1. Set $J=\left\{42,-6,\sqrt{3}\right\}$. If one of the members of Set J is selected at random, find the probability that the member selected is a rational number. Express your answer as a common fraction reduced to lowest terms.

2. If $x=-5$, and $y=3$, find the value of $2x^2+3y^2$.

3. If $x=4.5$, find the value of $\left(2x+7\right)^{(-0.75)}$. Express your answer as a common fraction reduced to lowest terms.

4. A farmer was asked how many cows he had in his herd. He answered: "If $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{8}$ of the herd were added together, they would total 178. How many total cows were in the farmer's herd?

5. The sequence: $13, 73, 14, 34, 74, \cdots$, is a sequence of reversals of consecutive prime numbers in ascending order. Find the sum of the next 5 members of the sequence after 74.

6. If x is an integer, find the sum of all distinct values of x such that $2<\left|x-3\right|<8$.

7. From the set $\left\{2.56, 3.15, 5.87, 10.15\right\}$, a member is selected at random and substituted for x in the expression $\left(104.13-2.43x\right)$. Find the probability that the expression will have a value more than 91.59. Express your answer as a common fraction reduced to lowest terms.

8. The sum of 87 **positive** integers is an even integer. If $k > 0$, let k be the **minimum** number of these integers that could be odd, and let w be the **maximum** number of these integers that could be odd. Find the value of $(w - k)$.

9. If shaving lotion is made up only of alcohol and water, find the number of ounces of water that must be added to reduce 9 ounces of shaving lotion mixture containing 50% alcohol to a shaving lotion mixture containing 30% alcohol.

10. A lady has 30 coins in her purse consisting only of nickels, dimes, and quarters. If the coins have a total value of $5**.**10, and there is at least 1 coin of each type, find the maximum number of nickels the lady could have in her purse.

11. Margie Mathlete runs from the Illini Union to her math contest at a constant rate of x mph. After the contest, she returns to the Illini Union along the same route at a constant rate of $0.25x$ mph. Her average rate for the two journeys is kx mph. Find the value of k. Express your answer as a common fraction reduced to lowest terms.

12. Find the value of x such that $\frac{3}{x-2} = \frac{4}{x-3} - \frac{6}{x^2 - 5x + 6}$.

13. Find the smallest positive integer x such that the four numbers represented by $\frac{x}{3}$, $\frac{x+2}{4}$, $\frac{x+3}{7}$, and $\frac{x+5}{11}$ are all integers.

14. If k, w, and f are positive numbers such that $3k = 4w$ and $2w = 5f$, arrange the letters k, w, and f as an **ordered triple** beginning with the letter with the smallest value and ending with the letter representing the largest. Be certain to use parentheses for your **ordered triple**.

15. Let x and y be non-equal positive integers. It is known that $\frac{1}{x}$ of the population of Johnston Village has hazel eyes and red hair and that $\frac{1}{y}$ of the population of Johnston Village has brown hair and is far-sighted. If the two groups comprise $\frac{1}{93}$ of the population of Johnston Village, find the smallest possible value of $(x+y)$.

16. For all positive integers a and b, let $a \oplus b = a^2 + 3b(2a)$ and let $a \otimes b = (b!)(2a) - 5a$. Find the value of $(3 \oplus 6) - (4 \otimes 3)$.

17. Let x and y be real numbers. The system: $\begin{cases} |x| + 4x + |y| + 4y = 300 \\ |x| - 4x + |y| - 4y = 60 \end{cases}$ has precisely two distinct solutions, namely, $(x, y) = (a, b)$ and $(x, y) = (c, d)$. Find the value of $(a+b+c+d)$.

18. If $x-3$ is a factor of $x^2 - 3kx - 72$, find the value of k.

19. All ages in this problem are in whole numbers of years. Jack is now 15 years older than Jill. Twelve years from now Jill will be half as old as Jack will be then. Find the number of years in Jill's age now.

20. The set of all real values for p such that both roots for x of the equation $x^2 + 5p(5p - 2x) - 121 = 0$ are greater than -31 and less than 236 is $\{p : w < p < k\}$. Find the value of $(k + w)$.

2012 RA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: **All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.**

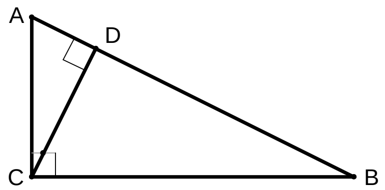
1. $\frac{2}{3}$ (Must be this reduced common fraction.)
2. 77
3. $\frac{1}{8}$ (Must be this reduced common fraction.)
4. 240 (cows optional)
5. 239
6. 30
7. $\frac{1}{2}$ (Must be this reduced common fraction.)
8. 84
9. 6 (Ounces optional.)
10. 9 (Nickels optional.)
11. $\frac{2}{5}$ (Must be this reduced common fraction.)
12. 5
13. 270
14. (f, w, k) (Must be this ordered triple.)
15. 496
16. 89
17. 60
18. -7
19. 3 (Years optional.)
20. 41

1. Points A, B, C, and D are four distinct collinear points such that $AB = BC = CD$. If 1 of these 4 points is selected at random, find the probability that the point selected is one of the trisection points of the segment that joins A with D. Express your answer as a common fraction reduced to lowest terms.

2. If the area of the rectangle with side-length as shown is 44, find the perimeter of the rectangle.

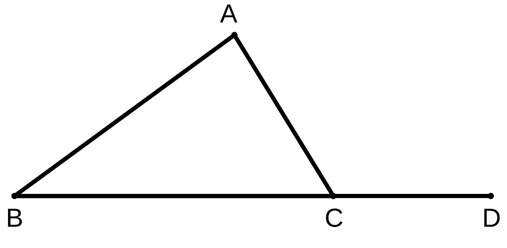
3. If one of two complementary angles is twice as large as the other, find the degree measure of the larger angle.

4. The diagram shows a right triangle with $\overline{CD}$ as the altitude to the hypotenuse. If $AC = 16$ and $BC = 25$, find AB.



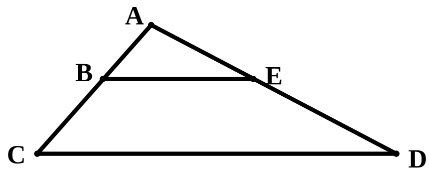
5. An isosceles triangle has side-lengths of 10, 10, and 16. Find the area of the isosceles triangle.

6. In the diagram, points B, C, and D are collinear. $\angle BAC = (kx - 177)°$, $\angle ABC = (3x)°$, and $\angle ACD = (7x - 42)°$. If $\angle ACB = 33°$, find the value of k.



7. The length of a diagonal of a rectangle is 17, and the lengths of all sides of the rectangle are integers. From among the diagonals and the sides of this rectangle, one segment is selected at random. Find the probability that the segment selected has a length that is less than 11. Express your answer as a common fraction reduced to lowest terms.

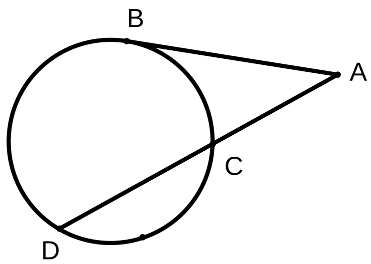
8. In the diagram, points A, B, and C are collinear, and points A, E, and D are collinear. $\overline{BE} \parallel \overline{CD}$. $AB = 13$, $BC = 6$, $AE = 12$, and $BE = 8$. Find ED. Express your answer as an improper fraction reduced to lowest terms.



9. An arc of a circle has a measure of $67°48'$. If the radius of the circle is 8.1, find the length of the given arc. Express your answer as a **decimal** rounded to the nearest thousandth.

10. A triangle has vertices at $A(2,4)$, $B(6,7)$, and $C(0,15)$. The triangle is revolved one complete revolution about its shortest side. Find the total surface area of the 3 dimensional figure formed by the rotation. Express your answer as a **decimal** rounded to the nearest tenth.

11. In the diagram, $\overline{AB}$ is tangent to the circle at B. Points A, C, and D are collinear, and C and D lie on the circle. If $AC = 4$ and $CD = 5$, find AB.



12. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

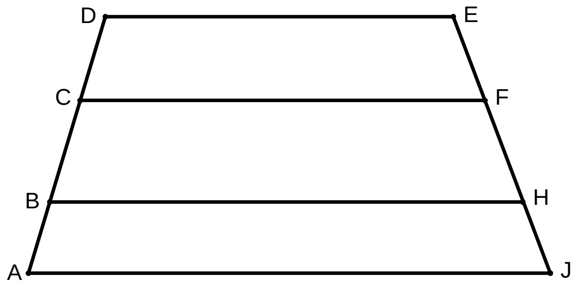
The graph of $y < 2x + 5$ is:

A) The interior of a circle.
B) The region to the right of a vertical line.
C) The region above a line with a negative slope.
D) The region above a horizontal line.
E) None of the choices (A—D) is correct.

Note: Be sure to write the correct capital letter as your answer.

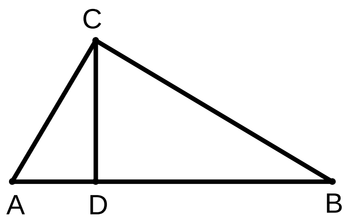
13. A sphere with center at M is inscribed in a right circular cone with a peak at P. The sphere is tangent to the lateral edges of the right circular cone at C and D, and the sphere is also tangent to the base of the cone at O, the center of the base of the cone, with $\overline{CD}$ intersecting $\overline{OP}$. If $\angle CPD = 60°$ and the length of a radius of the right circular cone is 6, find the volume of the sphere. Express your answer as a **decimal** rounded to the nearest tenth.

14. In the diagram, points D, C, B, and A are collinear. Points E, F, H, and J are collinear. $DC = 4$, $CB = 6$, and $BA = 3$. $\overline{DE} \parallel \overline{CF} \parallel \overline{BH} \parallel \overline{AJ}$. If $HF = 24$, find EJ.



15. The area of ΔABC with $A(-2,6)$, $B(5,18)$, and $C(x,10)$ is 118. If $x > 0$, find the value of x.

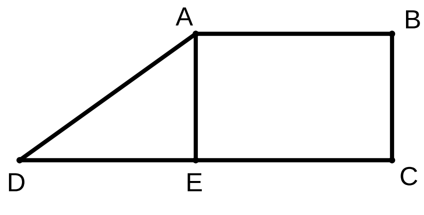
16. In the diagram, $\overline{AC} \perp \overline{BC}$, and D lies on $\overline{AB}$ such that $\overline{CD} \perp \overline{AB}$. If $AC = 3\sqrt{5}$ and $BD = 12$, find AB.



17. If the area of a regular octagon is $1200\sqrt{2} + 1200$, then, when expressed in simplest radical form, the length of a side of the regular octagon is $k\sqrt{w}$. Find the value of $(k + w)$.

18. A line contains the point $(7,k)$ and the point $(-2,-5)$. The slope of this line is $-\frac{3}{4}$. Find the value of k. Express your answer as a **exact decimal**.

19. In Quadrilateral $ABCD$, point E lies on $\overline{DC}$, $\angle ADC = 30°$, $\overline{AB} \parallel \overline{DC}$, $\overline{AE} \perp \overline{DC}$, and $\overline{BC} \perp \overline{DC}$. If $AB = \sqrt{27}$, and $BC = 12$, find DC.



20. A triangle has sides of lengths 60, 64, and 70. A circle is inscribed in this triangle. Find the area of the largest triangle that can be inscribed in this circle. Express your answer as a **decimal rounded** to the nearest **tenth** of a square unit.

2012 RA
Geometry

Name ____**ANSWERS**____

School ______________________
(Use full school name – no abbreviations)

_______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)

2. 30

3. 60 (Degrees optional.)

4. $\sqrt{881}$ (Must be this exact answer.)

5. 48

6. 9

7. $\frac{1}{3}$ (Must be this reduced common fraction.)

8. $\frac{72}{13}$ (Must be this reduced Improper fraction.)

9. 9.585 (Must be this decimal.)

10. 665.4 (Must be this decimal.)

11. 6

12. E (Must be this capital letter.)

13. 174.1 (Must be this decimal.)

14. 52

15. 20

16. 15

17. 16

18. -11.75 (Must be this decimal.)

19. $15\sqrt{3}$ (Must be this exact answer.)

20. 441.5 (Must be this decimal.)

1. Jingles has exactly 4 pennies and 1 nickel in his pocket. June has a penny, a nickel, and a dime. June selects one of these three coins at random and places this coin in the right hand of Jingles. Jingles now takes the 5 coins out of his pocket and places these 5 coins in his right hand along with the coin he got from June. Find the probability that, from among these 6 coins, it is possible for Jingles to find **change** for a dime. Express your answer as a common fraction reduced to lowest terms.

2. Let $i = \sqrt{-1}$. Then $4(x+5i)(x-5i) = kx^2 + wx + f$ where k, w, and f are integers. Find the value of $\left(k + w + f\right)$.

3. $18,500 is invested for one year at an annual percentage rate of 5%. Find the number of dollars of **interest** earned on this one year investment.

4. Find the value of x such that $x\left(\log(27)\right) = \log(9)$. Express your answer as a common fraction reduced to lowest terms.

5. A boat sets out to travel north at 20.2 mph. A wind directly from the west moves the boat eastward at 7.3 mph. Determine the boat's velocity in mph. along the path it travels. Express your answer as a **decimal** rounded to the nearest tenth.

6. The initial dimensions of a rectangle are 4 units by 6 units. The length and width are each increasing at the rate of 1.92 units per second. Find the number of seconds it will take for the area to be at least 7 times its initial size. Express your answer as a **decimal** rounded to the nearest hundredth of a second.

7. From Condition A, Condition B, and Condition C, one condition is selected at random. Find the probability that from the condition selected, one can correctly compute the area of Circle O. Express your answer as a common fraction reduced to lowest terms.

   Condition A: The circumference of Circle O is 14π.
   Condition B: The radius of Circle O is 7.
   Condition C: A chord of Circle O has a length of 10.

8. Find the value of $\sum_{n=1}^{4}\left((2n)^n\right)$.

9. Let $i=\sqrt{-1}$. If $k \neq 0$, find the value of k for which $x^2+2ki\,x-9k=0$ has equal roots for x.

10. Let $S=\{a_1,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13}\}$. Note that a_2 is missing so S has just 12 distinct elements. Form sets, each of which contain one or more of the elements of S (including all the elements of S). The only restriction is that the subscript of each element in a specific set must be an integral multiple of the smallest subscript in the set. For example, $\{a_3,a_6\}$, $\{a_4,a_8,a_{12}\}$, and $\{a_5\}$ are all acceptable sets. Also, for example, $\{a_2,a_5\}$ would **not** be an acceptable set. Find the total number of distinct such possible sets that could be formed.

11. A circle with center at $(-2,4)$ contains the point $(4,-4)$. Find the length of an arc of the circle if the measure of the arc is $37°12'$. Express your answer as a **decimal** rounded to the nearest tenth.

12. Find the 22nd term of the arithmetic sequence: $-4,-1,2,5,\cdots$.

13. The largest of 4 concentric circles has an area of 15π, and the smallest of the 4 concentric circles has an area of π. Let a, b, c, and d be four numbers that form an arithmetic progression in that order. The area of the smallest of the 4 concentric circles is a; the area of the next smallest circle is $(a+b)$; the area of the third circle is $(a+b+c)$; and the area of the largest circle is $(a+b+c+d)$. The length of the radius of the third circle, expressed in simplest radical form, is $\frac{\sqrt{k}}{w}$ where k and w are positive integers. Find the value of $(k+w)$.

14. Find the value of x such that $\log_{32}(8) = \frac{3}{5} - \log_{32}(x)$.

15. Let $f(x+2) = \frac{\sqrt[3]{9x-18}}{3}$. Then $f^{-1}(x) = kx^w + f$ where k, w, and f are positive integers. Find the value of $(2k+3w+4f)$.

16. Give the length of the major axis of the ellipse whose equation is:

$$\frac{4(x-3\frac{1}{2})^2}{225} + \frac{(y-\frac{5}{6})^2}{16} = 1$$

17. All points in this problem are coplanar. A variable point (x, y) is the center of a varying circle which (i) passes through the point $P(1,0)$ and (ii) is tangent to the circle C whose equation is $(x+1)^2 + y^2 = 36$. The locus for all possibilities for these points of the form (x, y) can be expressed as an equation in the form $kx^2 + wy^2 = f$ where k, w, and f are positive integers. Find the smallest possible value of $(k+w+f)$.

18. How many distinct numbers that are integral multiples of 7 are there between 104 and 516?

19. Let $f(x) = x^3 - 6x^2 + kx + 2k$. Find the value of k such that $f(5) = 17$.

20. When written in base five, a positive integer has three terminal zeroes. When written in either base three or base four, this same positive integer has two terminal zeroes. In how many positive integral bases greater than one and other than the three previously given **must** the representation of this integer have at least two terminal zeroes?

2012 RA
Algebra II

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [   ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 104

3. 925 ($ or dollars optional.)

4. $\frac{2}{3}$ (Must be this reduced common fraction.)

5. 21.5 (Must be this decimal, mph optional.)

6. 4.17 (Must be this decimal.)

7. $\frac{2}{3}$ (Must be this reduced common fraction.)

8. 4330

9. 9

10. 2071

11. 6.5 (Must be this decimal.)

12. 59

13. 36

14. 1

15. 31

16. 15

17. 89

18. 59

19. 6

20. 8

1. There are 4 aisles from the back to the front of the auditorium. Exactly one person is standing in each aisle. The first name of each person is known and given in the following sentence. Jerry is standing in one of these 4 aisles, Judy is standing in a second aisle, Christie is standing in a third aisle, and Katie is standing in the fourth aisle. Bob selects one of these aisles at random. Find the probability that the person who is standing in Bob's chosen aisle has a first name beginning with J. Express your answer as a **common fraction** reduced to lowest terms.

2. The point represented by $(3, y)$ lies on a given line represented by $y = 3x + 2$. At $(3, y)$, a perpendicular segment is drawn to the given line. If the perpendicular segment passes through $(2, p)$, find the value of p. Express your answer as an **improper fraction** reduced to lowest terms.

3. The common binomial factor of $x^2 - 2x - 8$ and $x^2 - x - 6$ is $x + k$. Find the value of k.

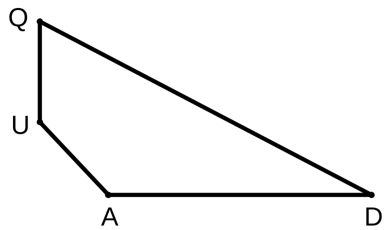
4. If $\log(\tan(x)) = k$, find, in terms of k, $\log(\cot(x))$.

5. The sum of a number and its reciprocal is 2.2025. If the number is larger than one, find the number. Express your answer as an **exact decimal**.

6. From physics, the height s, in feet, of an object after t seconds that has been thrown straight up from a point s_o feet above ground level with an initial velocity of v_o is: $s = -16t^2 + v_o t + s_o$. If a baseball is thrown straight up from 3 feet above ground level with initial velocity of 91 feet per second, find the number of seconds that will elapse before the ball first hits the ground. Express your answer as a **decimal rounded to the nearest hundredth of a second**.

7. Find the value of $\log_3(\tan(225°))$.

8. In convex quadrilateral $QUAD$, $QU = 55$, $UA = 26$, and $AD = 48$. $\angle QUA$ and $\angle UAD$ are obtuse such that $\sin(\angle QUA) = -\cos(\angle UAD) = \frac{12}{13}$. Find QD.



9. The infinite repeating decimal $0.\overline{48}$ can be written as an infinite geometric series. Find the sum of that infinite geometric series. Express your answer as a **common fraction** reduced to lowest terms.

10. The three real roots of $8x^3 - 42x^2 + kx - 27 = 0$ are in geometric progression. Find the value of k.

11. From an urn that contains only orange and blue marbles, a marble is drawn at random, and the marble drawn is orange. That marble is then discarded. The urn originally contained 61 marbles. After the first discard, the probability that a second marble drawn at random from the urn is blue is $\frac{3}{4}$. Find the number of orange marbles that were originally in the urn.

12. The transformations needed to produce the graph of $y = -5x^2 + 100x - 23$ from the graph of $y = x^2$ are **in order**: a vertical stretch by a factor of 5, a reflection over the *x*-axis, a horizontal shift k to the right, and a vertical shift w upward. Find the value of $(k + w)$.

13. Find the sum of the infinite series: $\frac{1}{3} + \frac{4}{9} + \frac{7}{27} + \frac{10}{81} + \frac{13}{243} + \cdots + \frac{3n-2}{3^n} + \cdots$ Express your answer as an improper fraction reduced to lowest terms.

14. Let $A = \{2,3,4,5,7,8,9,10,11,12,14,15,16,17,19,20,21,23,24,25,27,28,88,167\}$ Let m_A be the arithmetic mean and M_A be the median for the 24 numbers in A. Replace the element 8 with a 44 to get set B. Then let m_B be the arithmetic mean and M_B be the median of the 24 numbers in set B. Find the **exact decimal** value of $|m_A - m_B| + |M_A - M_B|$.

15. On a flat horizontal surface stands a vertical flagpole. Cindy noticed that as the angle of elevation of the sun increased from $32.13°$ to $46.41°$, the pole's shadow on the horizontal surface decreased by 19.14 feet. Find the number of feet in the height of the flagpole. Express your answer as a **decimal** rounded to the nearest hundredth.

16. Find the seventh term of the geometric sequence whose first term is 1 and 2nd term is $-\frac{1}{3}$. Express your answer as a **common fraction** reduced to lowest terms.

17. Find the sum of the first 100 positive integral multiples of 3.

18. Find the value of $\log_8\left((16)\left(\left(\frac{1}{4}\right)^{(-2)}\right)\right)$. Express your answer as an **improper fraction** reduced to lowest terms.

19. After the interest was credited at the end of 8 years, a sum of money invested at 10.89% annual percentage rate and compounded quarterly had grown to \$8432. Find the number of dollars in the original investment. Express your answer **rounded to the nearest whole number of dollars**.

20. Let a, b, and c be **integral** side-lengths of **scalene** ΔABC (with a opposite $\angle A$, etc.). If the perimeter is the smallest possible **even** number such that the largest side-length is an integral multiple of five and $14a^2 - 7ac = 10ab - 5bc$, then, in simplest radical form, $\tan(B) = \frac{k\sqrt{w}}{19}$. Find the value of $(k+w)$.

2012 RA
Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)
2. $\frac{34}{3}$ (Must be this reduced improper fraction.)
3. 2
4. $-k$
5. 1.5625 (Must be this exact decimal.)
6. 5.72 (Must be this decimal, seconds optional.)
7. 0 OR zero
8. 97
9. $\frac{16}{33}$ (Must be this reduced common fraction.)
10. 63
11. 16
12. 487
13. $\frac{5}{4}$ (Must be this reduced improper fraction.)
14. 2.5 (Must be this exact decimal.)
15. 29.89 (Must be this decimal, feet optional.)
16. $\frac{1}{729}$ (Must be this reduced common fraction.)
17. 15150
18. $\frac{8}{3}$ (Must be this reduced improper fraction.)
19. 3570 ($ or dollars optional.)
20. 68

NO CALCULATORS

1. From a shortstop (infielder), a center fielder (outfielder), and a right fielder (outfielder), one player is selected at random. Find the probability that the player selected is an infielder. Express your answer as a common fraction reduced to lowest terms.

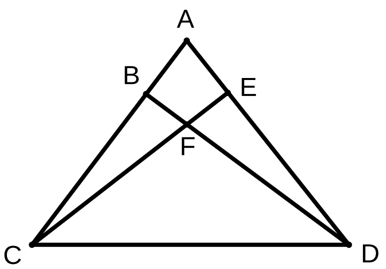
2. Find the slope of a line that is perpendicular to the line whose equation is $3x+7y+13=0$. Express your answer as an improper fraction reduced to lowest terms.

3. **(Always, Sometimes, or Never True)** For your answer, write <u>*the whole word*</u> **Always, Sometimes,** or **Never—**whichever is correct.

   If the perimeter of a square is equal to the circumference of a circle, then the area of the square is more than the area of the circle.

4. One side is selected at random among the sides of a regular triangle, a regular quadrilateral, and a regular pentagon. Find the probability that the side selected belonged to the regular pentagon. Express your answer as a common fraction reduced to lowest terms.

5. In the diagram, A, B, and C are collinear, A, E, and D are collinear, and altitudes $\overline{BD}$ and $\overline{CE}$ of ΔACD intersect at F. If $\angle CAD = 84°$, and $\angle ACD = 62°$, find the degree measure of $\angle ECD$.



6. How far is the center of the circle $x^2+2=2y-y^2-6x$ from the origin?

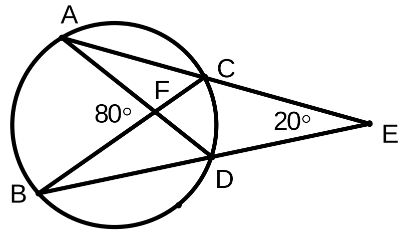
7. How many distinct ordered pairs (x, y) exist such that $x<y<30$, x and y are positive integers, and y^2-x^2 is the square of a positive integer?

8. Find the degree measure of the acute angle formed by the minute hand and the hour hand of a clock at 3:24.

9. If x is an integer, find the sum of all distinct values of x such that $|3+4x| \leq 9$.

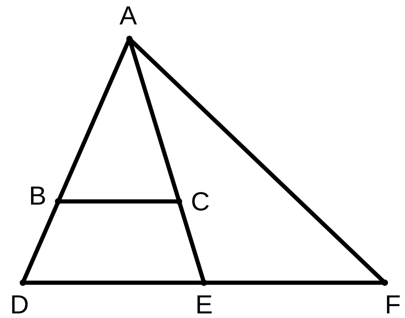
NO CALCULATORS

10. In the diagram, points A, B, C, and D lie on the circle. C lies on $\overline{AE}$, and D lies on $\overline{BE}$. $\overline{BC}$ and $\overline{AD}$ intersect at F. If $\angle AFB = 80°$ and if $\angle AEB = 20°$, find the degree measure of the minor arc from A to B.



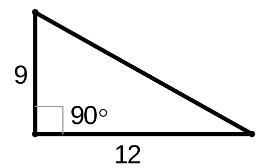
11. Find the sum of all distinct values of x such that $(x-2.2)(x-3.6)(x-12.2)=0$.

12. In the diagram, A, B, and D are collinear. C is the centroid of ΔADF, and E is the midpoint of $\overline{DF}$. If $\overleftrightarrow{BC} \parallel \overleftrightarrow{DF}$, then the ratio of the area of quadrilateral $BCED$ to the area of ΔADF can be expressed as $\frac{k}{w}$ where k and w are relatively prime integers. Find the value of $(k+w)$.



13. The ratio of the degree measure of an interior angle of an equiangular convex polygon to an exterior angle of that polygon is $9:1$. Find the number of sides of the polygon.

14. In the triangle with lengths as shown, a point is selected at random on the perimeter of the triangle. Find the probability that the point selected was on the longest side of the triangle. Express your answer as a common fraction reduced to lowest terms.

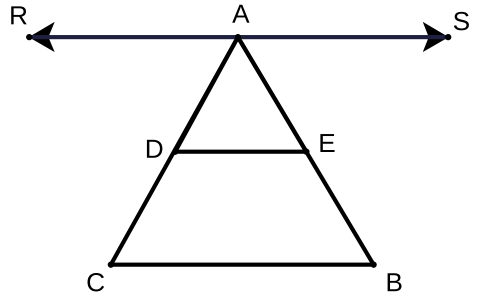


15. $\overline{PR}$ is the base of an isosceles triangle in which the lengths of each side of the triangle are positive integers. If the perimeter of the isosceles triangle is 98, find the longest possible length of $\overline{PR}$.

NO CALCULATORS

NO CALCULATORS

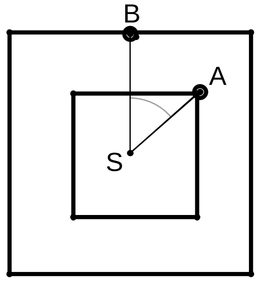
16. In the diagram, $\overleftrightarrow{RS} \parallel \overleftrightarrow{CB}$, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{AB}$, $\overline{DC} \cong \overline{EB}$, $\angle DAB \cong \angle SAB$. Find the degree measure of $\angle RAD$.



17. An up-going escalator rises at the constant rate of some number of steps per second. In addition, Tom also walks up this up-going escalator by stepping up at the rate of 1 step per second and reaches the top in 20 seconds. If he had stepped up at the rate of 2 steps per second, he would have reached the top in 16 seconds. How many steps are there in the escalator?

18. Find the value of $(2012-0)(2011-1)(2010-2)\cdots(2-2010)(1-2011)(0-2012)$.

19. In a different galaxy, two planets revolve around their sun in concentric square orbits. Planet A travels in a clockwise direction at 2 million miles per hour in an orbit of length 8 million miles. Planet B travels in a clockwise direction at 5 million miles per hour in an orbit of length 16 million miles. Initially, Planet A has a 45° head start (see diagram). Find the number of hours it will be before the two planets are, for the first time, separated by the maximum possible distance.



m∠BSA=45°

20. The seven integers from one through seven are randomly drawn (without replacement) one at a time from a bag. It is known that the first integer drawn was a 4. In how many distinct ways will the sequence be alternately decreasing and then increasing?

2012 RA

Fr/So 8 Person

School ______ **ANSWERS** ______

(Use full school name – no abbreviations)

_______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{3}$ (Must be this reduced common fraction.)

2. $\frac{7}{3}$ (Must be this reduced improper fraction.)

3. NEVER (Must be the whole word.)

4. $\frac{5}{12}$ (Must be this reduced common fraction.)

5. 56 (Degrees optional.)

6. $\sqrt{10}$ (Must be this exact answer.)

7. 20

8. 42 (Degrees optional.)

9. -5

10. 100 (Degrees optional.)

11. 18

12. 23

13. 20

14. $\frac{5}{12}$ (Must be this reduced common fraction.)

15. 48

16. 60 (Degrees optional.)

17. 80 (Steps optional.)

18. 0 OR zero

19. 10 (Hours optional.)

20. 46

NO CALCULATORS

1. Sally has eight coins in her purse. Exactly 2 are pennies, exactly 2 are nickels, and exactly 4 are quarters. If Sally selects one of these coins at random, find the probability that the value of the coin selected is an integral multiple of 5 cents. Express your answer as a **common fraction** reduced to lowest terms.

2. If $(2,15)$ is the vertex of the parabola whose equation is $y=-2x^2+8x+k$, find the value of k.

3. If $\csc(\theta)=\dfrac{13}{12}$ and $\cos(\theta)$ is negative, find $\tan(\theta)$. Express your answer as a **decimal**.

4. Find the value of $\sum_{k=0}^{\infty} 3^{(-k)}$. Express your answer as a **decimal**.

5. A right triangle has one leg of length 140. If the length of the other leg is selected at random from the set $\left\{284, 412, 300, \dfrac{4043}{13}\right\}$, find the probability that the area of the right triangle will be greater than 21560. Express your answer as a **common fraction** reduced to lowest terms.

6. $(\sqrt[3]{9})(\sqrt{6}) = k\sqrt[6]{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

7. It is known that y varies directly as the sine of x. If the constant of variation is 14, find the value of y when x is $60°$.

8. If $x=(\log_7 625)\left(\log_2(\frac{1}{7})\right)(\log_3 8)\left(\log_5 \sqrt{27}\right)$, find the value of x.

NO CALCULATORS

NO CALCULATORS

9. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

Which one of the following statements is **not** logically equivalent to the other four statements?

A) x implies y.
B) Not y implies not x.
C) x only if y.
D) y only if x.
E) y is a necessary condition for x.

Note: Be sure to write the correct capital letter as your answer.

10. In a plane, a circle has the same center as an ellipse. The circle intersects the ellipse in 4 distinct points, one of which is C. The circle also passes through points A and B which are the foci of the ellipse. The length of the minor axis of the ellipse is 8. Find the area of ΔCAB.

11. Find the value of $\log_2 8$.

12. Each interior angle of a regular polygon with x sides is $6°$ less than each interior angle of a regular polygon with y sides, and each interior angle of a regular polygon with y sides is $6°$ less than each interior angle of a regular polygon with z sides. Find the largest possible value of $(x+z)$.

13. Find the coordinates of the focus of the parabola $y=\frac{1}{8}x^2$. Express your answer as an **ordered pair of the form (x, y).**

14. The length of one leg of a right triangle is 6, and the length of the second leg of the right triangle is selected at random from the set of integers $\{5, 6, 7, \cdots, 20\}$. Find the probability that the hypotenuse of this right triangle is greater than 10. Express your answer as a **common fraction** reduced to lowest terms.

NO CALCULATORS

15. If the foci of a hyperbola are $(-7,5)$ and $(-21,5)$, and the length of the transverse axis is $2\sqrt{33}$, find the length of the conjugate axis.

16. Let $i=\sqrt{-1}$ and let $k>0$. If $(1+k\,i)^3=-8$, find the value of k.

17. A positive integer k, with $k \le 300$, is chosen in such a way that if $k \le 100$, then the probability of choosing k is $5p$, and if $k > 100$, then the probability of choosing k is p. Find the probability that the number chosen is the cube of an integer. Express your answer as a **common fraction** reduced to lowest terms.

18. $\begin{bmatrix} k & 5 & -2 \\ 3 & 6 & 12 \end{bmatrix} - \begin{bmatrix} 6 & -3 & 1 \\ w & 7 & -2 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. If $ae-3bf=-356$ and if $d-2c=12$, find the value of $(k+w)$.

19. If $g(x)=\dfrac{19x+k}{x+14}$, then the transformations that will produce a complete graph of g from the graph of $y=\dfrac{1}{x}$ are: a horizontal shift 14 to the left, a vertical stretch by a factor of 207, and a vertical shift 19 upward. Find the value of k.

20. A triangle has an area of 2. The lengths of its medians equal the lengths of the sides of a second triangle. The lengths of the medians of the second triangle equal the lengths of the sides of a third triangle. In general, the medians of the n^{th} triangle have the same lengths as the sides of the $(n+1)^{st}$ triangle. Find the limit of the sum of the areas of all the triangles thus formed in this infinite sequence.

NO CALCULATORS

2012 RA
Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{3}{4}$ (Must be this reduced common fraction.)
2. 7
3. -2.4 (Must be this decimal.)
4. 1.5 (Must be this decimal.)
5. $\frac{1}{2}$ (Must be this reduced common fraction.)
6. 27
7. $7\sqrt{3}$ (Must be this exact answer.)
8. -18
9. D (Must be this capital letter.)
10. 16
11. 3
12. 144
13. $(0,2)$ (Must be this ordered pair.)
14. $\frac{3}{4}$ (Must be this reduced common fraction.)
15. 8
16. $\sqrt{3}$ (Must be this exact answer.)
17. $\frac{11}{350}$ (Must be this reduced common fraction.)
18. 23
19. 473
20. 8

1. One of the degree measures of a rhombus is 136.13°. Find the degree measure of an angle of the rhombus that is not congruent to the given angle.

2. From four angles—two of which are acute and two of which are obtuse—two angles are selected at random without replacement. Find the probability that both angles selected are acute. Express your answer as a **common fraction reduced to lowest terms.**

3. Three numbers in the ratio of $14:19:89$ have a sum of 4148. Find the largest of the three numbers. **Express your answer as an exact integer.**

4. For the following system, find the value of $(x+y)$: $\begin{cases} 5.23x - 7.14y = 10.81 \\ 1.96x + 4.32y = 7.21 \end{cases}$

5. The area of a triangle is 986.4. A second triangle is formed with vertices at the midpoints of the sides of the original triangle. Find the area of this second triangle.

6. If $i = \sqrt{-1}$, find $\left| 7.248i^2 - (3.584i - 4.689) \right|$

7. If the area of an equilateral triangle is 998.9, find the length of a segment whose endpoints are the midpoints of two sides of the equilateral triangle.

8. A cubic function of the form $y = x^3 + bx^2 + cx + d$ has zeroes at $\frac{2}{5}$, $8\frac{1}{2}$, and at $-3\frac{1}{3}$. If x and y are both positive integers and if $x < 23$, find the sum of all distinct possible values of y. **Express your answer as an exact integer.**

9. The sum of the first k positive integers is 666. Find the value of k. **Express your answer as an exact integer.**

10. A regular dodecagon is inscribed in a circle with a radius whose length is 14.28. Find the area of the regular dodecagon.

11. If $y = 3.458x^3 - 28.54x^2 - 15.73x + 4.562$ and if $-1 \le x \le 105.3$, find the smallest possible value of y.

12. Consider the three points $(5,14)$, $(7,26)$, and $(9,50)$. Given any straight line, we can calculate the sum of the squares of the three **vertical distances** from these points to the line. What is the **smallest** possible value this sum can be? **Express your answer as an exact integer.**

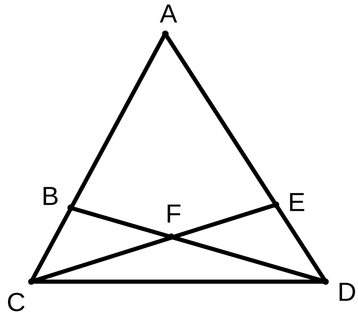
13. It is known that Wanda will have a hot fudge sundae while watching a baseball game 90% of the time. It is known that Paul will have a hot fudge sundae while watching a baseball game 80% of the time. It is known that Richard will have a hot fudge sundae while watching a baseball game 80% of the time. Find the probability that exactly one of the three will have a hot fudge sundae if each of them watches a baseball game today. Express your answer as an **exact decimal**.
Do not use 4 significant digits. Do not use scientific notation.

14. If $90° < x < 180°$, and $\sin(x) = 0.7314$, find the value of x. Express your answer in the form of **degrees, minutes, seconds**, with the value of x rounded to the nearest second.

15. Point O is the center of the circular lower base of a right circular upright cylinder, and point P is the center of the circular upper base of this cylinder. $\overline{BA} \parallel \overline{PO}$ and $\overline{BA}$ is a lateral edge of the cylinder. $OB = \sqrt{128}$, and $\angle OBA = 45°$. Find the total surface area of the cylinder.

16. If x is a radian measure, how many distinct solutions to $\cos(2x) = 2\cos(x)$ exist in $[-10\pi, 10\pi]$? Express your answer as an **exact integer**.

17. In the diagram, B lies on $\overline{AC}$, E lies on $\overline{AD}$, and $\overline{BD}$ and $\overline{CE}$ intersect at F. $AB = BD$, $AE = CE$, $AD = AC$, and $CF = FD$. If $\angle CAD = 47°$ and $AC = 24.48$, find the area of quadrilateral $ABFE$. Express your answer as a **decimal rounded to the nearest hundredth**. **Do not use scientific notation.**



18. If the average of 67.20, 63.13, 11.99, 48.79, and k is 56.13, find the value of k.

19. Two ships leave a port on the ocean at the same time. One travels in a direction $\text{N}75°\text{E}$, and the other in a direction $\text{S}40°\text{E}$. The ships travel at constant speeds of 24.74 mph. and 30.13 mph. respectively. Find the number of miles apart the ships will be 3 hours after having left the port. Assume a flat ocean surface.

20. Point P lies on the line whose equation is $4x + 3y = 60$. The absolute value of the distance from point P to the line whose equation is $4y = 3x + 20$ is 10. Find the sum of the two possibilities for the y-coordinate of point P. Express your answer as an **exact decimal**. **Do not use 4 significant digits. Do not use scientific notation.**

2012 RA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 43.87 OR 4.387×10^1 OR 4.387×10 (Degrees optional.)

2. $\frac{1}{6}$ (Must be this reduced common fraction.)

3. 3026 (Must be this integer.)

4. 3.135 OR 3.135×10^0

5. 246.6 OR 2.466×10^2

6. 4.404 OR 4.404×10^0

7. 24.01 OR 2.401×10^1 OR 2.401×10

8. 2454 (Must be this integer.)

9. 36 (Must be this integer.)

10. 611.8 OR 6.118×10^2

11. -372.1 OR -3.721×10^2

12. 24 (Must be this integer.)

13. 0.068 OR .068 (Must be this decimal.)

14. 132°59'46" (Must be in degrees, minutes, seconds form.)

15. 804.2 OR 8.402×10^2

16. 20 (Must be this integer.)

17. 135.92 (Must be this decimal.)

18. 89.54 OR 8.954×10^1 OR 8.954×10

19. 89.49 OR 8.949×10^1 OR 8.949×10

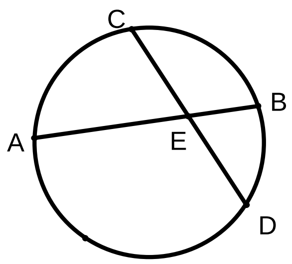
20. 20.8 (Must be this decimal.)

1. If $(2x^3y^3)^3(-6xy^2)$ is expressed in simplest form as kx^ay^b where k, a, and b are integers, find the value of $(k+a+b)$.

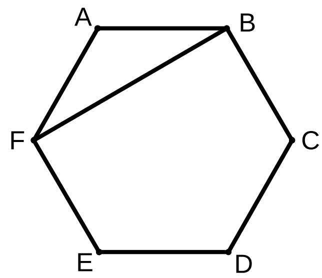
2. Two angles that are complementary have degree measures that differ by 78. Two other angles that are supplementary have degree measures that differ by 88. Find the sum of the degree measures of the two largest angles of these four angles.

3. Find the smallest possible value of x that is an **integer** so that $3 > -2(x-1)-3$.

4. In the circle in the diagram, chords $\overline{AB}$ and $\overline{CD}$ intersect at point E. If $AE = 2048$, $CE = 1536$, and $BE = 144$, let $DE = y$. Let k be the sum of all distinct integral values of x such that $|x-5| < 3.6$. Find the value of $(y+k)$.

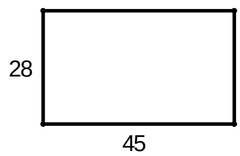


5. In the diagram, $ABCDEF$ is a regular hexagon. Find the degree measure of $\angle AFB$.



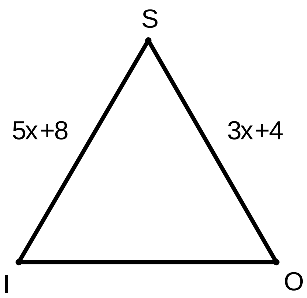
6. Let k be the length of a diagonal of the rectangle with side-lengths as shown. Let w be the value of $5x+2y$ for the system shown. Find the value of $(k+w)$

$$\begin{cases} 3x-4y=144 \\ x+3y=-69 \end{cases}$$



7. Let k be the sum of all distinct solutions to the two equations $|x-5| = 12$ and $|x+3| = 11$. Let w be the length of the altitude to the base of an isosceles triangle whose base is 32 and whose perimeter is 162. Find the value of $(k+w)$.

8. Let (x, y) be the solution to the system $\begin{cases} 5x+2y=1071 \\ 13x-y=3851 \end{cases}$. Let k be the length of the shortest side of a triangle whose perimeter is 20424, and whose sides have lengths in the ratio $89:786:827$. Find the value of $(x+y+k)$.



9. In the diagram, $IS = 5x + 8$, $SO = 3x + 4$, and the perimeter of ΔISO is 202. If $\angle ISO \cong \angle IOS$, let $w = IO$. One of the roots for x of the cubic equation $x^3 - 38x^2 + kx + 1748 = 0$ is 19. Find the value of $(k + w)$.

10. Let x be the smallest positive integer such that $x!(x+2)! = (x+3)!$. Let y be the length of the longest altitude of a triangle with side-lengths of 11, 13, and 20. Find the value of $(x + y)$.

2012 RA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. -27	
2. 218 (Degrees optional.)	
3. -1	
4. 227	
5. 30 (Degrees optional.)	
6. 59	
7. 67	
8. 1179	
9. 347	
10. 15	

TOTAL SCORE: ____

(*enter in box above)

Extra Questions:

11. $-\frac{25}{4}$ (Must be this reduced improper fraction.)

12. 225 (Degrees optional.)

13. 3

14. $96\sqrt{3}$ Must be this exact answer.)

15. 3750

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let $k = \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} - \begin{vmatrix} 8 & 4 \\ 3 & 1 \end{vmatrix}$. Let $4^{(w+7)} = (\frac{1}{8})^8$. Find the value of $(k+w)$.

2. Find the value of x such that $\log(x) - \log(3) + \log(5) = 2$.

3. The terms of an arithmetic sequence are: $16, 48, 80, \cdots$. By how much will the sum of the 8th through the 13th terms of this sequence exceed the sum of the first seven terms of this sequence?

4. Let $x^2 - x - 2 = 0$ and let $y^3 + 12y^2 + 47y + 60 = 0$. Find the sum of all **distinct** values that are roots of either or both of the given equations.

5. For the cubic equation $x^3 - 44x^2 + 531x + f = 0$, one of the three roots for x is 3. Find the absolute value of the difference between the other two roots.

6. Alice, Bob, Charles, Diane, Elsie, Frank, and George are trying out for a role in a one-act play that needs 4 actors. Two parts must be played by two of the four boys and 2 parts must be played by girls. How many different casts are possible if who plays what part does matter in the counting of the different casts?

7. Let $A = \{2, 3, 5, 12, 29\}$. Let k be the probability that the sum of two distinct elements of A selected at random is a prime integer. Let w be the probability that the sum of three distinct elements of A selected at random is a prime integer. Find the value of $(k+w)$. Express your answer as a **decimal**.

8. Let $\frac{5^k}{5^3} = 25^6$. Let $A = \{7, 25, 33, 110, 156, 14, 20, 9702, 6888\}$. Let S be the sum of all distinct values of A that can be expressed as the product of two consecutive integers. Find the value of $(k+S)$.

9. Let 1608 be the sum of the first sixteen terms of an arithmetic sequence whose first two terms are respectively $2k+4$ and $2k+7$. Let w be the first term of an infinite geometric sequence whose sum is 25 and whose common ratio is 0.2. Find the value of $(k+w)$.

10. A line whose equation is $y = \frac{3}{4}x + 6$ is a distance of 4 from a line whose equation is $y = \frac{3}{4}x + k$. If $k > 7$, find the value of k.

2012 RA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. –5	
2. 60	
3. 1136	
4. –11	
5. 7	
6. 72	
7. 0.9 OR .9 (Must be this decimal.)	
8. 10003	
9. 57	
10. 11	

TOTAL SCORE: ____

(*enter in box above)

Extra Questions:

11. 309
12. 49 (Feet optional.)
13. 36
14. 80
15. 0.5 OR .5 OR $\frac{1}{2}$

*** Scoring rules:**

Correct in 1st minute – 6 points

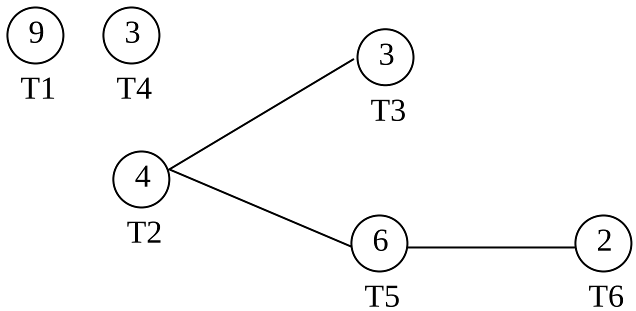
Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

2012 ICTM Regional
Div A Oral Competition

1. A company has 2 employees that need to complete production tasks described by the following order-requirement digraph with times as given and the priority list T1, T2, . . . T6



a. Find an optimal schedule for the two employees.

b. The company is hoping to speed up production by hiring a 3rd employee. However, the budget for labor costs are fixed and cannot be increased.

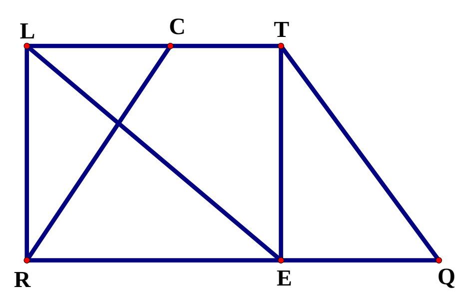
i. Show how the tasks can be scheduled for 3 employees. Is this an optimal schedule?

ii. All employees are paid based upon the completion time of an entire job. Is it possible to hire a 3rd employee and speed up production, while staying at or under labor cost budget? Explain your answer.

2. USA High School is hosting an invitational math contest. There are six different topics for the contest:

L – Linear Equations	T – Trigonometry	C – Conic Sections
R – Ratios	E – Exponents	Q – Quadratic Functions

The graph shown represents conflict information about scheduling the contest.



a. How many time slots are necessary to schedule all of the topics? Explain your answer.

b. Which events can take place in each time slot?

3. The table below shows chemical compounds which cannot be mixed without dangerous reactions.

	A	**B**	**C**	**D**	**E**
A		X	X	X	
B	X		X		X
C	X	X		X	X
D	X		X		X
E		X	X	X	

a. Show a graph that could be used to facilitate the scheduling of disposal containers for the compounds.

b. Give the chromatic number of this graph.

4. John is organizing the books he plans to take with him to college. The local store has storage bins on sale, each of which will hold 8 books. His book collection consists of:

5 math books
5 science books
4 novels
4 English books
4 Spanish books
3 sports books
3 music books
3 study skills books
3 biographies
2 history books

Books in a particular subject area must be in the same bin.

If John uses FFD, how many bins must he buy?

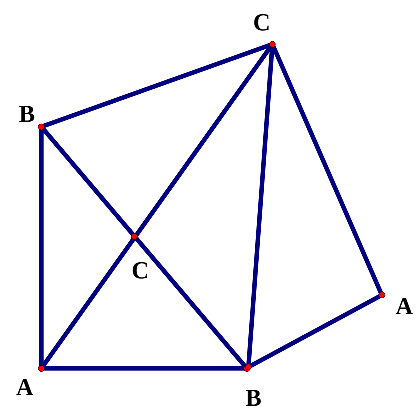
2012 ICTM Regional
Div A Oral Competition

Extemporaneous Questions

PLEASE GIVE THIS SHEET TO THE STUDENTS AT THE BEGINNING OF THE EXTEMPORANEOUS QUESTION TIME PERIOD.

1. Refer to question 4 of the original questions, where John is organizing the books he plans to take with him to college. If John uses NFD, how many bins must he buy?

2. Explain why the following vertex coloring, where the letter at each vertex indicates the color of the vertex, is not correct.



3. Three machines, each with a total time capacity of 12, are being used for independent tasks with times 2, 3, 3, 4, 4, 5, 5, 6 and 7. Any task can be done on any machine. Is it possible to complete these tasks on these three machines? Why or why not?

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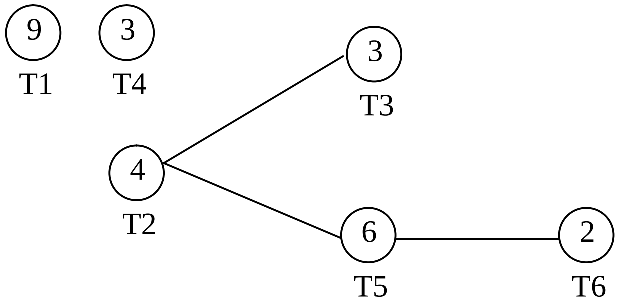
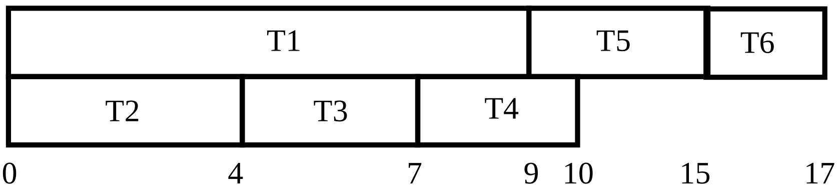
1. A company has 2 employees that need to complete production tasks described by the following order-requirement digraph with times as given and the priority list T1, T2, . . . T6



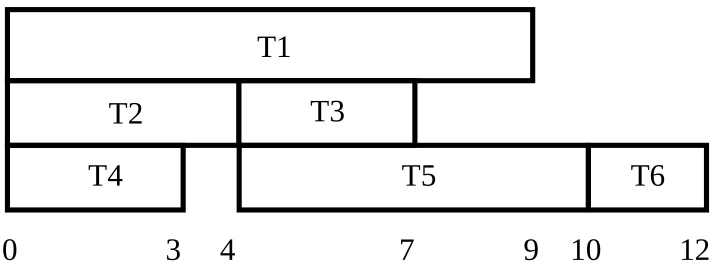
a. Find an optimal schedule for the two employees.



The completion time is 17 hours
(***Note: some credit should be given for students who complete the list-processing algorithm, even if they don't find the optimal schedule.)***

b. The company is hoping to speed up production by hiring a 3rd employee. However, the budget for labor costs are fixed and cannot be increased.

i. Show how the tasks can be scheduled for 3 employees. Is this an optimal schedule?



Since the critical path from the diagraph is 4 + 6 + 2 = 12, which is the same length as this schedule, this is an optimal schedule.

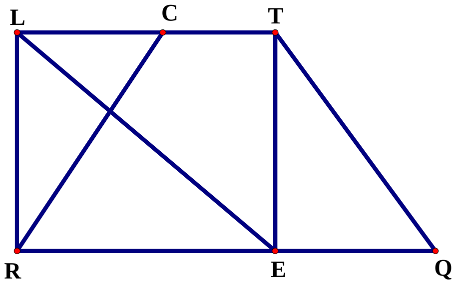
ii. All employees are paid based upon the completion time of an entire job. Is it possible to hire a 3rd employee and speed up production, while staying at or under labor cost budget? Explain your answer.

While a third employee would speed up production (12 instead of 17 hours), a net increase of 2 total man hours would go over the labor cost budget. (3 @ 12 instead of 2 @ 17)

2. USA High School is hosting an invitational math contest. There are six different topics for the contest:

L – Linear Equations	T – Trigonometry
C – Conic Sections	R – Ratios
E – Exponents	Q – Quadratic Functions

The graph below represents conflict information about scheduling the contest.



a. How many time slots are necessary to schedule all of the topics? Explain your answer.

3 time slots are necessary since the chromatic number of the graph is three.

b. Which events can take place in each time slot?

Time Slot 1 – Linear Equations and Quadratic Functions
Time Slot 2 – Trigonometry and Ratios
Time Slot 3 – Exponents and Conic Sections

OR

Time Slot 1 – Linear Equations and Trigonometry
Time Slot 2 – Quadratic Functions and Ratios
Time Slot 3 – Exponents and Conic Sections

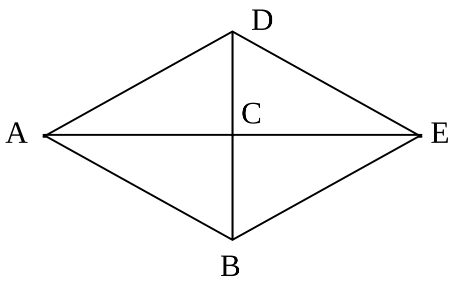
Note: While the topics must be in either of the sets of the pairs listed above, any time slot can be slated for a particular pair.

3. The table below shows chemical compounds which cannot be mixed without dangerous reactions.

	A	**B**	**C**	**D**	**E**
A		X	X	X	
B	X		X		X
C	X	X		X	X
D	X		X		X
E		X	X	X	

a. Show a graph that could be used to facilitate the scheduling of disposal containers for the compounds.

Below is a sample of a possible graph:



b. Give the chromatic number of this graph.

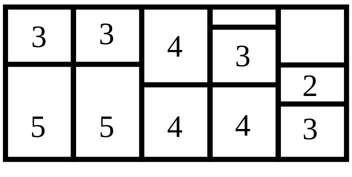
The chromatic number of the graph is 3.

4. John is organizing the books he plans to take with him to college. The local store has storage bins on sale, each of which will hold 8 books. His book collection consists of:

5 math books
5 science books
4 novels
4 English books
4 Spanish books
3 sports books
3 music books
3 study skills books
3 biographies
2 history books

Books in a particular subject area must be in the same bin.

If John uses FFD, how many bins must he buy?



Five bins must be purchased.

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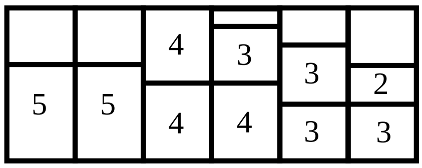
Extemporaneous Questions

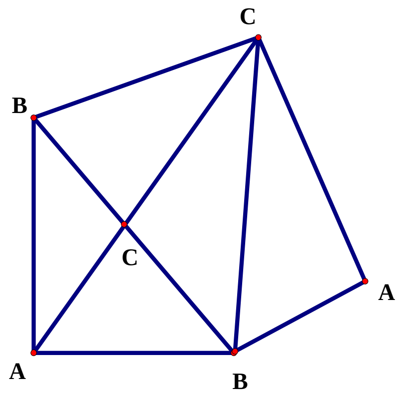
1. Refer to question 4 of the original questions, where John is organizing the books he plans to take with him to college. If John uses NFD, how many bins must he buy?



Six bins must be purchased.

2. Explain why the following vertex coloring, where the letter at each vertex indicates the color of the vertex, is not correct.

Two consecutive vertices are labeled C



3. Three machines, each with a total time capacity of 12, are being used for independent tasks with times 2, 3, 3, 4, 4, 5, 5, 6 and 7. Any task can be done on any machine. Is it possible to complete these tasks on these three machines? Why or why not?

NO – total time required is 39, and only 36 is available on the machines. Students may reference the idea of the area of the rectangle formed by the three machines (3 X 12) and smaller rectangles (1 X n) for each task of time n, with the smaller rectangles filling the larger rectangle.