of $(k+w+p)$.

17. Two fair, standard cubical dice are thrown. Find the probability that the sum of the numbers showing on the uppermost faces is less than or equal to 5. Express your answer as a common fraction reduced to lowest terms.

18. Let $C(n,k)=\frac{n!}{k!(n-k)!}$ where n and k represent positive integers. Find the **sum** of all distinct values of n such that $7<C(n,4)<106$.

19. If $f(x)=\frac{x}{x^2-x-2}$, the asymptotes are $x=k$, $x=w$, and $y=p$. Find the value of $(k+w+p)$.

20. In the diagram with coordinates as shown, $AC=14$, and $BC=15$. The ratio of the area of ΔPAB to the area of ΔPAC to the area of ΔPBC is $1:2:3$. Find the **ordered pair** that represents point P. Express your answer as an **ordered pair** with **each member** of the ordered pair expressed as an improper fraction reduced to lowest terms.



2011 RA

Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 37

2. 5

3. E (Must be this capital letter.)

4. 199.85 (Must be this decimal.)

5. 1

6. –4

7. 35

8. $-\frac{375}{112}$ or $\frac{-375}{112}$ (Must be this reduced improper fraction.)

9. 127

10. 3840

11. –105

12. 33

13. 291

14. 31

15. $\frac{e}{e-1}$ or $\frac{-e}{1-e}$ (Must be a single fraction.)

16. 50

17. $\frac{5}{18}$ (Must be this reduced common fraction.)

18. 21

19. 1

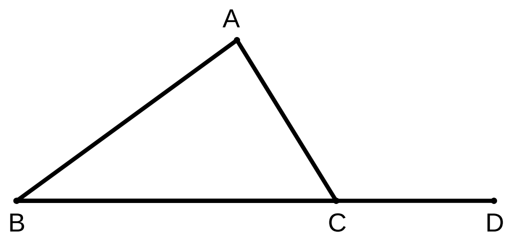
20. $\left(\frac{68}{13},\frac{28}{13}\right)$ (Must be this ordered pair with reduced improper fractions.)

NO CALCULATORS

1. Find the value for x for which $\frac{15}{2x+1}=6$. Express your answer as a common fraction reduced to lowest terms.

2. How many distinct integers are solutions for x if $-8<2x<13$?

3. In the diagram, points B, C, and D are collinear. $\angle BAC=(2x)°$, $\angle ABC=(3x)°$, and $\angle ACD=(7x-42)°$. Find the degree measure of $\angle ACB$.



4. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

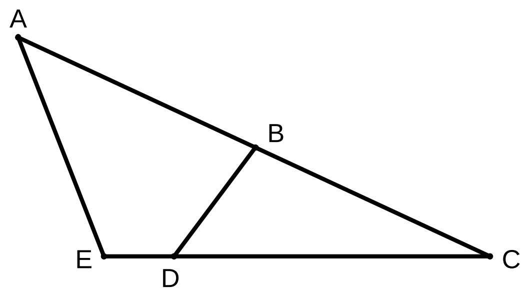
If $k=6w$, for how many distinct values of k can $k=w$?

A) 0 B) 1 C) 2 D) 3 E) 6 F) more than 6

Note: Be certain to write the correct capital letter as your answer.

5. Convert the base eight number 236_8 to a base seven number. Express your answer as a base seven numeral.

6. In the diagram, B lies on $\overline{AC}$, and D lies on $\overline{EC}$. If $\frac{AB}{BC}=\frac{5}{7}$ and $\frac{ED}{EC}=\frac{2}{7}$, then the ratio of the area of ΔBDC to the area of ΔAEC can be expressed in the form of $k:w$ where k and w are positive integers. Write $k:w$ as a reduced common fraction.



7. $A(8,2)$, $B(10,-8)$, and $C(7,3)$ are the vertices of ΔABC. Find the exact length of the median from C to $\overline{AB}$.

NO CALCULATORS

NO CALCULATORS

8. $\sqrt{79+20\sqrt{3}} = a + b\sqrt{c}$ where a, b, and c are integers. Find the value of $(a+b+c)$.

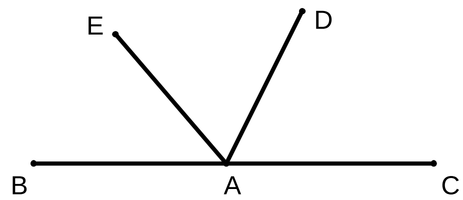
9. The system $\begin{cases} 3x + ay = 5 \\ bx + 6y = 15 \end{cases}$ has infinitely many solutions (x, y). Find the value of $(a+b)$.

10. Let a, b, c, and d represent 4 positive integers such that the sum of the 4 integers is 108. If 5 were added to the first, 5 subtracted from the second, 5 multiplied by the third, and 5 divided into the fourth, all the results would be equal. Find the product $(abcd)$.

11. Find the degree measure formed by the minute hand and hour hand of a clock 90 seconds after $9:00$. Express your answer as a decimal.

12. Given the system: $\begin{cases} x + y = 94 \\ y + z = 45 \\ x + z = 23 \end{cases}$. Find the value of $(x+y+z)$.

13. In the diagram, points B, A, and C are collinear. $\overrightarrow{AE}$ bisects $\angle BAD$, $\angle CAD = (2x+16)°$, and $\angle EAD = (4x-3)°$. Find the degree measure of $\angle EAD$.

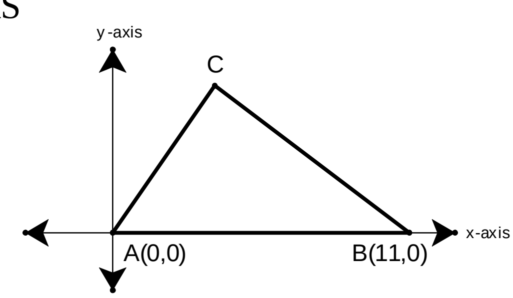


14. Find the area of a triangle whose sides have lengths of $\sqrt{5}, \sqrt{10},$ and $\sqrt{13}$.

15. Craig averaged 85 on his first 3 exams. He studied very hard so he could raise his average and scored 3 points more on his fifth exam than his fourth exam. He succeeded in raising his average to 90. What was his score on the fifth exam?

NO CALCULATORS

16. In the diagram with coordinates as shown, $AC = 12$ and $BC = 13$. Find the $x - coordinate$ for the ordered pair that represents point C. Express your answer as an improper fraction reduced to lowest terms.



17. The equation $5x^2 - 25x - 8 = 0$ can be transformed into an equivalent equation $(x-k)^2 = w$. Find the sum $(k+w)$, expressed as a reduced improper fraction.

18. The degree measures of the angles of a triangle are in the ratio of $3:4:5$. If one of the angles of this triangle is selected at random, find the probability that the degree measure of the angle selected is an integral multiple of 10. Express your answer as a common fraction reduced to lowest terms.

19. Assume that Tom and Kay are playing a game with matchsticks. Three separate piles of matchsticks are formed. Tom and Kay have alternate turns. At each turn, the person must remove either 1 or 2 matchsticks from any **one** of the remaining piles. The winner is the person who takes the last matchstick. Suppose the game gets to the point where it is Kay's turn to remove matchsticks, and there are 4 matchsticks in one pile and 1 matchstick in each of the other two piles. If Kay plays perfectly, she can win by taking one matchstick from the big pile leaving the ordered triple $(3,1,1)$, or she could also win by taking 1 matchstick from either of the single piles leaving the ordered triple $(4,1,0)$. For the $(4,1,0)$ ordered triple, there would only be two remaining piles. At the present time, it is Kay's turn to remove matchsticks. There are 5 matchsticks in one pile, 4 matchsticks in the second pile, and 1 matchstick in the third pile. If Kay plays perfectly, list any and all ordered triples she should leave immediately after her present turn. List any and all answers as **ordered triples** of the form (a,b,c) where $a \ge b \ge c$.

20. $(1,9)$, $(1,3,6)$, and $(1,2,3,4)$ are 3 examples of distinct groups of **positive** integers for which the sum of the members of each group is 10. Within each group, the members must be arranged in ascending order. In other words, $(2,3,5)$ is an acceptable group, but $(3,2,5)$ would **not** meet the conditions. Find the number of distinct groups of **positive** integers (with the **positive** integers arranged in a strictly ascending order) for which the sum of the members of each group is 15. Assume that each group must contain at least two members.

NO CALCULATORS

2011 RA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{3}{4}$ (Must be this reduced common fraction.)

2. 10

3. 75 (Degrees optional.)

4. B (Must be this capital letter.)

5. 314_7 OR 314_{seven} OR 314

6. $\frac{5}{12}$ (Must be this reduced common fraction.)

7. $2\sqrt{10}$ (Must be this exact answer.)

8. 10

9. 11

10. 45000

11. 98.25 (Must be this decimal, degrees optional.)

12. 81

13. 65 (Degrees optional.)

14. $\frac{7}{2}$ OR $3\frac{1}{2}$ OR 3.5

15. 99

16. $\frac{48}{11}$ (Must be this reduced improper fraction.)

17. $\frac{207}{20}$ (Must be this reduced improper fraction.)

18. $\frac{1}{3}$ (Must be this reduced common fraction.)

19. $(4,3,1)$ (Must be this and only This ordered triple.)

20. 26

NO CALCULATORS

1. If p and q are two statements, then $p \vee q$ is read as "p or q" called "the **disjunction** of p and q." Let p be the statement that "$7 > 4$" and let q be the statement that "$-2 > -3$". Write the whole word **True or False** for the following statement: "$p \vee q$ is a true disjunction."

2. How many distinct numbers are in the **range** of the relation: $\{(5,2),(4,0),(6,2),(854,2)(6,7),(4,2)\}$?

3. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   On Doc's geometry test, the arithmetic mean score was 80 and the median score was 84. Three students took the test after these numbers were computed. Their scores were, 60, 84, and 90 and the mean and median were recomputed. Choose the best response below and report the capital letter of your choice.

   A) Both mean and median increased.
   B) Both mean and median decreased.
   C) The mean increased but the median did not.
   D) The mean decreased but the median did not.
   E) Both mean and median stayed the same.
   F) There is not enough information to select from (A) through (E).

   Note: Be certain to write the correct capital letter as your answer.

4. An ellipse has a vertex at $(10,0)$, its center at $(0,0)$, and has a focus at $(-6,0)$. The equation of this ellipse can be written in the form $\frac{x^2}{k} + \frac{y^2}{w} = 1$. Find the value of $(2k+3w)$.

5. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   How many real values for x satisfy the equation: $|2x-3| = -3$?

   A) 0 B) 1 C) 2 D) 3 E) more than 3 but finite F) infinite

   Note: Be certain to write the correct capital letter as your answer.

NO CALCULATORS

NO CALCULATORS

6. If the lines with equations $3x+2y=k$ and $2x-wy=12$ (k and w are constants) intersect at the point $(4,-3)$, find the value of the product (kw).

7. If $\log_7 3 = P$, $\log_7 8 = Q$, and $\log_7 10 = T$, find the value of $\log_7\left(\frac{15}{32}\right)$ in terms of P, Q, and T. Use capital letters when writing your answer.

8. Two circles are externally tangent and have radii whose lengths are 35 and 2. The sine of the angle that is 25% of the angle formed by the 2 common external tangents can be expressed as $\frac{\sqrt{k}-\sqrt{2}}{\sqrt{w}}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

9. The vector of length 40 in the direction opposite of the vector $[4,-3]$ can be represented by $[k,w]$. Find the value of $(k+w)$.

10. From four cards—one of which is a club, one of which is a diamond, one of which is a heart, and one of which is a spade—two cards are selected at random without replacement. Cindy is told (truthfully) that at least one of the cards selected is a red suit. Using this information, find the probability that Cindy can correctly identify the suits of the two cards that were selected. Express your answer as a common fraction reduced to lowest terms.

11. Find the value of $\sqrt{(101)(102)(103)(104)+1}$.

12. How many distinct **rational** roots for *x* exist if $x^3 - x^2 - \frac{x}{4} = -\frac{1}{4}$?

13. A parabola has the equation $3x^2 - 9x - 5y - 2 = 0$. The equation of the directrix for this parabola can be expressed in the form $y = k$. Find the value of k. Express your answer as an improper fraction reduced to lowest terms.

14. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

    If $x^4 = 16$, then which of the following expression(s) would **NOT** be equal to x^3?

    (A) -8 (B) $-8i$ (C) $4i$ (D) 8

    Note: Be certain to write the correct capital letter as your answer.

NO CALCULATORS

15. The terms of an arithmetic sequence are: $2,5,8,\cdots,3n-1$. In terms of n, the sum of any such arithmetic sequence is $\dfrac{3n^k+wn}{p}$. Find the value of $(2k+3w+4p)$.

16. Given the hyperbola: $12y^2-4x^2+72y+16x+44=0$. Find the endpoint of the latus rectum of this hyperbola such that the *x*-coordinate is greater than zero and such that the *y*-coordinate is less than zero. Express your answer as an **ordered pair** of the form (x,y).

17. The f_5 operates on a sequence of letters and yields (or gives or reports) the number of different times in a sequence of letters that the fifth letter after any letter in the sequence is the same as itself. For example, in the sequence: $a,b,c,d,e,a,b,a,d,e,a,c,d,d,e$, f_5 of the sequence is 7. Find the f_5 of the sequence: $a,x,b,c,y,a,x,c,c,c,c,x,d,c,c,x,x,d$.

18. Find the value of k if the graph of $f(x)=\dfrac{3kx-10}{4x+k}$ has the line whose equation is $y=6$ as a horizontal asymptote.

19. In ΔABC, $BC=20$, $\sin(\angle ABC)=\dfrac{3}{5}$, $\sin(\angle CAB)=\dfrac{12}{13}$, and the area of ΔABC is 126. Find $\sin(\angle ACB)$. Express your answer as a common fraction reduced to lowest terms.

20. Each of the real solutions of the equation $\sqrt{9-\sqrt{9+x}} = x$ can be expressed in simplest radical form as $\dfrac{\sqrt{k}-w}{p}$ where k, w, and p are positive integers. Find the sum of all possible distinct values of k.

NO CALCULATORS

2011 RA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. True (Must be the whole word.)

2. 3

3. D (Must be this capital letter.)

4. 392

5. A (Must be this capital letter.)

6. 8

7. $T + P - 2Q$ (Or equivalent commuted-term answer, must be capital letters.)

8. 109

9. -8

10. $\frac{1}{5}$ (Must be this reduced common fraction.)

11. 10505

12. 3

13. $-\frac{13}{6}$ OR $\frac{-13}{6}$ (Must be this reduced improper fraction.)

14. C (Must be this capital letter.)

15. 15

16. $(8, -7)$ (Must be this ordered pair.)

17. 8

18. 8

19. $\frac{63}{65}$ (Must be this reduced common fraction.)

20. 33

1. How much is 20% of 50% of 184? Express your answer as a **decimal**.

2. Let $f(x) = x^2 - 10x + 25$ and $g(x) = \ln(x) - 2$. Find the value of k so that the distance between $P(k, f(k))$ and $Q(k, g(k))$ is a minimum.

3. How many seconds are there in the month of April in the year 2011? Express your answer as an **exact integer**. Do **not** use scientific notation.

4. If the ratio of 0.2 to x is equal to 0.001, find the exact value of x.

5. Freddie Freethrow makes 80% of his free throws in any basketball game. Find the probability that in a particular game, he doesn't make his first free throw until his fourth or fifth attempt. Answer as an exact decimal (without scientific notation.)

6. A box is formed by removing squares of side length x from the corners of a rectangular piece of sheet metal 8.5 feet wide and 10.25 feet long. If the resulting box is to have a volume of at least 40 cubic feet, then $k \le x \le w$. Find the value of $(k + w)$.

7. A triangle has sides of lengths 5.003, 6.004, and 7.007. The largest angle of the triangle is bisected and divides one side of the triangle into **two segments**. Find the product of the lengths of these **two segments**.

8. From a standard deck of 52 cards with 4 suits and 13 ranks per suit, 13 cards are dealt at random without replacement. Find the probability that among those 13 cards, there is no suit that contains 5 or more cards.

9. Find the value of $\sum_{n=1}^{200}\left[(\sin n^\circ)(\cos n^\circ)\right]-\sum_{n=1}^{100}\left[(\sin n^\circ)(\cos n^\circ)\right]$.

10. An observer in a building that rises vertically notes that two objects on a horizontal road below have respective angles of depression of 26° and 19° respectively. If the distance from the base of the building to the far object is 1827 feet and if the horizontal road runs directly away from the observer, find the number of feet in the distance between the two objects.

11. The circumference of a circle is $38.74\pi x$. The area of this same circle is kx^2. Find the value of k.

12. A six-sided die is weighted so that the probability distribution $P(x)$ is as shown:

x:	1	2	3	4	5	6
$P(x)$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	k

. The value, denoted by x in the table, of a single roll of a die is defined to be the number that appears on the top of the die when it comes to rest. Find the expected value on a single roll of this die. **Express your answer as an improper fraction reduced to lowest terms**.

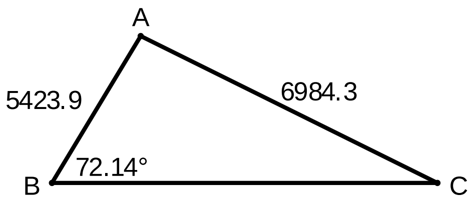
13. Find the circumference of the circle whose equation is $x^2+y^2-\frac{3}{2}x-\frac{5}{2}y+\frac{3}{2}=0$.

14. A surveyor stands on flat land at point A and measures distances to the two ends, points B and C, of a lake to be 1230 feet and 1570 feet respectively. She then finds the measure of $\angle BAC$ to be 51.00°. Find the number of feet between the two ends of the lake from B to C.

15. At the end of each month starting with the first investment of $400 at the end of January, 2010, Roger invests $400 in an annuity that pays 7.18% annual percentage rate compounded monthly. Find the value of Roger's annuity after the interest has been credited and immediately after his monthly investment of $400 has been made at the end of December, 2011. Round your final answer to the nearest dollar, and express your answer as a **whole number**. Do **not** use scientific notation.

16. On a level surface a man leaves point A and travels in a direction of $N14.34°E$ for 1.342 miles. He then changes and goes in a direction of $S56.57°W$ for a total of 13.57 miles. At this finish, find the number of miles the man is from point A.

17. Find the length of the radius of the inscribed circle of the triangle shown in which $AB = 5423.9$, $AC = 6984.3$, and $\angle ABC = 72.14°$



18. Christy bought a truckload of cantaloupe for $405.81, and Marnie bought a truckload of cantaloupe for $379.42 to stock their vegetable stand. There were 889 cantaloupes in Christy's truckload, and there were 881 cantaloupes in Marnie's truckload. Find the average number of **cents** paid per cantaloupe. Round your answer to the nearest hundredth of a cent.

19. Let A and B be the foci of the ellipse whose equation is $\frac{(x+1)^2}{100} + \frac{(y-2)^2}{36} = 1$. Let M be one of the endpoints of the minor axis of the ellipse. Find the degree measure of $\angle AMB$. Express your answer as a **decimal** rounded to the nearest hundredth of a degree.

20. Let a, b, c, and d represent non-zero integers (not necessarily different integers) such that $|a| = |b| < 15$ and $|c| = |d| < 15$. If c and d are the solutions for x of $x^2 + ax + b = 0$ and if a and b are the solutions for x of $x^2 + (2-c)x - 8d = 0$, find the ordered quadruple (a,b,c,d). Be sure to express your answer as that ordered quadruple (a,b,c,d).

2011 RA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 18.4 (Must be this decimal.)

2. 5.098 OR 5.098×10^0

3. 2592000 (Must be this integer, seconds optional.)

4. 200 (Must be this exact answer.)

5. 0.00768 OR .00768 (Must be this decimal.)

6. 3.329 OR 3.329×10^0

7. 12.17 OR 1.217×10 OR 1.217×10^1

8. 0.3508 OR $.3508$ OR 3.508×10^{-1}

9. -24.18 OR -2.418×10 OR -2.418×10^1

10. 537.2 OR 5.372×10^2 (Feet optional.)

11. 1179 OR 1.179×10^3

12. $\frac{187}{60}$ (Must be this reduced improper fraction.)

13. 4.967 OR 4.967×10^0

14. 1244 OR 1.244×10^3 (Feet optional.)

15. 10290 (Must be this whole number, dollars optional.)

16. 12.61 OR 1.261×10 OR 1.261×10^1 (Miles optional.)

17. 1751 OR 1.751×10^3

18. 44.36 OR 4.436×10 OR 4.436×10^1 (Cents optional.)

19. 106.26 (Must be this decimal, degrees optional.)

20. $(-4,4,2,2)$ (Must be this ordered quadruple.)

1. Find the length of an altitude of a rhombus if the lengths of the diagonals of the rhombus are respectively 40 and 42. Express your answer as an improper fraction reduced to lowest terms.

2. If $x<0$, find the value of x such that the point $(x,2)$ is $2\sqrt{5}$ units from $(-1,4)$.

3. The endpoints of the diameter of a circle are $(-4,-8)$ and $(2,0)$. The point $(x,-1)$ lies on the circle. Find the **product** of all distinct values of x.

4. Let x, y, and z be consecutive **odd** integers such that $x>y>z$. Find the value of $(x-y)(y-z)(x-z)$.

5. Find the value of x for which $3^{(x^2+2x)} = \frac{1}{3}$.

6. ΔABC is isosceles with perimeter 306. Two sides have lengths $4x+8$ and $3x+4$. Find the sum of the two possible values for x.

7. Let k be the number of distinct diagonals that can be drawn in a dodecagon. Let w be the numeric value of the average degree measure of an exterior angle of a dodecagon. Find the value of $(k+w)$.

8. The longest altitude of an isosceles triangle whose sides have lengths of 73, 73, and 110 has length $\frac{A}{B}$ when written as a reduced improper fraction. $kA-401B=2011$. Find k as an exact decimal.

9. A triangle and a square both have perimeter 2011. The sides of the triangle are in the ratio $5:7:11$. Let k be the average length of a side of the triangle. Let w be the length of the diagonal of the square. Find the sum $(k+w)$. Round your answer to the nearest whole number and answer as that integer.

10. Let x represent integers between -20 and 20, inclusive. If $\frac{2x-15}{x}>4$, find the sum of all distinct x values that are solutions.

2011 RA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. $\frac{840}{29}$ (Must be this reduced improper fraction.)	______
2. -5	______
3. -15	______
4. 16	______
5. -1	______
6. 55	______
7. 84	______
8. 5.925 (Must be this decimal.)	______
9. 1381 (Must be this integer.)	______
10. -28	______

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. 41

12. $-2\sqrt{26}$ (Must be this exact answer.)

13. 3

14. 81

15. 12

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Find the value of k such that the lines with equations of $(3k-35)x+10y=17$ and $15x+(3k-45)y=11$ are perpendicular.

2. The sum of the terms of an infinite geometric sequence is $\frac{2}{3}$, and the first term of this sequence is $\frac{1}{4}$. Find the fourth term of this geometric sequence. Express your answer as a common fraction reduced to lowest terms.

3. Find the length of the segment joining the vertices of the graph of $y^2-4y=4x^2-24x+36$.

4. Find the vector that has the opposite direction as the vector $(-4,-3)$ and which has a magnitude of 15. Express your answer as an ordered pair of the form (x,y).

5. From the set $\{8,27,36,45\}$, one number is selected at random. Find the probability that the number selected is one of the terms of the geometric sequence: $\frac{2}{27},\frac{2}{9},\frac{2}{3},\cdots$

6. If x represents an integer greater than -1 and less than 259, find the sum of all distinct x such that either $\log_3 x$ or $\cos(x°)$ is a rational number.

7. The graph of $f(x)=|2x-3|$ forms an angle. Find the measure of that angle rounded to the nearest minute. Write your answer in the form $d°m'$.

8. In the game of "seven come eleven" a person can only score 7 points or 11 points if (s)he makes a shot. Find the largest integer score that is unattainable regardless of the number of shots made.

9. Rounded to the nearest whole number of pounds, what force is necessary to pull a 9200 pound truck up a $4°$ incline? Disregard any effect of friction.

10. Let $i=\sqrt{-1}$ and let x and y be real numbers. If $216i+\log(x)-1=3^y i-27i$, find the value of $(x+y)$.

2011 RA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		Score (to be filled in by proctor)
1. 13		
2. $\frac{125}{2048}$	(Must be this reduced common fraction.)	
3. 4		
4. $(12,9)$	(Must be this ordered pair.)	
5. 0 OR zero		
6. 1054		
7. $53°8'$	(Must be in $d°m'$ form.)	
8. 59		
9. 642	(Pounds optional.)	
10. 15		

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 70 (Base ten optional.)

12. 2413 ($ optional.)

13. 98

14. 15

15. 5

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

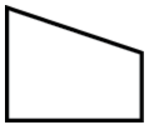
ICTM Regional 2011
Division A Oral

Symmetry and Patterns

1. What is the geometric mean of 2, 9 and 12?

2. Explain the difference in strip patterns described by *p1a1* and *p1m1*.

3. With the given figure and rigid motion for the pattern, find the 2nd figure in each of the following patterns:

Rigid Motion	1st figure	2nd figure
Reflection over a vertical line		
Rotation of 90° counterclockwise about the right angle		

4. What are the dimensions of a golden rectangle that has a perimeter of 12? Give the dimensions rounded to the nearest tenth.

5. A recursive sequence can be considered a Fibonacci-type sequence if $t_n = t_{n-1} + t_{n-2}$. In a certain Fibonacci-type sequence, the 4th term is 9 and the 7th term is 39. List the first 10 terms of this sequence.

ICTM Regional 2011
Division A Oral

Extemporaneous questions:

1. How many lines of symmetry are there for each of the following figures?

a. 

b. 

2. Explain why each of the following is or is not an isometry of a triangle.

   a. a clockwise rotation of 45° about the center of the triangle

   b. an expansion to double the size of the triangle

   c. a reflection along a vertical line outside the triangle

   d. a reflection along a vertical line through the triangle, but not through the center

   e. an decrease in the height of the triangle and increase in the base of the triangle to keep the same area.

3. The figure below is the symbol for the Nigerian currency, the naira. Can the figure be classified as either a cyclic rosette or dihedral rosette? If so, give the notation (*cn* or *dn*) for the rosette. If not, explain why not.

ICTM Regional 2011
Division A Oral **SOLUTIONS**

Symmetry and Patterns

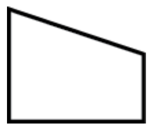
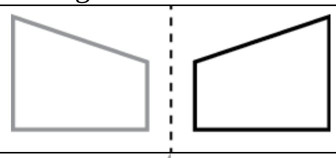
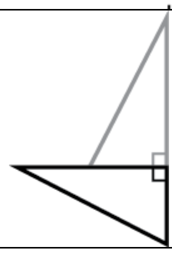
1. What is the geometric mean of 2, 9 and 12?

   The geometric mean of three numbers is the cube root of their product, so the geometric mean of 2, 9, and 12 is $\sqrt[3]{2 \bullet 9 \bullet 12} = \sqrt[3]{216} = 6$.

2. Explain the difference in strip patterns described by *p1a1* and *p1m1*.

   The pattern *p1m1* has a horizontal reflection line, whereas *p1a1* does not (it has a glide reflection). Both patterns do not have a vertical reflection line.

3. With the given figure and rigid motion for the pattern, find the 2nd figure in each of the following patterns:

Rigid Motion	1st figure	2nd figure
Reflection over a vertical line		
Rotation of 90° counterclockwise about the right angle		

4. What are the dimensions of a golden rectangle that has a perimeter of 12? Give the dimensions rounded to the nearest tenth.

   The lengths of the sides of a golden rectangle are in the ratio $1 : \frac{1+\sqrt{5}}{2}$, so let the length of the shorter side by x and the longer side be $\frac{1+\sqrt{5}}{2}x$. The perimeter is $2x + 2\frac{1+\sqrt{5}}{2}x = \left(3+\sqrt{5}\right)x = 12$, so $x = \frac{12}{3+\sqrt{5}} \approx 2.3$ and $\frac{1+\sqrt{5}}{2}x = \frac{12\left(1+\sqrt{5}\right)}{2\left(3+\sqrt{5}\right)} \approx 3.7$. The dimensions of the rectangle are 2.3 by 3.7.

5. A recursive sequence can be considered a Fibonacci-type sequence if $t_n = t_{n-1} + t_{n-2}$. In a certain Fibonacci-type sequence, the 4th term is 9 and the 7th term is 39. List the first 10 terms of this sequence.

The first and second terms are t_1 and t_2, respectively. The third term is $t_3 = t_1 + t_2$. The fourth term is $9 = t_4 = t_2 + t_3 = t_2 + (t_1 + t_2) = t_1 + 2t_2$. The seventh term is $39 = t_7 = t_5 + t_6 = t_5 + (t_4 + t_5) = t_4 + 2t_5 = t_4 + 2(t_3 + t_4) = 2t_3 + 3t_4$ $= 2(t_1 + t_2) + 3(9) = 2t_1 + 2t_2 + 27$, so $12 = 2t_1 + 2t_2$. Subtracting the equation for the fourth term from this gives $t_1 = 3$. Using either equation again to solve for t_2 gives $t_2 = 3$. So, the first 10 terms are 3, 3, 6, 9, 15, 24, 39, 63, 102, and 165.

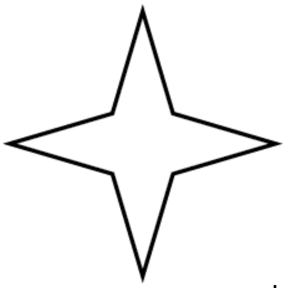
Students may have alternate methods of finding terms of the sequence, including noting that the given terms are 3 times the terms in the Fibonacci sequence.

Extemporaneous questions:

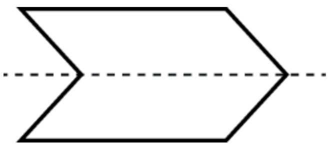
1. How many lines of symmetry are there for each of the following figures?

a.

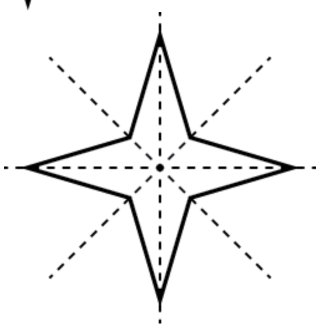
b.



a. 1



b. 4



2. Explain why each of the following is or is not an isometry of a triangle.

   a. a clockwise rotation of 45° about the center of the triangle
      It is an isometry; all rotations are isometries.

   b. an expansion to double the size of the triangle
      It is NOT an isometry; isometries preserve size.

   c. a reflection along a vertical line outside the triangle
      It is an isometry; all reflections are isometries.

   d. a reflection along a vertical line through the triangle, but not through the center
      It is an isometry; all reflections are isometries.

   e. an decrease in the height of the triangle and increase in the base of the triangle to keep the same area.
      It is NOT an isometry; isometries preserve lengths.

3. The figure below is the symbol for the Nigerian currency, the naira. Can the figure be classified as either a cyclic rosette or dihedral rosette? If so, give the notation (*cn* or *dn*) for the rosette. If not, explain why not.

The figure does not have reflection symmetry, so it cannot be a dihedral rosette. It does have 2-fold (180°) rotational symmetry, so it is a cyclic rosette, *c2*.