1. The circumference of Circle O is 34, the circumference of Circle P is 42, and the circumference of Circle T is 53. If one of the three circles is selected at random, find the probability that the radius of that circle is more than 6. Express your answer as a common fraction reduced to lowest terms.

2. The sum of two numbers is 106, and their difference is 18. Find the larger of the two numbers.

3. If $17.4x+34.2=5.8(3x+y)$, find the value of y. Express your answer as an improper fraction reduced to lowest terms.

4. For all values of x such that $x \neq 3$, $\dfrac{5x-15}{12} \div \dfrac{4x-12}{15} = k$. Find the value of k. Express your answer as a **decimal**.

5. If the sum of two numbers is 15 and the product of the two numbers is 5, find the sum of the reciprocals of the two numbers.

6. The repeating decimal $0.\overline{522}$ (where the 522 repeats) is expressed as a common fraction reduced to lowest terms. Find that common fraction.

7. Three different fruits lie on a table: an orange, an apple, and a banana. If two different fruits are selected at random from among these three, find the probability that the first letter of each fruit selected is a vowel. Express your answer as a common fraction reduced to lowest terms.

8. Assume that men and women are adults, and assume that boys and girls are **not** adults. At a Halloween party, only men, women, boys, and girls attended. There were 17 girls, 11 adults without costumes, 14 women, 10 girls with costumes, 29 people without costumes, 11 women with costumes, and 16 males with costumes. Find the total number of people who attended this Halloween party.

9. Emily travels at a rate of $4k$ mph. for 4 miles, $5k$ mph. for 5 miles, etc., and finally at the rate of wk mph. for w miles. For what value of w will the total distance traveled divided by the total time it took (average rate) be 12 times her starting rate?

10. When Sam started to work at Harriet's Hamburgers, he asked Harriet what his hourly pay would be. Harriet said Sam could pick any positive dollar amount he wanted (such as \$17.42), and then Harriet would subtract the square (percentage-wise) of that amount from twice his picked dollar amount to arrive at his net pay per hour. For example, if Sam picked \$4.00, Harriet would subtract $(4.00)^2$ per cent of \$4.00 from \$8.00, and Sam's net pay per hour would be \$7.36. Find the amount Sam should pick in order to maximize his net hourly pay. After you have found that amount, round that amount to the nearest cent. Then express your answer in **dollars and cents with, of course, the cents rounded to the nearest cent**.

11. If $\sqrt{x} = \sqrt{600+\sqrt{600+\sqrt{600+\sqrt{600+\cdots}}}}$, find the value of x.

12. When $\dfrac{a^{-1}b^{-1}+ab}{ab^{-1}+(ab)^{-1}}$ is expressed in simplest form with positive exponents, the result is $\dfrac{1+a^k b^w}{a^p+g}$ where k, w, p, and g are positive integers. Find the smallest possible value of $(k+w+p+g)$.

13. Let $f(x)=\dfrac{4kx}{3x+4}$ where k is an **integer** and where $x>0$. If $f\left(f(x)\right)=x$ for all $x>0$, then find the value of k.

14. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

If $5x=7y=z$, and x, y, and z are positive integers, then all of the following **must** be an integer **except**:

A) $\dfrac{z}{xy}$ B) $\dfrac{z}{7}$ C) $\dfrac{z}{35}$ D) $\dfrac{z}{5}$ E) $\dfrac{y}{5}$

Note: Be certain to write the correct capital letter as your answer.

15. Let N be a four-digit number (with the thousands digit non-zero) such that the units digit of N^2 is 9, and such that the sum of the digits of N is 10. How many distinct possibilities exist for the value of N ?

16. Let x represent a positive number base such that $129_x = 233_{ten}$. Find the value of x. Express your answer in base ten.

17. Candle A is 5 times as long as Candle B. Candle A is consumed uniformly in 4 hours, and Candle B is consumed uniformly in 6 hours. If the 2 candles are lit at the same time, find the absolute value of the difference between the number of hours it will take for Candle A to be 3 times as long as Candle B and the number of hours it will take for Candle B to be 3 times as long as Candle A. Express your answer as an improper fraction reduced to lowest terms.

18. If $x = 2$ and $y = -3$, find the value of $\left(x^y\right)^{\left(3-x^{(-y)}\right)}$. Express your answer as an exact **integer**.

19. Two positive integers each of which is less than 10 form a two-digit number. The digits are then switched to form a second two-digit number. When the second two-digit number is subtracted from the original two-digit number, the result is a number that is one less than the second two-digit number. Find the original number.

20. Serena was rowing **upstream** one day when her cap blew off into the stream. She failed to notice it was missing until 18 minutes after it blew off. She immediately turned around and recovered the cap 2.64 miles **downstream** from where it initially blew off. Assume Serena's physical rate of rowing was constant, the rate of the current was constant, and that it took no time to turn around. Find the number of miles per hour in the rate of the current. Express your answer as an **exact decimal.**

2012 RAA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 62

3. $\frac{171}{29}$ (Must be this reduced improper fraction.)

4. 1.5625 (Must be this decimal.)

5. 3

6. $\frac{58}{111}$ (Must be this reduced common fraction.)

7. $\frac{1}{3}$ (Must be this reduced common fraction.)

8. 66

9. 92

10. 8.16 (Must be in dollars and cents, $ optional.)

11. 625

12. 7

13. –1

14. A (Must be this capital letter.)

15. 34

16. 14 OR 14_{ten} OR 14_{10}

17. $\frac{160}{129}$ (Must be this reduced improper fraction, hours optional.)

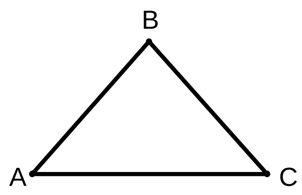
18. 32768

19. 73

20. 4.4 (Must be this decimal, mph optional.)

1. Triangle ABC has sides of lengths 6, 8, and 10. Triangle DEF has sides of lengths 5, 12, and 13. Triangle GHJ has sides of lengths 3.5, 12, and 12.5. If one of these triangles is selected at random, find the probability that the triangle selected has an area greater than 25. Express your answer as a common fraction reduced to lowest terms.

2. In ΔABC, if $AB = BC$, and $\angle BAC = 32°$, by how many degrees does the angle measure of $\angle ABC$ exceed that of $\angle BCA$?



3. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   The midpoint of the hypotenuse of a right triangle is what center of the triangle?

   A) Circumcenter B) Incenter C) Center of gravity D) Orthocenter

   Note: Be sure to write the correct capital letter as your answer.

4. Two triangles have congruent bases. The heights of each of these two triangles on these two congruent bases are also congruent. **Must** these two triangles be congruent? For your answer, write the <u>*whole word*</u> **Yes** or **No**—whichever is correct.

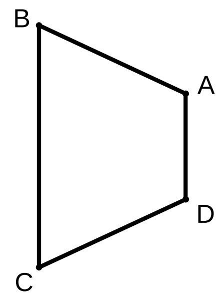
5. **(Always, Sometimes, or Never True)** For your answer, write the <u>*whole word*</u> **Always, Sometimes,** or **Never**—whichever is correct.

   If the slope of a line is a positive number, then the line does **not** pass through (intersect) Quadrant IV.

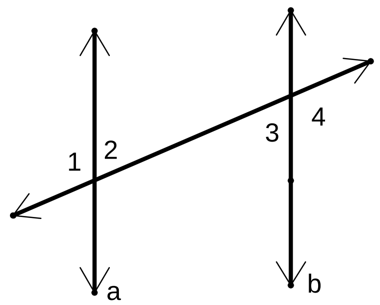
6. In rectangle $RECT$, M is the midpoint of $\overline{RE}$, N is the midpoint of $\overline{EC}$, O is the midpoint of $\overline{TC}$, and P is the midpoint of $\overline{RT}$. If $RP = 6$, and $TO = 17.5$, find the perimeter of quadrilateral $MNOP$.

7. From all the interior angles of an equiangular pentagon, an equiangular hexagon, and an equiangular heptagon, one of these interior angles is selected at random. Find the probability that the angle selected has a degree measure that is an integer. Express your answer as a common fraction reduced to lowest terms.

8. In the diagram, $ABCD$ is an isosceles trapezoid with $\overline{BC}$ as one of the bases. $AD = kx + 4$, $DC = 7.6x - 5.1$, $BC = 19x$, and $AB = 3.2x + 21.3$. If k is a positive integer, find the smallest possible value of k.

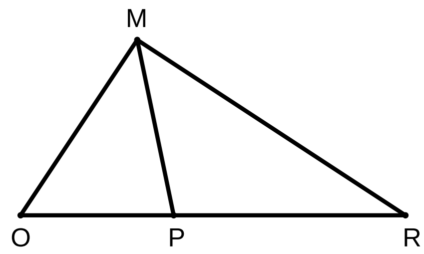


9. In the diagram, $a \parallel b$ with a non-perpendicular transversal. If two of the numbered angles are selected at random, find the probability that the two angles selected are congruent. Express your answer as a common fraction reduced to lowest terms.



10. Given the points $D(1,2)$, $E(10,2)$, and $F(9,-2)$. A lattice point is defined as a point for which all coordinates are integers. Find the number of distinct lattice points that are in the **interior** of ΔDEF.

11. In the diagram, $\angle OMP \cong \angle RMP$. $MO = 8$, $OP = 6$, and $MR = 12$. O, P, and R are collinear. The area of ΔMOR can be expressed as $\frac{w\sqrt{k}}{4}$. If k and w are positive integers, find the smallest possible value of $(k + w)$.



12. If $43°20'$ were expressed as an improper fraction in terms of degrees, the answer would be $\frac{130}{3}$ (° optional). By how much does $64\frac{2}{3}°$ exceed $60°25'30"$? Express your answer as an improper fraction in terms of degrees.

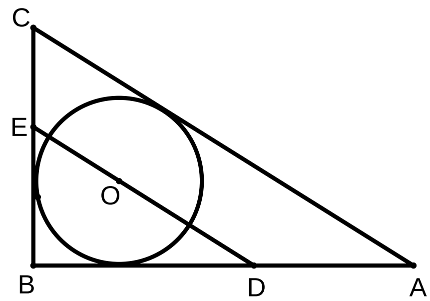
13. $\overline{AM}$, $\overline{BN}$, and $\overline{CP}$ are the respective medians of Triangle ABC. If $AM = 37.02$, $BN = 33.18$, and $CP = 39.51$, find the length of $\overline{MN}$. Express your answer in **decimal form** rounded to 4 significant digits.

14. A rectangular solid has dimensions of 5, 6, and 8. From one of the faces with smallest area, a circular hole of diameter 4 is drilled at a right angle from the face halfway through to the opposite face. When the circular hole reaches that halfway point, a semi-circular hole of diameter 4 is drilled the rest of the way to the opposite face and at right angles with that opposite face. Find the total surface area of the rectangular solid with the hole. Express your answer as a **decimal** rounded to the nearest tenth.

15. **(Always, Sometimes, or Never True)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

If the hypotenuse of a right triangle is parallel to one of the axes, then the product of the slopes of the two legs is -1.

16. A circle with center at O is inscribed in ΔABC. $AB = 12$, $BC = 5$, and $AC = 13$. Point E lies on $\overline{BC}$, and point D lies on $\overline{AB}$ such that $\overleftrightarrow{DE} \parallel \overleftrightarrow{AC}$ and such that $\overleftrightarrow{DE}$ passes through point O. Find DE. Express your answer as an improper fraction reduced to lowest terms.



17. The circle whose equation is $x^2 + y^2 - 6x - 10y = 18.2729$ is tangent to the line $x = k$ where $k > 0$. Find the value of k. Express your answer as a **decimal**.

18. The number of degrees in the sum of the degree measures of the interior angles of a convex polygon is 15858 more than the number of distinct diagonals that can be drawn in that polygon. If the number of sides of the polygon is **not** an integral multiple of 17, find the number of sides of the polygon.

19. If the area of a circle is 196π, the length of a $144°$ arc can be expressed as $k\pi$. Find the value of k. Express your answer as a **decimal**.

20. In ΔABC, the vertices are at $A(4,3)$, $B(14,5)$, and $C(-3,1)$. Find the length of the median from C to $\overline{AB}$.

2012 RAA

Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{3}$ (Must be this reduced common fraction.)

2. 84 (Degrees optiona.)

3. A (Must be this capital letter.)

4. NO (Must be the whole word.)

5. Sometimes (Must be the whole word.)

6. 74

7. $\frac{11}{18}$ (Must be this reduced common fraction.)

8. 5

9. $\frac{1}{3}$ (Must be this reduced common fraction.)

10. 12

11. 1468

12. $\frac{509}{120}$ (Must be this reduced improper fraction, degrees optional.)

13. 19.38 (Must be this decimal.)

14. 314.8 (Must be this decimal.)

15. Always (Must be the whole word.)

16. $\frac{221}{30}$ (Must be this reduced improper fraction.)

17. 10.23 (Must be this decimal.)

18. 159

19. 11.2 (Must be this decimal.)

20. $3\sqrt{17}$ (Must be this exact answer.)

1. From the set $\{2,5,8\}$, one member is selected at random and substituted for x in the expression $3x+8$. Find the probability that after the substitution is made, the expression will have a value greater than 20. Express your answer as a common fraction reduced to lowest terms.

2. A piece of property is assessed at $46,000. The tax rate for a certain year on the assessed property value is $2.50 per $1000. Find the number of dollars of tax paid on this piece of property for that certain year.

3. For all real values of x except for $x=\pm\sqrt{7}$,
$\frac{2x^5+x^4+3x^3-x+5}{x^2-7}=2x^3+x^2+17x+7+\frac{kx+w}{x^2-7}$. Find the value of $(k+w)$.

4. Clerk A, who sorts at a constant rate, sorts B letters per hour. Clerk C, who sorts at a constant rate, sorts D letters per hour. Every ten minutes, the number of letters sorted by Clerk A exceeds the number of letters sorted by Clerk C by 10. During a certain day each of these two clerks sorted letters for k hours. For that day, Clerk A sorted 420 more letters than Clerk C. Find the value of k.

5. If $x^4+3x^3-8x^2+2x-12$ is factored with respect to the integers, one of the factors is x^3+kx^2+wx+f. Find the value of $(k+w+f)$.

6. On a flat planar surface, a semicircle is drawn in the exterior of a square with a side of the square as its diameter. If the sum of the areas of the square and the semicircle is 240, find the length of the diameter. Express your answer as a **decimal** rounded to the nearest hundredth.

7. Four circles have radii of respective lengths 5.13, 2.15, 7.86, and 4.57. If two of the circles are selected at random without replacement, find the probability that each of the circles selected has an area greater than 61.65. Express your answer as a common fraction reduced to lowest terms.

8. The equation of the hyperbola whose foci are at $(6,0)$ and $(-6,0)$ and which has a latus rectum whose length is 18 can be expressed in the form $\frac{x^2}{k}-\frac{y^2}{w}=1$. Find the value of $(3k+2w)$.

9. Let $f(x)=x^2+5$ and let $g(x)=kx+w$ where k and w are positive integers. If $f(g(6))=1686$, find the possible values of $(k+w)$. For your answer, give the smallest possible value of $(k+w)$.

10. Let $7,8,4,9,3,21,59,\cdots$ be a sequence of numbers such that each present term after the fifth term is 3 more than twice the sum of the preceding term and the number of the present term. If n is the number of the term that equals 671088573, find the value of n.

11. The points $A(1,-3)$, $B(4,-2)$, and $C(8,16)$ are the vertices of ΔABC. The length of the longest altitude of ΔABC can be expressed as $k\sqrt{w}$, where k and w are positive integers. Find the smallest possible value of $(k+w)$.

12. Let $i=\sqrt{-1}$. If the reciprocal of $7-2i$ is written in $x+yi$ form where x and y are real numbers, find the value of $(4x+2y)$. Express your answer as a common fraction reduced to lowest terms.

13. Let k and w be real numbers such that $x^4-10ix^3+kx^2+wix+296=0$ where $i=\sqrt{-1}$. If one of the solutions for x of the given 4^{th} degree equation is $7+5i$ and at least two of the solutions for x of the given 4^{th} degree equation are real numbers, find the value of w.

14. Find the 50^{th} term of the arithmetic sequence $5,\frac{24}{5},\frac{23}{5},\cdots$. Express your answer as a **decimal**.

15. The equation of the graph which represents the locus of all points P in a plane such that the ratio of the absolute value of its distance from the point $(0,4.5)$ to the absolute value of its distance from the line $y=\frac{1}{18}$ is 9 can be expressed in the form $ky^2-x^2=w$. Find the value of $(k+w)$.

16. Let $i = \sqrt{-1}$. If x and y are real numbers, find the value of $(x+y)$ when

$$3x + 6yi \;=\; 18 + \begin{vmatrix} 1 & -1 & 0 \\ 2 & 0 & -3 \\ 3 & 6\frac{1}{3} & -2 \end{vmatrix} i$$

17. A positive integer, if divided by 12, leaves a remainder of 11; if divided by 11, leaves a remainder of 10; if divided by 10, leaves a remainder of 9; if divided by 9, leaves a remainder of 8; if divided by 8, leaves a remainder of 7; if divided by 7, leaves a remainder of 6; if divided by 6, leaves a remainder of 5; and if divided by 5, leaves a remainder of 4. Find the smallest such positive integer.

18. If x represents a positive integer, find the sum of all distinct values of x such that $x(x-4)(x+2) \le 0$.

19. Find the value of $\sum_{n=5}^{12}\left((n-7)^2\right)$.

20. Let ABC be a right triangle with $\angle ACB = 90°$, $AC = 6$, and $BC = 2$. E is the midpoint of $\overline{AC}$, and F is the midpoint of $\overline{AB}$. If $\overline{CF}$ and $\overline{BE}$ intersect at G, then $\cos(\angle CGB)$, in simplest radical form, is $\frac{k\sqrt{w}}{f}$ where k, w, and f are positive integers. Find the value of $(k+w+f)$.

2012 RAA

Algebra II

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 115 ($ or dollars optional.)

3. 172

4. 7

5. 13

6. 13.13 (Must be this decimal.)

7. $\frac{1}{2}$ (Must be this reduced common fraction.)

8. 81

9. 11

10. 30

11. 15

12. $\frac{32}{53}$ (Must be this reduced common fraction.)

13. 40

14. –4.8 (Must be this decimal.)

15. 100

16. 10

17. 27719

18. 10

19. 60

20. 267

1. In Bag A are 2 red marbles, 2 green marbles, and 2 white marbles. In Bag B, there are 2 red marbles and 1 green marble. Martha selects one of the marbles at random from Bag B and places this marble in Bag A. After Martha places this marble in Bag A, find the probability that there are more red marbles in Bag A than there are green marbles in Bag A. Express your answer as a **common fraction** reduced to lowest terms.

2. The distance between two points represented by $(x,5)$ and $(-3,26)$ is 29. Find the largest possible value of x.

3. In which quadrant is cotangent negative and cosine positive? Express your answer as a **Roman numeral**.

4. Let $i=\sqrt{-1}$. The two non-real roots of $x^4-x^2-12=0$ are wi and ki. Find $|w-k|$.

5. If the vector $\vec{u}=[4,-7,1]$ and the vector $\vec{v}=[2,-3,-2]$, then find the magnitude of the sum vector $\overrightarrow{u+v}$.

6. The graph of $y=\dfrac{2x^3-7x^2+5x-6}{x^2-5x+6}$ has an oblique or slant asymptote of $y=kx+w$. Find the value of $(k+w)$.

7. Let $i=\sqrt{-1}$. If x and y are real numbers such that $3x+5+7yi=2y+3xi$, find the value of y.

8. In Triangle ABC, altitude $\overline{AH}$ and median $\overline{BM}$ intersect inside the triangle and have equal lengths. If $\angle ABM=45°$ and $AB=16$, then, in simplest radical form, the length of $\overline{AH}$ is $k\sqrt{w}+f\sqrt{p}$ where k, w, f, and p are positive integers. Find the value of $(k+w+f+p)$.

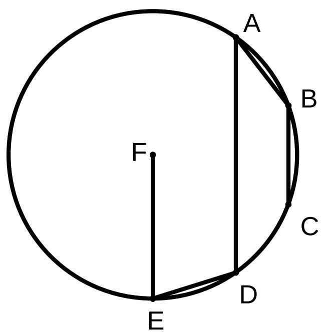
9. From a $20''\times 32''$ rectangular sheet of cardboard, $x''\times x''$ squares are cut from two corners, and $x''\times 16''$ rectangles are cut from the other 2 corners. The remaining cardboard is then folded to form a box with a lid. Find the value of x that yields the box of maximum volume. Express your answer as a **decimal** rounded to the nearest hundredth of an inch.

10. The graph of $f(x)=\dfrac{x^4-5x^3+2x^2+11x-1}{x^2-10x+3}$ has vertical asymptotes at $x=k$ and at $x=w$. If $k<w$, find the value of k. Express your answer as a **decimal** rounded to the nearest ten-thousandth.

11. Cindy has three marbles—one is red, one is white, and one is green. She has two containers—A and B. She selects a marble at random and then chooses a container at random into which she puts that marble. She then repeats this process for each of the two remaining marbles. Find the probability that the red marble was the first marble selected and that the white marble is in the same container as the red marble. Express your answer as a **common fraction** reduced to lowest terms.

12. $\left(\sqrt[4]{2}\right)\left(\sqrt[12]{6}\right) = \sqrt[24]{k}$ where k is a positive integer. Find the value of k.

13. In the diagram (**not drawn to scale**), Points A, B, C, D, and E lie on a circle with center at F. $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD} \parallel \overleftrightarrow{FE}$, $AB=BC=DE$, and $FE=8$. Find the **exact** value of $(AD-DE)$.



14. A student received a grade of 76 on a final examination in math for which the mean grade was 70 and the standard deviation was 5.
Find the “standard score” or “z-score” on this exam for this student. Express your answer as a **decimal**.

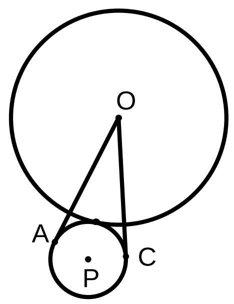
15. A right circular cylinder has a total surface area of 154π. Find the length of the radius that is needed for the right circular cylinder to have maximum volume. Express your answer as a **decimal** rounded to the nearest hundredth.

16. In ΔABC, $AB = 7$, and $AC = 8$. $\cos(\angle BAC)$ is a rational number that can be expressed as $\frac{k}{w}$ where k and w are relatively prime positive integers. It is known that $(k+w)$ is the cube of a positive integer and that BC is a positive integer. Find the sum of all possible distinct values of BC.

17. If $(x+y)^5$ is expanded and completely simplified, find the sum of the numerical coefficients.

18. If $i = \sqrt{-1}$, find $|21-72i|$.

19. In the diagram, a circle with center at P is externally tangent to the circle with center at O. Circle P is also tangent to $\overleftrightarrow{OA}$ at A and to $\overleftrightarrow{OC}$ at C. $\angle AOC = 26°$, and the length of a radius of the circle with center at P is 2.416. Find the length of a radius of the circle with center at O. Express your answer as a **decimal** rounded to the nearest thousandth.



20. The Polar Coordinates of points D, E, and F are $D(42, 154°)$, $E(93, 117°)$, and $F(67, 1°)$. Find the radius of the inscribed circle of Triangle DEF. Round your answer for the radius to the nearest integer and express your answer as that **integer**.

2012 RAA

Pre-Calculus

Name ANSWERS

School ______________________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 17

3. IV (Must be this Roman numeral.)

4. $2\sqrt{3}$ (Must be this exact answer.)

5. $\sqrt{137}$ (Must be this exact answer.)

6. 5

7. -1

8. 16

9. 4.00 (Must be this decimal, inches optional.)

10. 0.3096 OR .3096 (Must be this decimal.)

11. $\frac{1}{6}$ (Must be this reduced common fraction.)

12. 2304

13. 8

14. 1.2 (Must be this decimal.)

15. 5.07 (Must be this decimal.)

16. 19

17. 32

18. 75

19. 8.324 (Must be this decimal.)

20. 22

NO CALCULATORS

1. From the four names—Truman, Ryan, Daley, and Bush—1 name is selected at random. Find the probability that the name selected was composed of exactly four letters. Express your answer as a common fraction reduced to lowest terms.

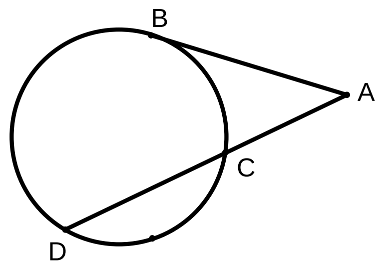
2. **(Always, Sometimes, or Never True)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

   The median of a triangle divides the triangle into two triangles of equal area.

3. If x is an integer, find the sum of all distinct values of x such that $|x-3| \le 5$.

4. Given four triangles with side-lengths of each triangle as shown by the ordered triples: $(3,4,5)$, $(5,12,14)$, $(\sqrt{43},\sqrt{53},4\sqrt{6})$, $(\sqrt{7},1,3)$. If one of the four triangles is selected at random, find the probability that the triangle selected is a right triangle. Express your answer as a common fraction reduced to lowest terms.

5. In the diagram, $\overline{AB}$ is tangent to the circle at B. Points A, C, and D are collinear, and C and D lie on the circle. If $AC = 3$ and $AB = \sqrt{78}$, find CD.



6. A line contains the point $(2,2)$ and is perpendicular to a second line that contains the points $(5,-1)$ and $(2,8)$. The equation of the first line can be expressed as $x - ky + w = 0$. Find the value of $(2k+3w)$.

7. Given the 4 points: $A(2,4)$, $B(20,7)$, $C(10,17)$, $D(18,12)$. Find the area of the region that is in the exterior of ΔACD but is in the interior of ΔABC.

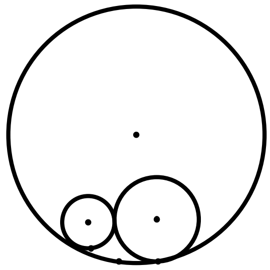
8. The perimeter of a **scalene** triangle is 22, and the longest side has a length of 10. Find the longest possible length of the shortest side if all sides have lengths that are whole numbers.

NO CALCULATORS

NO CALCULATORS

9. On a plane surface, two secants of a circle intersect at point P in the exterior of the circle. The angle formed by the two secants intercepts arcs which are respectively $\frac{2}{5}$ and $\frac{1}{4}$ of the circle. Find the degree measure of the acute angle formed by these two secants.

10. Each of the three circles shown in the diagram is tangent to the other two. If the radii of the three circles have lengths of 1, 2, and 9, find the exact area of the triangle formed by connecting the centers of the three circles.



11. $\frac{3-\sqrt{5}}{3+\sqrt{5}}=\frac{a-b\sqrt{c}}{d}$ where a, b, c, and d are positive integers. Find the smallest possible value of $(a+b+c+d)$.

12. The sides of a triangle have lengths of 5, 7, and 8. The area of the circumscribed circle of this triangle can be expressed as $\frac{k\pi}{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

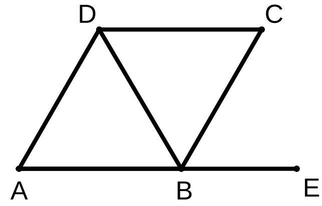
13. There are three consecutive **odd** integers. If the largest is multiplied by 4 and then the result is added to the smallest, the result is 181. Find the smallest of the three consecutive **odd** integers.

14. From five angles—two of which are acute, one of which is right, and two of which are obtuse, two angles are selected at random without replacement. Find the probability that neither of the angles selected has a cosine which is negative. Express your answer as a common fraction reduced to lowest terms.

NO CALCULATORS

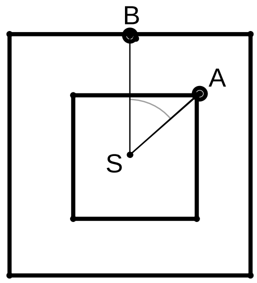
NO CALCULATORS

15. In the diagram, $\overleftrightarrow{DC} \parallel \overleftrightarrow{AB}$, $\overleftrightarrow{BD}$ bisects $\angle ADC$ and $\angle ABC$, and points A, B, and E are collinear. $\angle CBE = 60°$, and $ABCD$ has a perimeter of 240. Find the perimeter of ΔABD.



16. If m and n are the two distinct roots of $x^2 + 154x - 5 = 0$, find the value of $(m-n)^2$.

17. In a different galaxy, two planets revolve around their sun in concentric square orbits. Planet A travels in a clockwise direction at 2 million miles per hour in an orbit of length 8 million miles. Planet B travels in a clockwise direction at 5 million miles per hour in an orbit of length 16 million miles. Initially, Planet A has a 45° head start (see diagram). Find the number of hours it will be before the two planets are, for the first time, separated by the maximum possible distance.

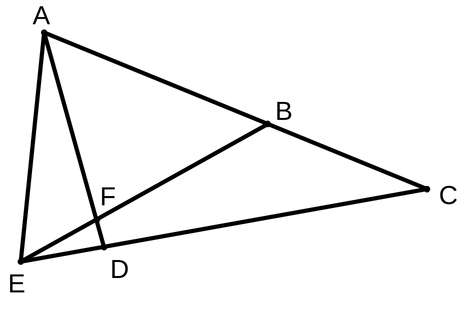


m∠BSA=45°

18. Use each of the four digits 2, 7, 8, and 9 exactly once to form the numbers k and w in the equation $\sqrt{k} - w = 10$. Find the value of the product (kw).

19. The length of an edge of a regular tetrahedron is 60. The absolute value of the difference between the surface area of the circumscribed sphere of this tetrahedron and the surface area of the inscribed sphere of this tetrahedron is $k\pi$. Find the value of k.

20. In the diagram, ΔAEC is acute, B lies on $\overline{AC}$, D lies on $\overline{EC}$, and $\overline{AD}$ and $\overline{BE}$ intersect at F. If $DC = 4(DE)$ and $AB : BC = 3 : 2$, then the ratio of the area of ΔEFD to the area of quadrilateral $BCDF$ is $k : w$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.



NO CALCULATORS

2012 RAA

School ANSWERS

Fr/So 8 Person

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)
2. Always (Must be this whole word.)
3. 33
4. $\frac{1}{2}$ (Must be this reduced common fraction.)
5. 23
6. 18
7. 42
8. 5
9. 27 (Degrees optional.)
10. $6\sqrt{3}$ (Must be this exact answer.)
11. 17
12. 52
13. 33
14. $\frac{3}{10}$ (Must be this reduced common fraction.)
15. 180
16. 23736
17. 10 (Hours optional.)
18. 2023
19. 4800
20. 17

NO CALCULATORS

1. Let $A = \{1, 2, 3\}$. If two distinct members of A are selected at random, find the probability that the sum of those two distinct members is 5. Express your answer as a **common fraction** reduced to lowest terms.

2. Find the value of $\dfrac{\log(128)}{\log(256)}$. Express your answer as a common fraction reduced to lowest terms.

3. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   Define the difference set $A - B$ as the set $\{x : x \in A \text{ and } x \notin B\}$. $(A - B) =$

   A) $B - A$ B) $B' - A'$ C) $A \cup B$ D) $A \cap B$ E) None of the previous.

   Note: B' is defined to mean the complement of set B.
   Note: Be sure to write the correct capital letter as your answer.

4. At Urbana, 33 teams entered the single elimination basketball tournament in which a team is eliminated after it loses one game. At Champaign, 13 teams entered the double elimination basketball tournament in which a team is eliminated after it loses two games. If there will be a champion after k games in Urbana and after a **maximum** of w games in Champaign, find the value of $(k + w)$.

5. Two fair, standard cubical dice are rolled one at a time. If the number on the uppermost face of the first die rolled is a five, find the probability that sum of the numbers on the two uppermost faces is more than eight. Express your answer as a **common fraction** reduced to lowest terms.

6. Find the sum of the first seventeen terms of an arithmetic sequence whose ninth term is 3 and whose seventeenth term is 27.

7. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   For statements x and y, the implication $x \to y$ is logically equivalent to which of the following?

   A) $y' \to x'$ B) $y' \to x$ C) $y \to x$ D) $x' \to y'$ E) $x' \to y$

   Note: x' denotes the negation of statement x.
   Note: Be sure to write the correct capital letter as your answer.

NO CALCULATORS

8. The coordinate axes of the graph of the equation $9x^2 - 3xy - 5y^2 - 267 = 2x^2 + xy - 9y^2$ are rotated through a positive acute angle (θ) so as to eliminate the xy term. The value of $\sin(\theta)$ can be expressed as $\frac{k\sqrt{w}}{f}$ where k, w, and f are positive integers. Find the smallest possible value of $(k + w + f)$.

9. It is known that x varies directly as y and inversely as the square of z. If $x = 5$ when $y = 100$ and $z = 2$, find the value of x when $y = 100$ and $z = 10$. Express your answer as a common fraction reduced to lowest teems.

10. Let k be the square of an integer. Let k have exactly 8, exactly 9, or exactly 15 distinct positive integral divisors. If $k < 261$, find the sum of all distinct values of k.

11. If $\begin{bmatrix} 4 & 1 & 0 \\ 2 & k & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ -2 & -2 \end{bmatrix}$, find the value of k.

12. The range of the real-valued function $y = \sqrt{3-x} + \sqrt{3+x}$ is given by $\{y : k \le y \le w\}$. Find the value of $(k + w)$.

13. The successive class marks (or class midpoints) in a frequency distribution of the weights of students are: 123, 135, 147, 159, 171, 183, 195, and 207. The third lowest class of weights ranges from k to w where k and w are positive integers. Find the value of $(k + w)$.

14. Bob is a gambler. Two orange and four blue marbles are placed in a container. From these six marbles, two are drawn at random without replacement. Bob is a winner if both marbles are blue or if one marble is orange and the other marble is blue. Find the probability that Bob is a winner. Express your answer as a **common fraction** reduced to lowest terms.

NO CALCULATORS

15. A ball is dropped from a height of 54 inches and strikes a flat surface. Each time the ball strikes the flat surface, it rebounds to $\frac{2}{3}$ of its previous height. Find the total number of inches the ball will travel before it eventually comes to rest.

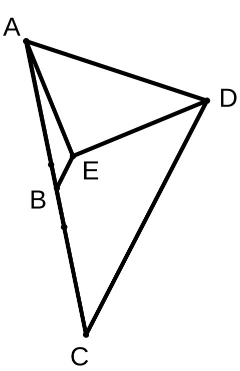
16. Find the radian period of the graph of $y = -2\tan(-\frac{1}{4}\theta)$.

17. Let $x = 3^a + 2^b$, where a and b are chosen independently (i.e., with replacement) from the twenty-five integers from 1 through 25 inclusive (with each integer having an equal likelihood of being chosen). Find the probability that x is an integral multiple of 5. Express your answer as a **common fraction** reduced to lowest terms.

18. One card has blue on each side, and a second card has orange on one side and blue on the other. One of these cards is selected at random and placed on a table. If the shown side of the card on the table is blue, find the probability that the other side of that card is also blue. Express your answer as a **common fraction** reduced to lowest terms.

19. All ages in this problem are in years. Dick is one and a half times as old as he was when he was one and a half times as old as he was when he was five years younger than half as old as he is now. Find the number of years in Dick's present age.

20. In the coplanar diagram, points A, B, and C are collinear, $AD = 16$, and $AC = 21$. $\angle ADE \cong \angle CDE$, $\angle AED = 90°$, $\angle DAB = 60°$, and $AB = BC$. Find BE. Express your answer as a **decimal**.



2012 RAA

School ___ANSWERS___

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{3}$ (Must be this reduced common fraction.)
2. $\frac{7}{8}$ (Must be this reduced common fraction.)
3. B (Must be this capital letter.)
4. 57
5. $\frac{1}{2}$ (Must be this reduced common fraction.)
6. 51
7. A (Must be this capital letter.)
8. 12
9. $\frac{1}{5}$ (Must be this reduced common fraction.)
10. 957
11. -3
12. $\sqrt{6}+2\sqrt{3}$ OR $2\sqrt{3}+\sqrt{6}$
13. 294
14. $\frac{14}{15}$ (Must be this reduced common fraction.)
15. 270 (Inches optional.)
16. 4π (Radians optional.)
17. $\frac{157}{625}$ (Must be this reduced common fraction.)
18. $\frac{2}{3}$ (Must be this reduced common fraction.)
19. 90 (Years optional.)
20. 1.5 (Must be this decimal.)

1. If $g(x) = x^6 + x^5 - x^4 + 24.54$, find the value of $g(3.111)$.

2. A regular polygon with 37 sides is inscribed in a circle with a diameter whose length is 15.47. Find the area of this regular polygon.

3. (x, y) is the solution for the following system,: $\begin{cases} 6.25x + 3.48y = 33.87804 \\ 12.96x - 4.32y = 11.14992 \end{cases}$. Find the value of $(x + y)$

4. The length of an arc of a circle is 123.4. If the radius of the circle is 36.2, find the degree measure of the central angle subtended by the arc.

5. Quadrilateral $QUAD$ is inscribed in a circle. $\angle DQU = (243x + 13)°$, $\angle QUA = (517x - 186)°$, and $\angle UAD = (113x + 7)°$. Find the degree measure of $\angle ADQ$. **Express your answer in the form $k°w'$ with the value of w rounded to the nearest minute.**

6. A triangle has a base of length 45.12 and a height on that base of length h. To form a new triangle, the old triangle has its base of length 45.12 decreased by w and its height, on that base, is the length h increased by 984.1. If the area of the old triangle is the same as the area of the new triangle, then $w = \dfrac{p}{h + q}$. Find the value of $(p + q)$. Express your answer as an **exact decimal**. Do **not** use scientific notation.

7. If the vector $(1.384, -2.841, 3.856)$ is perpendicular to the vector $(2.845, 5.997, k)$, find the value of k.

8. A graph of a curve of the form $y = ax^3 + bx^2 + cx + d$ with a, b, c, and d rational numbers, passes through the following points with (x, y) coordinates: $(-20, -16)$, $(-8, 12)$, $(10, 3)$ and $(28, 16)$. Let (k, w) be an ordered pair of the form (x, y) that lies on the given curve. If k and w are both positive integers and if $29 < k < 99$, find the sum of all possible distinct values of w. **Express your answer as a whole number**.

9. If $x = 34.53$, find the value of $x^2 + x^{-2} + x^{-1.584}$. Express your answer as a **decimal rounded to the nearest thousandth**. Do **not** use scientific notation.

10. In Triangle ABC, $AB = 245$, and $\angle ABC = 60°$. If all sides of Triangle ABC have integral lengths, find the smallest possible perimeter of Triangle ABC such that the perimeter is greater than 1560. Express your answer for the perimeter as an **exact integer**.

11. One diagonal of a rectangle has a length of 48.42. Find the largest possible area of any such rectangle.

12. A six-sided die is weighted so that the probability distribution is as shown:

x	1	2	3	4	5	6
$P(x)$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	k

. The value, denoted by x on the table, of a single roll of a die is defined to be the number that appears on the top of the die when it comes to rest. Find the expected value on a single roll of a die. **Express your answer as an improper fraction reduced to lowest terms.**

13. Rounded to 5 decimal places, the total population standard deviation (σ_X) of a set of k numbers is 2.47861. Rounded to 5 decimal places, the sample population standard deviation (S_x) of the same set of k numbers is 2.48195. Naturally, k is a positive integer. Find the value of k. **Express your answer as an exact integer.**

14. Sheila, the seamstress, buys her cloth in strips that are 30 inches wide and 864 inches long. She estimates that in making a shirt there is a waste amount of 8.96% of each 30-inch wide rectangular piece of cloth that she cuts from her long strip. If the finished shirt actually contains 752 square inches of cloth, find the number of shirts that Sheila can make from one 30 inch wide by 864 inch strip of cloth. Round your answer to the nearest integer and express your answer as that **integer**.

15. Find the amplitude of the graph of $y = 3.459\sin(\theta) + 7.248\cos(\theta)$.

16. The radius of the wheel of an exercise bike is 10 inches. If the wheel is turning 3 times per second, determine the **linear speed in mph.** of a point on the circumference of the wheel.

17. The length of a slant height of a regular square pyramid is 65. The length of a side of the square base is an even integer, and the length of the altitude is an integer. Find the absolute value of the difference between the largest and the smallest possible volume of such a pyramid. Express your answer as an **exact integer**.

18. Find the slope of a line that passes through $(13.17, -19.25)$ and $(68.86, 1728.1)$.

19. Assume that in the world's general population, 90% of the persons are predominantly right-handed, 9% of the persons are predominantly left-handed, and 1% are ambidextrous. If Jim, Bill, and John are 3 persons selected at random from the world's general population, find the probability that at least 2 of those 3 persons are predominantly left-handed. **Express your answer as a decimal rounded to the nearest thousandth.** Do **not** use scientific notation.

20. A particle must travel from $(6, -5, 10)$ to a point R on the plane whose equation is $x - 2y + 2z = 9$, and then from point R to the point $(3, -8, 13)$. The particle travels at constant rates of 3 units per second from $(6, -5, 10)$ to point R and 4 units per second from point R to $(3, -8, 13)$. Find the minimum possible number of seconds necessary to complete the required journey. **Express your answer as a decimal rounded to the nearest hundred-thousandth**. Do **not** use scientific notation.

2012 RAA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 1129 OR 1.129×10^{3}

2. 187.1 OR 1.871×10^{2}

3. 7.691 OR 7.691×10^{0}

4. 195.3 OR 1.953×10^{2} (Degrees optional.)

5. $133°38'$ (Must be in degree and minutes form.)

6. 45386.692 (Must be this decimal.)

7. 3.397 OR 3.397×10^{0}

8. 3362 (Must be this integer.)

9. 1192.325 (Must be this decimal.)

10. 1568 (Must be this integer.)

11. 1172 OR 1.172×10^{3}

12. $\frac{8}{3}$ (Must be this reduced improper fraction.)

13. 372 (Must be this integer.)

14. 31 (Must be this integer.)

15. 8.031 OR 8.031×10^{0}

16. 10.71 OR 1.071×10 OR 1.071×10^{1} (mph. optional.)

17. 119104 (Must be this integer)

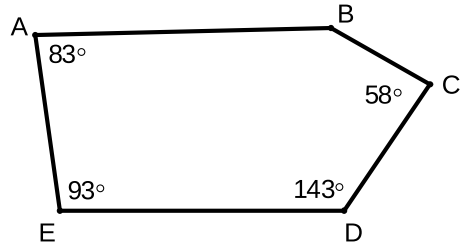
18. 31.38 OR 3.138×10 OR 3.138×10^{1}

19. 0.023 OR .023 (Must be this decimal.)

20. 6.11873 (Must be this decimal.)

1. Find the value of x such that $4^{(2x+5)} = \frac{1}{8}$. Express your answer as a **decimal**.

2. In the figure with degree measures as shown, let k be the degree measure of $\angle ABC$. The roots of $(5x-4)^2-(x-3)^2=3$ are equivalent to the roots of $ax^2-17x+b=0$. Find the value of $(a+b+k)$.



3. Find the sum of the distinct roots of the following four equations:

$5x-8=112$ $y^2-4y-12=0$ $z^2=288$ $|x-1| = 10$

4. In ΔABC with $A(1,-5)$, $B(-3,7)$, and $C(4,4)$, find the length of the median from C to $\overline{AB}$.

5. Working alone at constant rates , it takes the respective number of hours to clean Room 379: Karen, 3; Kay, 7; Tom, 13. With no loss of efficiency, let k and w be the respective number of hours required if the first two work together and then if the last two work together. Expressed as a **decimal**, find the value of $(k+w)$.

6. Let x be an integer such that $80 \le x \le 89$. Let S be the sum of all possible distinct values of x such that it is impossible to use exactly x American coins that will have a monetary value of one dollar. Find the value of S.

7. An equilateral quadrilateral has diagonals whose lengths are 56 and 90. Find the perimeter of the equilateral quadrilateral.

8. Let y vary inversely as x. If $y=3$ when $x=2$, find the value of y when $x=5$. Let q vary jointly as z and h. If $q=12$ when $h=3$ and $z=2$, find the value of q when $z=5$ and $h=4$. For your answer, give the value of $(y+q)$ as a **decimal**.

9. Some right triangles with three integer side lengths satisfy the property that area and perimeter are numerically equal. Find the sum of the perimeters of all such non-congruent triangles.

10. Players stand in a circle. Player 1 stays in. Player 2 is knocked out. Player 3, in; Player 4, out. This continues, knocking every other Player out, until only one Player remains. With 9 Players, let k be the number of the last Player remaining; with 11 players, let w be the number of the last Player remaining. Find the value of $\left(2k+3w\right)$.

ANSWERS

1. -3.25 (Must be this decimal.)

2. 177

3. 30

4. $\sqrt{34}$

5. 6.65 (Must be this decimal.)

6. 341

7. 212

8. 41.2 (Must be this decimal.)

9. 54

10. 27

2012 RAA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. -3.25 (Must be this decimal.)	______
2. 177	______
3. 30	______
4. $\sqrt{34}$	______
5. 6.65 (Must be this decimal.)	______
6. 341	______
7. 212	______
8. 41.2 (Must be this decimal.)	______
9. 54	______
10. 27	______

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. 39.5 (Must be this decimal, degrees optional.)

12. 32

13. 95

14. 60

15. 2

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let $\begin{vmatrix} k & -7 & 1 \\ 0 & 2 & 3 \\ -4 & 0 & 6 \end{vmatrix} = 152$. Let w be the area of a triangle with vertices at $(0,0)$, $(0,10)$, and $(-4,3)$. Find the value of $(k+w)$.

2. Let $a>1$ and use $\log_a 8 = 5.7132$ as an exact decimal in this problem. Find the value of $\log_a 2$. Express your answer as an **exact decimal**.

3. Find the smallest possible positive value for x for any point that lies on the conic whose equation is $\frac{(x-3)^2}{16} - \frac{y^2}{25} = 1$.

4. If $\begin{vmatrix} 1 & k & -1 \\ 2 & w & 3 \\ 3 & 2 & -1 \end{vmatrix} = 5$ and $\begin{vmatrix} 1 & -4 & 2 \\ 3 & k & 1 \\ 5 & w & 5 \end{vmatrix} = 110$, find the value of $(k+5w)$.

5. A line passes through the foci of the parabolas whose equations are $y^2 = 4x$ and $x^2 = -8y$. If the equation of that line is expressed in the form $y = mx + b$, find the value of $(5m+3b)$.

6. Find the sum of $0.\overline{4}$ (where the "4" repeats) and $0.6\overline{72}$ (where the "72" repeats). Express your answer as an improper fraction reduced to lowest terms.

7. If $90° < x < 720°$, let S be the sum of all distinct values of x such that $\cos(x°) = \sin(x°)$. Let k be the sum of the terms of an infinite geometric progression for which the first term is 784 and the common ratio is $\frac{1}{3}$. Find the value of $(S+k)$.

8. Let k be the tangent of the smallest angle of a right triangle whose hypotenuse has a length of 89 and one of whose legs has a length of 39. $[x]$ is defined as the greatest integer which is not greater than x. If $2 < x < 3$, let w be the largest possible **integral** value of $[x^2] - [x]^2$. Find the value of $(1600k + w)$.

9. Let f be a function for which $f(1) = 12$, $f(3) = 5$, and $f(k) = 3(f(k-1)) + f(k+1)$ for all positive integral k. Let (x, y) be the focus of the parabola whose equation is $y = \frac{1}{8}x^2 + 16x + 1$. Find the value of $(x + y + f(4))$.

10. By substituting 1, 2, 3, 4, and 5 for x in $P(x)$, the first five values for $P(x)$ are respectively 5, 18, 35, 56, and 81. By substituting 1, 2, 3, 4, and 5 for x in $R(x)$, the first five values for $R(x)$ are respectively 3, 17, 33, 51, and 71. If $P(x)$ and $R(x)$ are polynomial expressions of lowest degree satisfying the given, find $P(25)+R(25)$.

ANSWERS

1. 25
2. 1.9044 (Must be this decimal.)
3. 7
4. 64
5. 4
6. $\frac{553}{495}$ (Must be this reduced improper fraction.)
7. 2391
8. 784
9. -691
10. 2312

2012 RAA

Jr/Sr 2 Person Team

School **ANSWERS**

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. 25	______
2. 1.9044 (Must be this decimal.)	______
3. 7	______
4. 64	______
5. 4	______
6. $\frac{553}{495}$ (Must be this reduced improper fraction.)	______
7. 2391	______
8. 784	______
9. -691	______
10. 2312	______

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 4

12. 9

13. $(-2,-8)$ (Must be this ordered pair.)

14. $\sqrt{91}$

15. 8

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

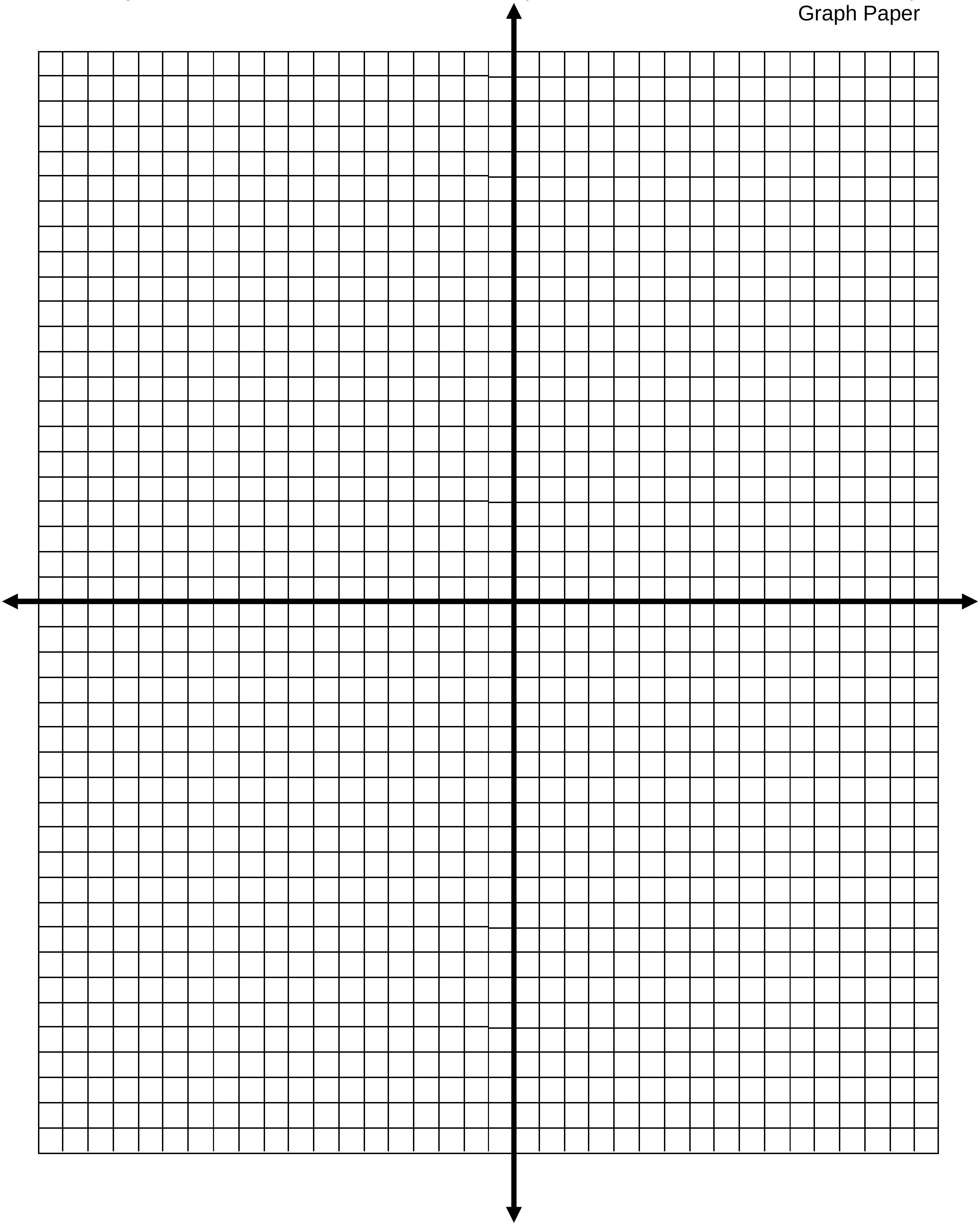
<u>The answers to all of the following problems can fit onto a single copy of the attached coordinate grid. You will be provided with a single copy of the grid on a transparency when you are presenting.</u>

1. In relation to *A*(– 4,8) and *B*(– 9,5), show graphically all points that are taxicab equidistant from *A* and *B*. What name might you use to describe these points?

2. Given *S*(– 10,– 3) and *T*(– 4,– 5), describe and sketch the set of points *P*, $\left\{P \mid d_T\left(SP\right) + d_T\left(TP\right) = 12\right\}$.

3. Ideal City's School District 2 is represented as the 1st quadrant of the plane. The four high schools in the district are located at *K*(4,8), *L*(10,4), *M*(12,10), and *N*(5,13), and everyone in District 2 attends the school closest to their home. Heather lives at *H*(10,11), but wants move so that her children will be in the part of the district served by high school *N* and be within one block of the boundary of the school. If she also wants a corner house (at a lattice point), what is the minimum number of blocks that she will have to move? Where could her new home be located?

ICTM Regional Contest 2012 Div AA Oral Competition Taxicab Geometry Graph Paper



Extemporaneous Questions

THIS SHEET SHOULD BE GIVEN TO STUDENTS AT THE BEGINNING OF THE EXTEMPORANEOUS QUESTION PERIOD.

<u>You may continue to use the transparency to answer these questions.</u>

E1. Given the points S $(4,-2)$ and T $(1,-6)$, compute the Euclidean distance and the taxicab distance between the two points.

E2. With two given points E (x_1, y_1) and F (x_2, y_2), the Euclidean distance between the points and the taxicab distance between the two points are equal. What must be true about the relationship(s) between any or all of x_1, y_1, x_2, y_2?

E3. Given point *C* (6, – 14), graphically show all points *P* such that the taxicab distance CP is equal to 4 and all points *R* such that the Euclidean distance *CR* is equal to 4. What taxicab and Euclidean shapes are described by these points?

SOLUTIONS

1. In relation to *A*(– 4,8) and *B*(– 9,5), show graphically all points that are taxicab equidistant from *A* and *B*. What name might you use to describe these points?

 All points equidistant from two points would form the taxicab midset or the taxicab perpendicular bisector of $\overline{AB}$ ***.***

 See quadrant II graph.

2. Given *S*(– 10,– 3) and *T*(– 4,– 5), describe and sketch the set of points *P*, $\left\{P \mid d_T\left(SP\right)+d_T\left(TP\right)=12\right\}$.

 The shape is a taxicab ellipse, since the sum of the distances to the two fixed points is constant. Students may note that the points S and T are the foci of the ellipse. (Note: the center of the taxi-ellipse is at (- 7, - 4). Students do not need to give this point; it is included for reference only.)

 See quadrant III graph.

3. Ideal City's School District 2 is represented as the 1st quadrant of the plane. The four high schools in the district are located at *K*(4,8), *L*(10,4), *M*(12,10), and *N*(5,13), and everyone in District 2 attends the school closest to their home. Heather lives at *H*(10,11), but wants move so that her children will be in the part of the district served by high school *N* and be within one block of the boundary of the school. If she also wants a corner house (at a lattice point), what is the minimum number of blocks that she will have to move? Where could her new home be located?

 3 blocks. Three different points are possible for the new home, as shown on the graph, at (7, 11), (8, 12) and (9, 13).

 Students should be given credit for finding any one of the possibilities and may just point out the location(s) on the graph. They should not be penalized if they do not include the coordinates of the new home.

 Note: only the boundary between N and M is necessary to solve the problem, so students should not be penalized for omitting the others.

 See quadrant I graph.

SOLUTIONS – Extemporaneous Questions

E1. Given the points S $(4,-2)$ and T $(1,-6)$, compute the Euclidean distance and the taxicab distance between the two points.

Euclidean distance is 5, using distance formula or Pythagorean Theorem ($\sqrt{3^2+4^2}$ ***). The taxicab distance is 7 (*** $3+4$ ***).***

See quadrant IV graph.

E2. With two given points E (x_1, y_1) and F (x_2, y_2), the Euclidean distance between the points and the taxicab distance between the two points are equal. What must be true about the relationship(s) between any or all of x_1, y_1, x_2, y_2?

If the distances are the same, the points must lie on a horizontal or vertical line. Students may give either $x_1 = x_2$ ***or*** $y_1 = y_2$ ***or both.***

E3. Given point *C* (6, – 14), graphically show all points *P* such that the taxicab distance CP is equal to 4 and all points *R* such that the Euclidean distance *CR* is equal to 4. What taxicab and Euclidean shapes are described by these points?

See quadrant IV graph. The graphs are a taxicab circle and a Euclidean circle.

SOLUTIONS and NOTES FOR JUDGES