1. If $\frac{5}{3}x = 35$, find the value of x.

2. If $a = 3$, $b = 4$, and $c = 5$, find the value of $\frac{a^9 - b^6}{c^5}$. Express your answer as a reduced improper fraction.

3. Find the value of x such that $\frac{x-4}{2} - \frac{2x-7}{6} = 3$.

4. Find the largest number which can be obtained as the product of odd positive integers, not necessarily distinct but each greater than three, if their sum is 43.

5. One negative real root of the equation $x^2 + 18x + k = 0$ is twice the other negative real root. Find the value of k.

6. Speedwalker Tom gives Speedwalker Dick a 5 minute head start. Tom walks 600 feet in 40 seconds while Dick walks 360 feet in 30 seconds. How many minutes will it take Tom to catch Dick assuming that each walks at his given constant rate?

7. If x is a positive integer such that $x < 100$, find the value of x such that $x!(x+1)! = (x+4)!$

8. Find the 2011th digit to the right of the decimal point in the decimal representation of $\frac{1}{13}$.

9. In the addition cryptarithm shown to the right, each letter stands for the same digit throughout the puzzle. No digit is represented by more than one letter. For your answer write the 4-digit-number represented by $LACA$.

$$\begin{array}{cccc} & C & A & T \\ + & C & L & T \\ \hline L & A & C & A \end{array}$$

10. If $\frac{5^{(2x+3)}-5\left(25^x\right)}{125\left(5^{(2x-1)}\right)}=\frac{k}{w}$ where k and w are positive integers, find the least possible value of $(k+w)$.

11. Find the sum of the 4 consecutive multiples of three if, when in increasing order, 4 times the third integer is 30 less than 5 times the second integer.

12. For all real numbers a and b, $a\oplus b=\frac{a+b+7}{2}$. $2\oplus(4\oplus k)=0$. Find the value of k.

13. Let x and y represent positive integers such that $x+y=31$. Find the sum of all possible distinct values of x if $\sqrt{(184-15)(x-5)(y-1)}$ is a positive integer.

14. Let A, B, and C be constants for which the following equation holds for all admissible values of x: $\frac{x^2+1}{x^3-6x^2+11x-6}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$. Find the value of $(6A+7B+8C)$.

15. One marble is drawn at random from a bag containing 9 orange, 18 blue, and 12 green marbles. Find the probability the marble was blue. Express you answer as a common fraction reduced to lowest terms.

16. A lab tech opened the drain valve on a full vat of liquid and emptied it completely in four hours. After cleaning the vat, she turned on the main filler pipe, which would normally fill the empty vat completely in three hours, but she forgot to close the drain. An hour later, she turned on an auxiliary filler pipe, which fills at half the rate of the main pipe, so both were filling together, but still didn't notice the drain was open. Two hours after opening the auxiliary pipe, she returned, realized the drain was open and closed the drain valve. From this point, how many minutes does it take to completely fill the vat?

17. Find the tens digit of 3^{2011}.

18. Find the sum of all the distinct positive integral factors of 2009.

19. If the solution set for x of $|kx-3| = 402$ is $\{-\frac{133}{4}, \frac{135}{4}\}$, find the value of k.

20. Mia was rowing **upstream** one day when her cap blew off into the stream. She failed to notice it was missing until 20 minutes after it blew off. She immediately turned around and recovered the cap 2.4 miles **downstream** from where it initially blew off. Assume Mia's rowing rate in still water was constant, the rate of the current was constant, and that it took no time to turn around. Find the number of miles per hour in the rate of the current. Express your answer as an **exact decimal.**

2011 RAA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 21

2. $\frac{15587}{3125}$ (Must be this reduced improper fraction.)

3. 23

4. 300125

5. 72

6. 20 (Minutes optional.)

7. 6

8. 0

9. 1898

10. 29

11. 174

12. –29

13. 70

14. 11

15. $\frac{6}{13}$ (Must be this reduced common fraction.)

16. 50 (Minutes optional.)

17. 4

18. 2394

19. 12

20. 3.6 (Must be this decimal, mph optional.)

1. Two angles are complementary and one of the angles has measure $10°24'51''$. Find the measure of the other angle. Compute your answer in the form $k°w'p''$ (degrees, minutes, seconds) and *write as your answer the* *ordered triple* (k, w, p).

2. Points A, B, and C lie on circle O and divide the circle into 3 minor arcs with degree measures in the ratio $5:8:11$. Find the degree measure of the largest arc in this circle determined by any two of these three points.

3. A segment of length 60 is divided into 3 segments in the ratio $3:4:5$. Find the length of the longest segment.

4. One circle has equation $x^2 + y^2 - 16x + 4y + 43 = 0$. A second circle has equation $2x^2 + 2y^2 + 12y - 1 = 0$. Find the exact distance between the centers of the two circles.

5. The degree measures of the angles of a triangle are in the ratio $3:4:5$. If one of these angles is selected at random, find the probability that the angle selected is acute.

6. Four angles of a convex pentagon have degree measures $45°$, $88°$, $124°$, and $132°$. Find the degree measure of the fifth angle.

7. One leg of a right triangle has length 8. The altitude to the hypotenuse of this right triangle has length 5. Find the length of the other leg of this right triangle. Write your answer in simplified radical form.

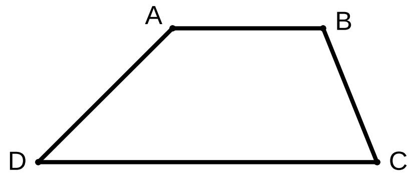
8. A chord of length 48 is drawn in a circle with diameter 52. Find the distance from the chord to the center of the circle.

9. A triangle whose sides have lengths of $(9+k)$, $(39+k)$, and $(48+k)$ has an area that is an integer. Find the smallest possible integer value of k.

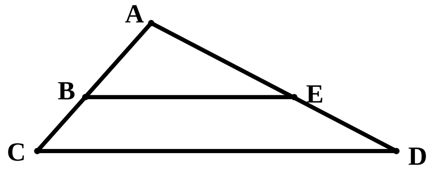
10. A rectangle has one side of length 56 and a diagonal of length 200. Find the area of this rectangle.

11. The diagonals of an isosceles trapezoid are each of length 25, the length of the altitude is 15, and the length of the lower base is 28. Find the perimeter of the trapezoid.

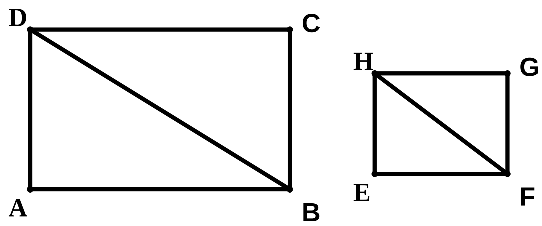
12. In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $AD = 12$, $AB = 10$, $BC = 9$, and $DC = 25$. Find the area of trapezoid $ABCD$.



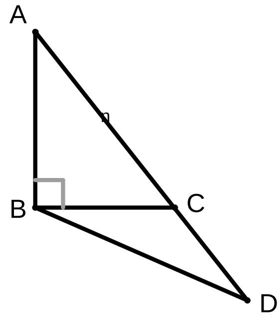
13. In the diagram, points A, B, and C are collinear, and points A, E, and D are collinear. $\overline{BE} \parallel \overline{CD}$. $AB = 10$, $BC = 4$, $AE = 12$, and $BE = 8$. Find CD. Express your answer as a decimal.



14. In the diagram, $ABCD$ and $EFGH$ are rectangles such that the lengths of all sides and diagonals are integers. The area of rectangle $ABCD$ is 385% of the area of Rectangle $EFGH$. If $BD = 65$, find HF.

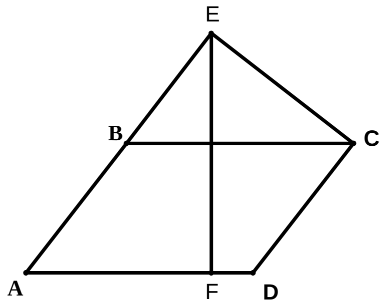


15. In the diagram, points A, C, and D are collinear. $AB = 15$, $BC = 8$, $\angle CBD = 45°$, and $\overline{AB} \perp \overline{CB}$. Find the length of $\overline{CD}$. Express your answer as an improper fraction reduced lowest terms.



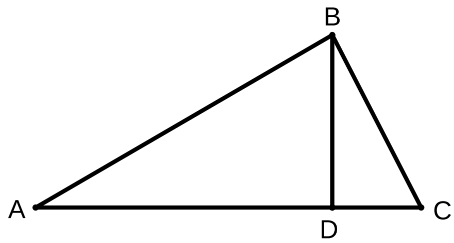
16. Two of the angles of a convex quadrilateral have respective degree measures of $151°$ and $42°$. One of the remaining angles of the quadrilateral has a degree measure that is 5 more than twice the degree measure of the other remaining angle of the quadrilateral. Find the degree measure of the larger of these two remaining angles.

17. In the diagram, $ABCD$ is a parallelogram, points A, B, and E are collinear, $\angle AEC = 90°$, F lies on $\overline{AD}$ such that $\overline{EF} \perp \overline{AD}$, $DC = 10$, $EC = 8$, and $\angle ECB = 30°$. Find the exact value of EF.



18. Find the exact area of the circle whose equation is $(x-8)^2 + y^2 + 10y + 13 = 0$.

19. In the diagram (not necessarily drawn to scale), $\angle ABC = 90°$, and D lies on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$. $AC = \frac{17}{15}$ and $AB = 1$. Then $BD = \frac{k}{w}$ where k and w are relatively prime positive integers. Find the value of $(2k + 3w)$.



20. In equilateral triangle ABC, the three altitudes are $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$, G is the trisection point of $\overline{CF}$ that is closer to C. H and J are the respective midpoints of $\overline{AD}$ and $\overline{BE}$. The area of ΔABC is $36\sqrt{3}$. The exact perimeter of ΔGHJ, in simplest radical form, is $k + p\sqrt{w}$. Find the value of $(k + w + p)$.

2011 RAA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $(79, 35, 9)$ (Must be this ordered triple.)

2. 285 (Degrees optional.)

3. 25

4. $\sqrt{65}$ (Must be this exact answer.)

5. 1 (Accept 100%)

6. 151 (Degrees optional.)

7. $\frac{40\sqrt{39}}{39}$ OR $\frac{40}{39}\sqrt{39}$ (Must be simplified radical form.)

8. 10

9. 52

10. 10752

11. 74

12. 126

13. 11.2 (Must be this decimal.)

14. 34

15. $\frac{136}{7}$ (Must be this reduced improper fraction.)

16. 113 (Degrees optional.)

17. $4+5\sqrt{3}$ OR $5\sqrt{3}+4$ (Must be this exact answer.)

18. 12π (Must be this exact answer.)

19. 67

20. 26

1. If $x^4 - x^2 - 20 = 0$, how many of the distinct roots are non-real?

2. Let $A = \left\{ \frac{1}{4}, \frac{2}{3}, -5, 12 \right\}$. If one of the members of A is selected at random and substituted for x, find the probability that $x > x^5$. Express your answer as a common fraction reduced to lowest terms.

3. Let x be a member of the set: $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$. Find the **sum** of all distinct values of x such that $\frac{x+1}{x} \le 1$.

4. A parabola has a vertical axis of symmetry and passes through the three points: $(2,5)$, $(-2,-3)$, and $(1,6)$. The equation of this parabola can be expressed in the form $y = ax^2 + bx + c$ where a, b, and c are integers. Find the value of $(a + 2b + 3c)$.

5. If $x < 0$, find the value of x such that $\left| 6 - (2x - 3) \right| = 19$.

6. Let k represent a negative integer. If the roots of $x^2 + 14x - k = 0$ are non-real, find the largest possible value of k.

7. Let $D = \{85, 83, 89, 72, 86, 95, 100, 88\}$. Find the absolute value of the difference between the arithmetic mean and the median. Express your answer as a **decimal**.

8. Reading from left to right, the first term in a row of Pascal's triangle is 1 and the second term is 5. Find the sum of the six terms in this row of Pascal's triangle.

9. Let a and b be real numbers. Let $x^3+b+8ax-ax^2=0$. Find the smallest possible value of the sum of the squares of the roots for x of the cubic equation.

10. If $x^2-10x+2=-y$ where x and y are real numbers, find the largest possible value of y.

11. Given the functions defined by $f=\{(2,7),(5,3),(7,11)\}$ and $g(x)=-x^2+1$. Find the value of $g(f(7))$.

12. Find the sum of all the distinct positive integral multiples of 11 if each multiple of 11 is less than 2010.

13. For all ordered pairs of positive integers (x,y):
    a) $f((x,1))=x$
    b) $f((x,y))=0$ if $y>x$
    c) $f((x+1,y))=2y[f(x,y-1)-f(x,y)]$
    Compute $f((6,6))$.

14. Let y be an integer such that $7<y<43$, and let nails have a regular price of y cents per 5. If nails were a whole number x cents per 5 less than the regular price of y cents per 5, one would pay $(7x+2)$ cents less for $10x$ nails than one would pay for $(7x+2)$ nails if they were $(x+2)$ cents per 5 more than the regular price of y cents per 5. Find the value of y.

15. Let $i=\sqrt{-1}$. If $(3i)(ki)=24$, find the value of k.

16. All points on the asymptotes of a certain hyperbola satisfy the equation of $x^2-4y^2=0$, and the point $(3,-1)$ lies on the hyperbola. If the equation of the hyperbola is expressed in the form $x^2+ky^2=w$, find the value of $(2k+5w)$.

17. Find the largest positive integer that divides 171, 237, and 395 with remainders R_1, R_2, and R_3, respectively, such that $R_2=R_1+1$ and $R_3=R_2+2$.

18. Find the sum of the first and last digits of 17^{2010}.

19. Let $i=\sqrt{-1}$. $\left|\frac{-2+3i}{1-i}\right|=\frac{\sqrt{k}}{w}$ where k and w represent positive integers. Find the smallest possible value of $(k+w)$.

20. On the Fairlong Math Team, there are 7 seniors, 9 juniors, 10 sophomores, and 6 freshmen. Each school day the Assistant Principal selects 1 member of the Math Team at random to read the announcements (the same person can be selected more than once.) Expressed as a decimal rounded to 4 significant digits, find the probability that on 6 consecutive school days there were 2 seniors, 2 juniors, 1 sophomore, and 1 freshman selected to read the announcements.

2011 RAA
Algebra II

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 2

2. $\frac{3}{4}$ (Must be this reduced common fraction.)

3. –6

4. 18

5. –5

6. –50

7. 0.25 OR .25 (Must be this decimal.)

8. 32

9. –64

10. 23

11. –120

12. 183183

13. 23040

14. 19

15. –8

16. 17

17. 13

18. 10

19. 28

20. 0.03992 OR .03992 OR 3.992×10^{-2} (Must be decimal, 4 sig. figs.)

1. Find the value of the following limit: $\lim_{x \to 5}\left(2^{(9-x)}\right)$.

2. A sequence is defined recursively by $a_{(n+1)} = 3a_n - 5$ for all integers $n \geq 1$. If $a_{10} = 16$, find the value of a_8.

3. Let $\vec{a}$ and $\vec{b}$ represent vectors such that $\vec{a} = (3, 2)$ and $\vec{b} = (-8, 13)$. Find the **ordered pair** that represents $3\vec{a} + 5\vec{b}$.

4. Find the value of the indicated sum: $\sum_{2}^{5}(k^3 + 3.782)$. Express your answer as an **exact decimal.**

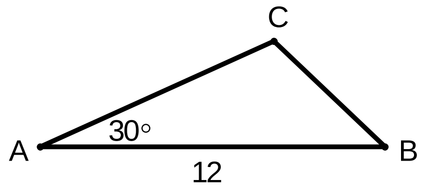
5. Find the tenth term of the geometric sequence whose first three terms are respectively: $\frac{1024}{5}$, $\frac{512}{15}$, and $\frac{256}{45}$. Express your answer as a common fraction reduced to lowest terms.

6. Given the relation defined by $f = \{(2,7),(5,3),(7,11),(19,w)\}$. Find the product of all the distinct real values of w for which f^{-1} is **not** a function.

7. Let k be a positive integer and let w be an integer such that two of the roots for x of the cubic equation $x^3 - 17x^2 + kx + w = 0$ are consecutive positive integers. Find the value of k if $|k - w|$ is a maximum.

8. The angle of elevation of the summit from the base of a ski lift is $35°$. If a skier rides 1000 feet on this lift to the summit, find the number of feet in the **vertical distance** between the base of the lift and the summit. Express your answer as a **decimal** rounded to the nearest tenth of a foot.

9. In the diagram, $\angle CAB = 30°$, and $AB = 12$. $AC = k\sqrt{3}$, and $BC = w$ where k and w are positive integers, and $w < 164$. Find the sum of all possible distinct values of w.



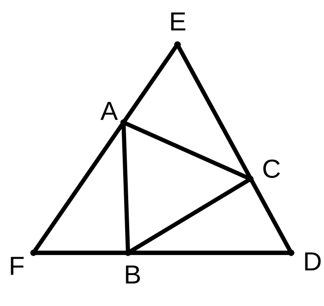
10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -3 \end{bmatrix} \bullet \begin{bmatrix} 5 & 4 \\ 1 & 7 \end{bmatrix}$, find the value of $\det(A)$.

11. If $(3x - 4y + z + 2a)^2$ is expanded and completely simplified, find the sum of the numerical coefficients.

12. Let x be an integer such that $0 < x < 150$. Find the sum of all possible distinct values of x such that all three of the following conditions hold: $\tan(3x + 17)° < 0$, $\sin(5x - 13)° > 0$, and $\cos(2x + 3)° < 0$.

13. The area of the ellipse $\frac{(x-3)^2}{36} + \frac{(y+2)^2}{k} = 1$ is 18 times the area of the circle $x^2 + y^2 = 4$. Find the value of k.

14. In the diagram, ΔDEF is equilateral, points A, B, and C lie on $\overline{EF}$, $\overline{DF}$, and $\overline{DE}$ respectively. $\overline{EA} \cong \overline{FB} \cong \overline{DC}$. The ratio of the area of ΔABC to the area of ΔDEF is $53:128$. If $EF = 256$, and $AE < AF$, find AF. Express your answer as a decimal rounded to the nearest tenth.



15. Given that $\sin(A) = \frac{2}{5}$ and $\frac{\pi}{2} < A < \pi$ (where A is measured in radians). Then $\sin(2A)$ can be expressed in simplest radical form as $\frac{k\sqrt{w}}{p}$ where k and p are relatively prime integers and $p > 0$. Find the value of $(k + w + p)$.

16. From a group of 8 men and 7 women, a committee of 3 is selected at random. Find the probability that the committee consists of 2 members of one sex and 1 member of the opposite sex.

17. If $\sin(x) = \frac{5}{13}$ and $\cot(y) = -\frac{3}{4}$, then there are two possible answers for $\tan(x - y)$. Find the sum of these two possible distinct answers. Express your answer as an improper fraction reduced to lowest terms.

18. Let $C(n,k) = \frac{n!}{k!(n-k)!}$ where n and k represent positive integers. Find the **sum** of all distinct values of n such that $97 < C(n,6) < 2114$.

19. The graph of the rational function $g(x) = \frac{x^2 - 5x + 6}{x - 4}$ has a slant (oblique) asymptote which can be expressed in the form: $f(x) = mx + b$. Find the value of $(m + b)$.

20. Let x and n represent two digit positive **odd** integers for which the tens digit is non-zero and for which $n > x$. Find the largest possible value for x for which the sum of the consecutive positive **odd** integers from 1 through x is 99 less than the sum of the consecutive positive **odd** integers from x through n.

2011 RAA

Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 16

2. 4

3. $(-31, 71)$ (Must be this ordered pair.)

4. 239.128 (Must be this exact decimal.)

5. $\frac{2}{98415}$ (Must be this reduced common fraction.)

6. 231

7. 26

8. 573.6 (Must be this decimal, feet optional.)

9. 216

10. –279

11. 4

12. 2535

13. 144

14. 187.9 (Must be this decimal.)

15. 42

16. 0.8 OR .8 OR $\frac{4}{5}$

17. $\frac{507}{112}$ (Must be this reduced improper fraction.)

18. 46

19. 0 OR “zero”

20. 43

NO CALCULATORS

1. Consider the ordered sequence of numbers 1, 2, 4, 10, 19, 32, 58, 96. Write as your answer the largest number in this sequence that is greater than the sum of all the numbers in the sequence that precede it.

2. Twelve times the sum of a number and 12 is equal to 96. Find the number.

3. If $(x^4+13)^2(x-8)^3$ is expressed as a polynomial in x, find the degree of that polynomial.

4. Find the sum of all solution(s) for x such that $(2x-3)^2-5(2x-3)=6(2x-3)$.

5. The ratio of the complement of an angle to the supplement of that angle is $11:26$. Find the measure of the angle.

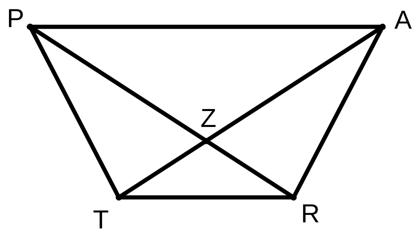
6. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   A bag contains exactly 12 marbles—some of them are orange, and the rest are blue. Which of the following could **not** possibly be the ratio of orange marbles to blue marbles?

   A) 1:5 B) 1:12 C) 2:1 D) 1:3 E) 5:7 F) 1:1

   Note: Be certain to write the correct capital letter as your answer.

7. In isosceles trapezoid $TRAP$, $TP = RA$. The diagonals of the trapezoid intersect at point Z. If $\angle ATR = (2x+4)^\circ$, $\angle PRA = (9x+23)^\circ$, and $\angle APR = (5x-17)^\circ$, find the degree measure of $\angle PZA$.



8. $\dfrac{\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}} = \dfrac{a+b\sqrt{2}+c\sqrt{3}+d\sqrt{6}}{e}$ where a, b, c, d, and e are positive integers. Find the smallest possible value of $(a+b+c+d+e)$.

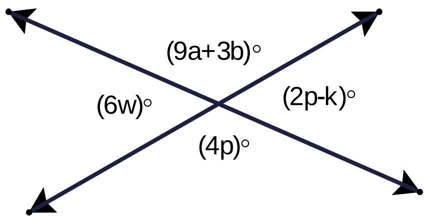
NO CALCULATORS

9. Find the exact value of $\sqrt{(350-26)(550+26)}$.

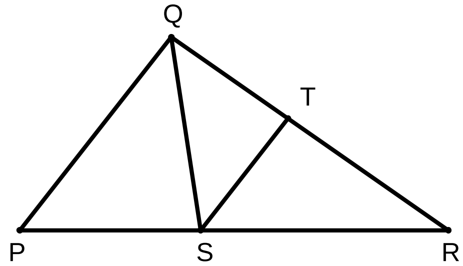
10. Find the area of a triangle whose sides have lengths of $\sqrt{13}$, $\sqrt{29}$ and $\sqrt{34}$.

11. The degree measures of three angles of a parallelogram are represented by $x+44$, $2x+20$, and $5x-8$. Find the degree measure of one of the larger angles of the parallelogram.

12. In the diagram below consisting of two intersecting straight lines with angle measures as shown, let a, b, k, p, and w represent positive integers. Find the largest possible value of $(a+b+k+p+w)$.



13. In the diagram, $\overline{PQ} \parallel \overline{ST}$, $\angle PQS \cong \angle RQS$, $\angle QPR = 72°$, and $\angle QRP = 64°$. Points P, S, and R are collinear, and points Q, T, and R are collinear. Find the degree measure of $\angle QST$.



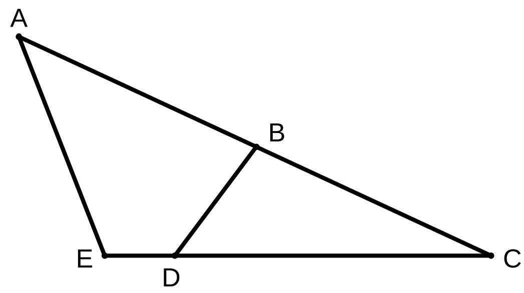
14. x varies inversely with y and directly with z. If $x=8$ and $y=12$ when $z=16$, find y when $x=9$ and $z=36$.

15. Christie biked for a mile at a constant rate of 12 miles per hour and returned on her bike one mile at a faster constant rate of 15 miles per hour. Find the number of minutes in Christie's round trip.

NO CALCULATORS

16. Let k and w be two **relatively prime positive integers** with $k < w$ such that $1 < k < 5$ and $3 < w < 8$. Let x and y be positive integers such that $\frac{3x+4y}{10x-3y} = \frac{k}{w}$. If $x < 42$, find the sum of all distinct possible values of x.

17. In the diagram, B lies on $\overline{AC}$, and D lies on $\overline{EC}$. If $\frac{AB}{BC} = \frac{5}{7}$ and $\frac{ED}{EC} = \frac{2}{7}$, then the ratio of the area of ΔBDC to the area of quadrilateral $ABDE$ can be expressed in the form of $k:w$ where k and w are positive integers. Write $k:w$ as a reduced common fraction.



18. A leaky faucet drips 12 times per minute (at a constant rate). If each drip is one-fourth of an ounce, how many gallons of water are wasted each day? Express your answer as an improper fraction.

19. Assume that Tom and Kay are playing a game with matchsticks. Three separate piles of matchsticks are formed. Tom and Kay have alternate turns. At each turn, the person must remove either 1 or 2 matchsticks from any **one** of the remaining piles. The winner is the person who takes the last matchstick. Suppose that the game gets to the point where it is Kay's turn to remove matchsticks, and there are 4 matchsticks in one pile and 1 matchstick in each of the other two piles. If Kay plays perfectly, she can win by taking one matchstick from the big pile leaving the ordered triple $(3,1,1)$, or she could also win by taking 1 matchstick from either of the single piles leaving the ordered triple $(4,1,0)$. For the $(4,1,0)$ ordered triple, there would only be two remaining piles. At the present time, it is Kay's turn to remove matchsticks. There are 5 matchsticks in one pile, 3 matchsticks in the second pile, and 3 matchsticks in the third pile. If Kay plays perfectly, list any and all ordered triples she should leave immediately after her present turn. List any and all answers as **ordered triples** of the form (a,b,c) where $a \ge b \ge c$.

20. In the system $\begin{cases} x+y+2z=45 \\ 3x-y+z=8 \\ kx-2y+5z=63 \end{cases}$, x, y, z, and k all represent positive integers. Find the ordered quadruple solution (x,y,z,k) and express your answer as that ordered quadruple.

NO CALCULATORS

2011 RAA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 19

2. -4

3. 11

4. $\frac{17}{2}$ OR $8\frac{1}{2}$ OR 8.5

5. 24 (Degrees optional.)

6. B (Must be this capital letter.)

7. 144 (Degrees optional.)

8. 16

9. 432

10. $\frac{19}{2}$ OR $9\frac{1}{2}$ OR 9.5

11. 112 (Degrees optional.)

12. 170

13. 22 (Degrees optional.)

14. 24

15. 9 (Minutes optional.)

16. 189

17. $\frac{5}{7}$ (Must be this reduced common fraction.)

18. $\frac{135}{4}$ (Must be this reduced improper fraction.)

19. $(5,3,2),(3,3,3)$ (Must be these two ordered triples, either order.)

20. $(2,13,15,7)$ (Must be this ordered quadruple.)

NO CALCULATORS

1. Write the whole word **Yes or No** to answer the question. Given the theorem: If a triangle is isosceles, then at least two sides of that triangle are congruent. Is the contrapositive of this theorem a true statement?

2. How many distinct numbers are in the **domain** of the relation: $\{(5,2),(4,0),(6,2),(854,2)(6,7),(4,2)\}$?

3. Find the value of k if the graph of $f(x)=\dfrac{2kx-3}{5x+k}$ has an x-intercept of 24. Express your answer as a common fraction reduced to lowest terms.

4. One of the vertices of the hyperbola whose equation is $\dfrac{(x-2)^2}{9} - \dfrac{(y+1)^2}{16} = 1$ is located at the point (x,y) where $x>y$. Find the **ordered pair** that represents this vertex.

5. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   For how many real values for x does $|2x|^2 = 4x^2$?

   A) 0 B) 1 C) 2 D) 3 E) more than 3 but finite F) infinite

   Note: Be certain to write the correct capital letter as your answer.

6. If $\log_7 3 = P$, $\log_7 8 = Q$, and $\log_7 10 = T$, find the value of $\log_7\left(\dfrac{15}{32}\right)$ in terms of P, Q, and T. Use capital letters when writing your answer.

7. Given that $f=\{(2,3),(0,4),(4,5),(7,5)\}$ and $g=\{(5,7),(2,4),(4,2)\}$. Find the value of $g(f(4))+f(f(0))$.

NO CALCULATORS

8. Nine women have different first names, and two of these women are named Peggy and Sue. If these nine women stand in a straight line, find the probability that there are two or more women between Peggy and Sue. Express your answer as a common fraction reduced to lowest terms.

9. Let $D = \{21, 4, 11, x\}$. If $x > 0$, and if the median of D is an integer one more than the arithmetic mean of D, find the value of x.

10. The equation of a parabola is $2y - 8 = (x+5)^2$. The vertex of this parabola is located at (h, k). Find the **ordered pair** (h, k).

11. A rectangular hyperbola has the equation $xy = -128$. One of the vertices of this rectangular hyperbola is located at (x, y) where $x > y$. Find this vertex. Express your answer as an **ordered pair** of the form (x, y) with exact value entries.

12. Find the value of $\left(\sqrt{16+6\sqrt{7}} - \sqrt{27+10\sqrt{2}} + \sqrt{6+\sqrt{32}}\right)^2$

13. Find the largest possible length of the hypotenuse of any right triangle that exists such that the length of one side is 12 and the sine of one of the angles is $\frac{1}{2}$.

14. Find the value of a so that the following four terms taken in that order form an arithmetic sequence: 7, $x+3y$, $2x+5y$, $3x+ay$.

NO CALCULATORS

15. Let a, b, and c be positive integers. If the graph of $y=\frac{2x^2+7x-4}{(ax+b)(x-c)}$ has a horizontal asymptote of $y=2$, and the only vertical asymptote of the graph is $x=3$, find the value of $(ab+c)$.

16. Knowing $\sin(18°)=\frac{\sqrt{5}-1}{4}$, the exact value of $\sin(12°)$ can be expressed as $\frac{\sqrt{k+w\sqrt{f}}-\sqrt{p}+\sqrt{g}}{8}$ where k, w, f, p and g are positive integers. Find the smallest possible value of $(k+w+f+p+g)$.

17. Let $i=\sqrt{-1}$. If $k=\frac{1+i\sqrt{3}}{2}$, find the value of $k+k^2+k^3+k^4+k^5$.

18. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

Let A be the set of all integral multiples of 2; let B be the set of all integral multiples of 3: let C be the set of all integral multiples of 11. Which of the following contains 88? [Note: A' is the complement of A.]

A) $A\cap B$ B) $B\cap C$ C) $A\cap C$ D) $A'\cup B$ E) $B\cup C'$ F) $A'\cup C'$

Note: Be certain to write the correct capital letter as your answer.

19. Let $i=\sqrt{-1}$. For some real number k, the polynomial $4x^3-12x^2+9x+48ix-36i-16ix^2$ is divisible by $(x-k)^2$. Find the value of k.

20. In ΔABC, $AB=17$, $\sin(\angle CAB)=\frac{8}{17}$, the area of ΔABC is 36, and $AB>BC$. Find $\sin(\angle CBA)$. Express your answer as a common fraction reduced to lowest terms.

2011 RAA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. YES (Must be the whole word.)

2. 4

3. $\frac{1}{16}$ (Must be this reduced common fraction.)

4. $(5,-1)$ (Must be this ordered pair.)

5. F (Must be this capital letter.)

6. $T+P-2Q$ (Or equivalent commuted-term answer, must be capital letters.)

7. 12

8. $\frac{7}{12}$ (Must be this reduced common fraction.)

9. 24

10. $(-5,4)$ (Must be this ordered pair.)

11. $\left(8\sqrt{2},-8\sqrt{2}\right)$ (Must be this ordered pair with exact entries.)

12. 7

13. 24

14. 7

15. 7

16. 35

17. -1

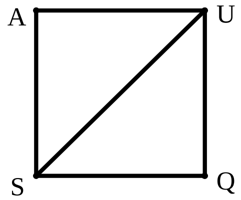
18. C (Must be this capital letter.)

19. $\frac{3}{2}$ OR $1\frac{1}{2}$ OR 1.5

20. $\frac{36}{85}$ (Must be this reduced common fraction.)

1. If 72.6% of x is 384, what is 36.3% of x?

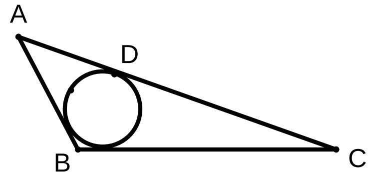
2. In square SQUA, $SU = 1.63x - y$, $m\angle ASU = (3x + y)^\circ$, and $m\angle AUS = (2x - 3y)^\circ$. Find the perimeter of square SQUA.



3. If $3.112^x = 7.143$, find the value of x.

4. Alpha Centauri is 4.3 light years away. A light year is the distance light travels at 186,000 miles per second in the $365\frac{1}{4}$ days of a year. Using these approximations and assuming constant rates, how long would it take a space shuttle traveling 1,000,000 miles per hour to reach Alpha Centauri? Round your answer to the nearest whole year.

5. In the diagram, the circle is inscribed in ΔABC, and the point of tangency with $\overline{AC}$ is D. If $AB = 23.14$, $BC = 55.26$, and $AC = 63.98$, find AD.



6. Let $1^\circ < x^\circ < 177^\circ$. Find the value of x° such that $4\cos(2x^\circ) + 3\cos(x^\circ) = 1$. Express your answer in the form $k^\circ w'$ (degrees and minutes with the minutes rounded to the nearest minute.)

7. If the length of a side of a regular heptagon is x, then the radius of the inscribed circle of the regular heptagon has a length of kx. Find the value of k.

8. The exact value of one of the terms of the Fibonacci sequence whose first 7 terms are 1, 1, 2, 3, 5, 8, 13 is 956722026041. Find the number of the term in the Fibonacci sequence whose value is 956722026041. (For example, 13 is term number 7.) Express your answer as an **integer**.

9. $\sum_{n=1}^{100}\left[\left(\sin\left(2n\right)^{\circ}\right)\left(\cos n^{\circ}\right)\right]=k\left(\sum_{n=1}^{100}\left[\left(\sin n^{\circ}\right)\left(\cos n^{\circ}\right)\right]\right)$. Find the value of k.

10. Point P is in the interior of rectangle $ABCD$. $PA=94.13$, $PB=198.5$, and $PC=272.4$. Find PD.

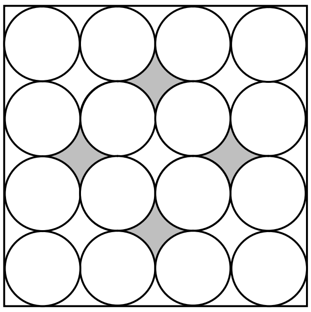
11. Members of the Math Club purchase boxes of candy for \$1.43 per box and sell them for \$3.00 per box. Assuming no other expenses, find the minimum whole number of boxes of candy they must sell to earn a net profit of at least \$500. Do **not** use scientific notation.

12. MTCI Corporation has a long term lease for its headquarters building. In year one, they pay \$1200. Each year, the price increases by 3% of the price for the previous year. Find the average payment per year for the first 48 years of the lease. Express your answer in **dollars** with the dollars rounded to the nearest hundred dollars. Do **not** use scientific notation.

13. If "ln" refers to natural logarithm, find the value of: $\ln(1)+\ln(2)+\ln(3)+\cdots+\ln(39)$.

14. A fair 8-sided die, with each face marked with distinct digits 1 through 8, is tossed 6 times and the resulting digit of the top face recorded. Find the probability that at least two of the recorded digits are the same.

15. In the diagram, there are 16 congruent circles. The circles are tangent to other circles and to the square as shown. If the area of the square is 400.0, find the area of the shaded region. NOTE: There are 4 shaded regions in the diagram.

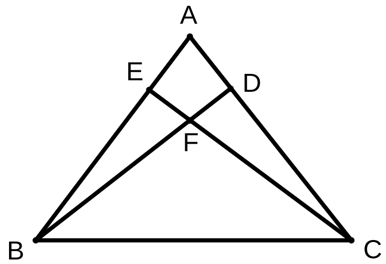


16. A regular nonagon (9-sided) with perimeter 90 is inscribed in a circle. The area of the nonagon is $k\%$ of the area of the circle. Find the value of k.

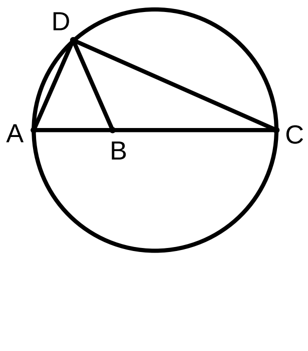
17. Let $A=\{1,3,5,7,9,11,13,15,17\}$. Let $B=\{1,2,3,4,5,6,7,8,9,10,11,12\}$. Let k be an element of A. From k^2, we subtract 1; from that result, we subtract 2; from that result, we subtract 3; and so on. (The quantities being subtracted continue to increase by 1.) This process continues until we reach results that are the squares of members of B or until the result is less than zero. During the process, if a result is reached that is the square of a member of B, that member of B is scratched out. Assume that once a square of a member of B is reached, the process continues until the result is less than zero. After all distinct members of A have been substituted for k and the process has been completed for all these substitutions, find the sum of all distinct members of B that have **not** been scratched out. Express your answer as an **integer**.

18. Christy bought a truckload of cantaloupe for \$405.81, and Marnie bought a truckload of cantaloupe for \$379.42. There were 889 cantaloupes in Christy's truckload, and there were 881 cantaloupes in Marnie's truckload. The average number of **cents** that Christy paid per cantaloupe was how much more than the average number of **cents** that Marnie paid per cantaloupe? Give your answer accurate to 4 significant digits.

19. In equilateral ΔABC, $\overline{BD}$ and $\overline{CE}$ meet at F. $BF=CF$, and $\angle BFC=89.04°$. E lies on $\overline{AB}$, and D lies on $\overline{AC}$. If $BC=74.58$, find the area of quadrilateral $ADFE$.



20. In the diagram, points A, D, and C lie on the circle whose radius has a length of 26.5. Point B lies on diameter $\overline{AC}$. $AD=28$, and the area of $\Delta DBC=344.7$. Find BD.



2011 RAA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 192 OR 192.0 OR 1.920×10^2

2. 87.01 OR 8.701×10 OR 8.701×10^1

3. 1.732 OR 1.732×10^0

4. 2879 (Must be this whole number, years optional.)

5. 15.93 OR 1.593×10 OR 1.593×10^1

6. $51°19'$ (Must be in degree-minute form.)

7. 1.038 OR 1.038×10^0

8. 59 (Must be this integer.)

9. 1.387 OR 1.387×10^0

10. 208.9 OR 2.089×10^2

11. 319 (Must be this whole number.)

12. 2600 (Must be this integer, dollars optional.)

13. 106.6 OR 1.066×10^2

14. 0.9231 OR .9231 OR 9.231×10^{-1}

15. 21.46 OR 2.146×10 OR 2.146×10^1

16. 92.07 OR 9.207×10 OR 9.207×10^1

17. 40 (Must be this integer.)

18. 2.581 OR 2.581×10^0 (Cents optional.)

19. 258.7 OR 2.587×10^2

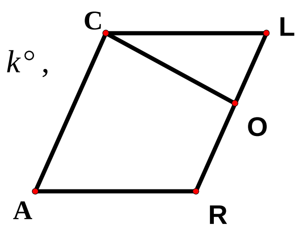
20. 25.49 OR 2.549×10 OR 2.549×10^1

1. A trapezoid has bases of respective lengths 34 and k, a height of 12, and an area of 456. Let w and p be the smallest and largest possible perimeters respectively of a triangle that has all integral side-lengths if two sides have respective lengths 14 and 16. Find the value of $(k+w+p)$.

2. If the degree measure of an interior angle of a regular polygon is 20 more than 3 times the degree measure of an exterior angle of that polygon, find the number of sides of the polygon.

3. Find the absolute value of the difference between the two real values of x such that $5^{(x^2-13x+12)}=1$.

4. $CARL$ is a parallelogram with $\overline{CO} \cong \overline{CL}$. If $m\angle R = 137°$, $m\angle ACO = k°$, $CL = 2k+9$, and $AR = 3k-34$, find the length of $\overline{CO}$.



5. $|3x-5|<16$. Find the sum of all integral values of x that are members of the solution set.

6. Let k be the largest positive integer less than 1000 that has exactly 4 distinct prime factors. Let w be the length of the altitude to the hypotenuse of an isosceles right triangle whose hypotenuse has a length of 12. Find the value of $(k+w)$.

7. Let k be the length of the shortest altitude of an isosceles triangle whose sides have lengths of 73, 73, and 110. Let w be the value of x such that $7(x-14.68)-(15.1-3x)=2442.14$. Find the value of $(k+w)$

8. Jeremy invests part of \$50,000 at 5% annual percentage rate of interest and the rest of his \$50,000 at 6% annual percentage rate of interest. If the interest that Jeremy earned in one year from these two investments was a total of \$2782, find the number of dollars that Jeremy invested at 5%.

9. The point $P(3.2, 2.4)$ lies on exactly k of the lines: $y=\frac{1}{2}x+4$, $6.3x+3.5y=28.56$, $2.28y+8.608=4.4x$, and $415x+260y=1952$. Point P also lies on the line $y=kx+w$. Find the sum $(k+w)$. Write your answer as an exact decimal.

10. $(2x-9)$ is a factor of $6x^2-kx+72$ when factored over the set of integers. w is the remainder when $x^3-2x^2-17x+k$ is divided by $(x-5)$. Find the value of w.

2011 RAA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		Score (to be filled in by proctor)
1. 134		______
2. 9		______
3. 11		______
4. 95		______
5. 15		______
6. 996		______
7. 304		______
8. 21800	($ optional.)	______
9. –4.2	(Must be this exact decimal.)	______
10. 33		______

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. 41

12. $-2\sqrt{26}$ (Must be this exact answer.)

13. 3

14. 81

15. 12

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let B represent the largest value of y such that $y=-|x+6|+3$ where x is a real number. Let A represent the smallest product of two real numbers whose difference is 6. Find the value of $(A+B)$.

2. In base x, $123_x = 227_{ten}$; in base y, $147_y = 628_{ten}$. Express the value of $(x+y)$ as a base ten numeral if both x and y are positive integers.

3. $9x^4 - Ax^3 - Bx^2 + Cx + 48 = 0$, where A, B, and C are positive integers. Let k be the ***number*** of possible distinct rational roots (using the Rational Root Theorem to generate the list). Let w be the largest possible number of negative roots. Find the product (kw).

4. $ABCD$ is a rectangle with $AB = 45$ and $BC = k$. Let $i=\sqrt{-1}$. If $|56+ki| = 65$, find the perimeter of the rectangle.

5. Find the sum of all distinct values of θ between 0° and 360° for which the following fraction is undefined: $\dfrac{\cos(\theta)}{\left(\sin(\theta)-\frac{1}{2}\right)\left(\tan(\theta)-1\right)}$.

6. Let k be the number of distinct positive integers that leave a remainder of 23 when divided into 1904. Let S be the sum of the first eleven terms of the geometric progression whose first term is 1 and whose fourth term is 8. Find the value of $(k+S)$.

7. The equations of the two vertical **asymptotes** of $y=\dfrac{x-4}{x^3-5x^2-52x+224}$ are $x=a$ and $x=b$. Let $(1,k)$ and $(w,1)$ be points on the graph in the (x,y) plane defined, parametrically by $\begin{cases} x = t-5 \\ y = 8t-23 \end{cases}$. Find the value of $(a+b+k+w)$.

8. One of the transformations needed to produce the graph of $y=-5x^2-120x-42$ from the graph of $y=x^2$ is a vertical shift k units upward. If $\log_2\left(\log_2\left(\log_2\left(\log_2\left(\log_2(w)\right)\right)\right)\right)=0$, find the integral value of w. For your answer, write the value of $(k+w)$.

9. Find the value of $\frac{x-3}{2x}+\frac{x}{2x+6}+\frac{7}{x+3}+\frac{9}{2x^2+6x}$ when $x=20.11$. Write your answer as a decimal rounded to four significant digits.

10. $\begin{bmatrix} k & 5 & 12 \\ 4 & -2 & 10 \end{bmatrix} + \begin{bmatrix} 14 & -3 & 7 \\ w & 6 & -2 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. If $ae-2cd=-310$ and if $2d-3a=-112$, find the value of $(k+w)$.

2011 RAA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. –6	
2. 37	
3. 80	
4. 156	
5. 810 (Degrees optional.)	
6. 2054	
7. 24	
8. 66214	
9. 1.173 (Must be this decimal.)	
10. 41	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 70 (Base ten optional.)

12. 2413 ($ optional.)

13. 98

14. 15

15. 5

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

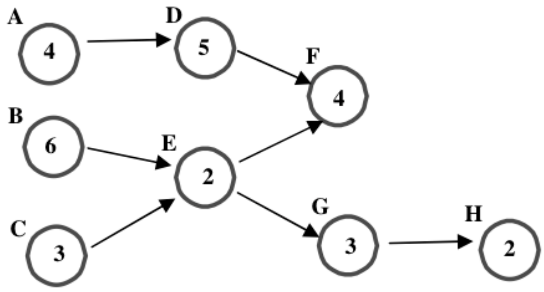
PLUS: 2 point bonus for being first In round with correct answer

ICTM Regional 2011
Division AA Oral

Planning, Scheduling and Linear Programming

1. Shelves are designed so that each can hold a maximum of 50 pounds. Given boxes with weights: 25, 20, 20, 20, 10, 10, 10, 8, 8, 5, 5, 5, compare the minimum number of shelves needed when using NFD (next fit decreasing) and FFD (first fit decreasing) bin-packing.

2. Below is an order-requirement digraph with the times for operations A through H.



   a. Using critical-path scheduling, determine a priority list for this digraph.

   b. Using the priority list from part a, determine the schedule produced by using the list-processing algorithm for 2 machines for this digraph.

   c. Are there any possible schedules with no idle time? Why or why not?

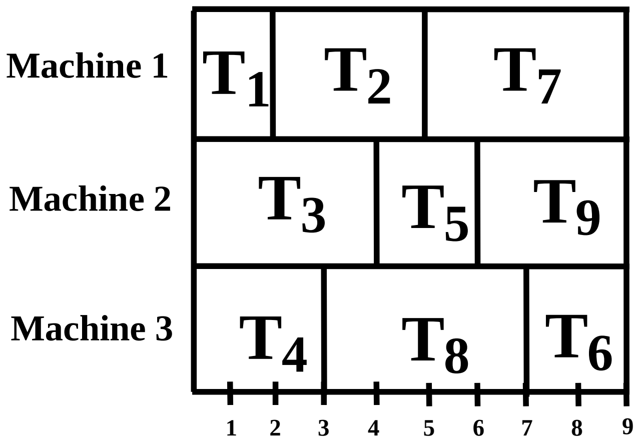
3. Yearbook pictures for 6 different clubs are being scheduled after school on Wednesday. Clubs are A, B, C, D, E, and F. Many students are members of more than one club. Clubs with students in common are A and B, A and D, A and F, B and C, B and D, B and E, C and E, D and E, D and F, and E and F.

   Display this information in a graph with edges. What is the minimum number of time slots that need to be scheduled to take all 6 club pictures? How many different photo set-ups are necessary?

ICTM Regional 2011
Division AA Oral

Extemporaneous questions:

1. Give the priority list with task times for the schedule shown below.



2. Given independent tasks with times 12, 8, 6, 5 and 3, use the decreasing-time algorithm to schedule these tasks on two processors.

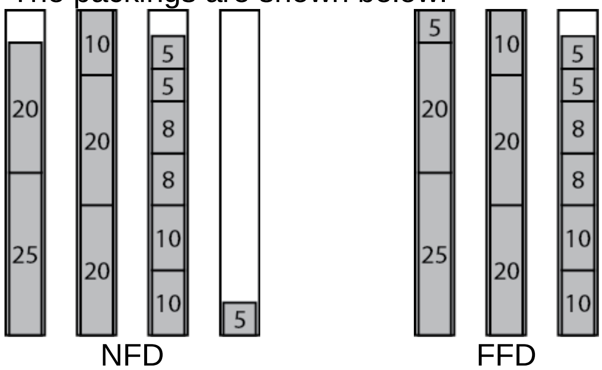
   What is the completion time for the schedule?

   Is this time optimal? Explain why or why not.

ICTM Regional 2011
Division AA Oral **SOLUTIONS**

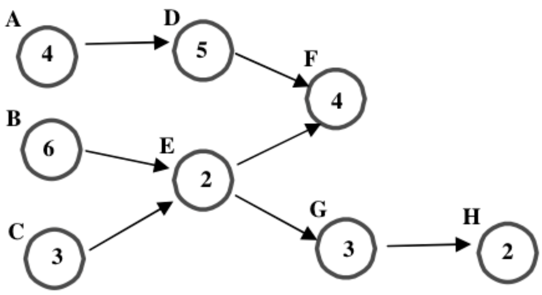
Planning, Scheduling and Linear Programming

1. Shelves are designed so that each can hold a maximum of 50 pounds. Given boxes with weights: 25, 20, 20, 20, 10, 10, 10, 8, 8, 5, 5, 5, compare the minimum number of shelves needed when using NFD (next fit decreasing) and FFD (first fit decreasing) bin-packing.
In NFD, a box is placed on the current shelf if it fits, and a new shelf otherwise. In FFD, a box is placed on the lowest shelf on which it fits, or a new shelf otherwise. The packings are shown below.



The two packings proceed identically until the boxes of weight 5. FFD requires 3 shelves (first box with weight 5 can fit in the empty space on the first shelf); NFD requires 4 shelves (last box doesn't fit on the third shelf, so must go on a fourth shelf).

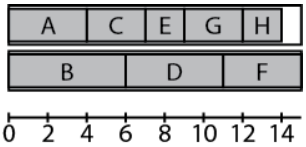
2. Below is an order-requirement digraph with the times for operations A through H.



a. Using critical-path scheduling, determine a priority list for this digraph.
The paths A-D-F and B-E-G-H are tied for the longest path, at 13, so choose the first-in-order of those two heads as the highest priority: A. Now, removing A, B-E-G-H is the longest path, so the next priority is B. Removing B, the longest path is a tie between D-F and C-E-G-H, at 9, so choosing the first-in-order, C is the next priority. Removing C, the longest path is D-F, so D is the next priority. E is now the head of all paths, so E is the next priority. G-H is now the longest path, so G is next. F is longer that H, so F gets higher priority, and H is last. The priority list is:
A, B, C, D, E, G, F, H

b. Using the priority list from part a, determine the schedule produced by using the list-processing algorithm for 2 machines for this digraph.

Place each task from the priority list onto the first available machine. The schedule is as shown below.



c. Are there any possible schedules with no idle time? Why or why not?

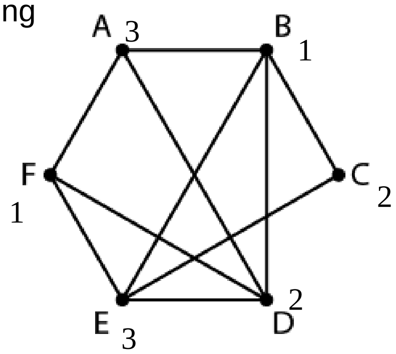
It is not possible to have a schedule with no idle time. The total time for all the tasks is odd and each task time is a whole number, so the time cannot be evenly split between two machines.

3. Yearbook pictures for 6 different clubs are being scheduled after school on Wednesday. Clubs are A, B, C, D, E, and F. Many students are members of more than one club. Clubs with students in common are A and B, A and D, A and F, B and C, B and D, B and E, C and E, D and E, D and F, and E and F.

Display this information in a graph with edges. What is the minimum number of time slots that need to be scheduled to take all 6 club pictures? How many different photo locations are necessary?

Students may (but do not need to) label the graph using numbers to represent different time slots (colors). One possible numbering is given in the graph. (Note: number 1 can be B/F or D/C or E/A; likewise for numbers 2 and 3)

(This an example of a possible graph; others are possible with edges connecting the same vertices)



Vertices B, D, and E all have valence 4 and no vertex has higher valence, so it is possible to schedule all the photos, avoiding conflicts, with at most 4 time slots. Since B, D, and E each have valence 4, for each of them, there is only one club that could be scheduled at the same time, so at most 2 locations can be used during the time slots when B, D, and E are being shot. Starting with 2 locations and 3 time slots, place B, D, and E in the first location in three different time slots. Only F can be shot at the same time as B; only C can be shot at the same time as D; only A can be shot at the same time as E. So, the schedule is:

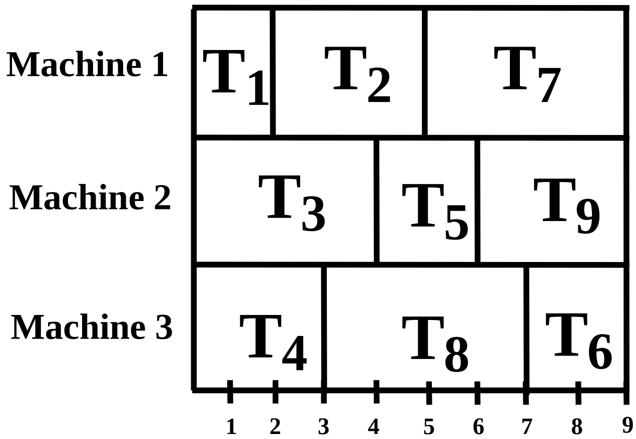
Time Slot	1	2	3
Location 1	B	D	E
Location 2	F	C	A

(Note: Time Slot 1 can be B/F or D/C or E/A; likewise for Time Slots 2 and 3)

ICTM Regional 2011
Division AA Oral **SOLUTIONS**

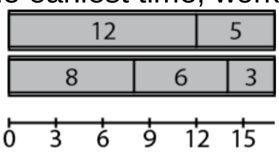
Extemporaneous questions:

1. Give the priority list with task times for the schedule shown below.



The priorities can be read by looking at earliest starting times and, for tasks starting at the same time, lower-numbered machines (higher in the chart). The times are the widths of the task boxes. The priorities (with times in parentheses) are: T_1(2), T_3(4), T_4(3), T_2(3), T_8(4), T_5(2), T_7(4), T_9(3), T_6(2).

2. Given independent tasks with times 12, 8, 6, 5 and 3, use the decreasing-time algorithm to schedule these tasks on two processors.
The list is already in decreasing order. Place the tasks into the lowest numbered machine at the earliest time, working through the list in decreasing-time order.



What is the completion time for the schedule?
The completion time is 17.

Is this time optimal? Explain why or why not.
This time is optimal because there is no idle time on either machine. The total time for all the tasks is 34, so divided equally across 2 machines would be 17 per machine.