1. If $w^7 + w^7 + 17 = k + 2w^7$, find the value of k.

2. Someone stole 5 candy bars and 2 bags of potato chips. Each bag of potato chips sells for \$2.19, and each candy bar sells for k **cents** each. If the value of the 5 candy bars and 2 large bags of potato chips that were stolen was \$7.83, find the value of k. Express your answer as an integral number of cents.

3. During a sale, Mrs. Wellness was able to buy every 5 bottles of vitamin pills for the usual cost of 3 bottles of vitamin pills. If Mrs. Wellness bought 75 bottles of vitamin pills during that sale, how many bottles of vitamin pills would she have been able to buy at the regular price?

4. Find the value of the product (ab) given that $\frac{1}{a} + \frac{1}{b} = \frac{5}{6}$ and $a + b = \frac{15}{2}$.

5. The lines $kx + 3y = 9$ and $8x + 2kx - 2y = 15$ do not have a point of intersection. Find the value of k.

6. Two numbers have an arithmetic mean of 38. When a third number is included, the arithmetic mean is decreased by 4. Find the third number.

7. For what value(s) of k will the equation $9x^2 = kx - 25$ have a single solution?

8. $92\frac{6}{7}\%$ of the residents of Urbana live in private homes. Of those who live in private homes in Urbana, 70% live in air-conditioned homes. Find the percent of the residents of Urbana who live in air-conditioned private homes. Be sure to express your answer as a **percent**.

9. The numerator of a fraction is a positive integer that is one more than its denominator. The sum of the fraction and its reciprocal is $\frac{25}{12}$. Find the numerator.

10. How many milliliters of a 15% copper sulfate solution must be added to 8 milliliters of a 40% copper sulfate solution to obtain a 20% copper sulfate solution?

11. If $\sqrt{x+14} = x$, then one solution for x is $x = \frac{k+\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.

12. On Monday, Jose withdraws 42% of the amount of money in his savings account. On Tuesday, Jose withdraws $4300 of the amount then in his savings account. On Wednesday, Jose withdraws $3440 of the amount then in his savings account. After the Wednesday withdrawal, there is exactly 22% of the amount that was in the savings account prior to Jose's withdrawal on Monday left in the savings account. Find the number of dollars left in the savings account after Jose's withdrawal on Wednesday.

13. What is the largest integral value for which $|3x-2| < x+11$?

14. Let x be a single digit non-zero positive integer and let y be a two digit positive integer. Find all ordered pairs of the form (x, y) such that the sum of x and y and the product of x and y each contain the same digits but in reverse order. Be certain to express all possible answers as **ordered pairs** of the form (x, y).

15. Solve for k if $\left(x^2x^k\right)^5 = x^{75}$.

16. A survey was given to 100 people, all of whom responded “for” or “against”. Among the 56 males who responded, there was a $1:3$ ratio of who responded “for” vs “against.” If 17 females responded to the survey with “for”, then how many total people responded “against”?

17. Al’s age is the same as Bruce’s age and Carl’s age together. In two years, Al will be twice as old as Bruce will be then. Six years ago, Carl was twice as old as Bruce was then. How old will Carl be in two years?

18. When the polynomial $2x^3 - 5x^2 + 3x + k$ is divided by $(x-2)$, the remainder is 5. Find the value of k.

19. The product of two numbers is 24, and the sum of the reciprocals of the two numbers is $\frac{7}{12}$. Find the **larger** of the two numbers.

20. Assume that it is possible to travel 1 mile at 8 mph., then 1 more mile at 12 mph., and then 1 more mile at k mph. where k is a positive integer. If the average rate for the 3 miles (total distance divided by total time) is w mph, where w is a positive integer, find the sum of all possible distinct values for k.

2013 RAA
Algebra I

Name ANSWERS

School ____________________

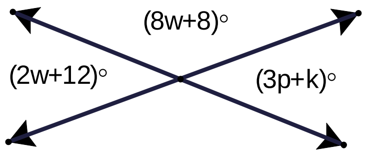
(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 17
2. 69 (Must be this integer, cents optional.)
3. 45 (Bottles optional.)
4. 9
5. -3
6. 26
7. $30, -30$ (Must have both answers, either order, solution set optional.)
8. 65 (% optional.)
9. 4
10. 32 (Milliliters or ml of copper sulfate optional.)
11. 60
12. 4730 ($ optional.)
13. 6
14. $(2, 47), (3, 24)$ (Must have both ordered pairs, either order.)
15. 13
16. 69 (People optional.)
17. 12 (Years old optional.)
18. 3
19. 12
20. 206

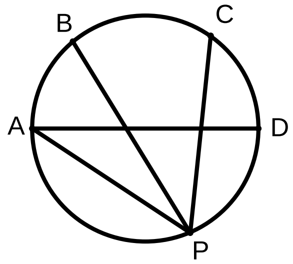
1. The diagram shows two intersecting lines with degree measures as indicated. Find the value of w.



2. Three vertices of parallelogram $ABCD$ are $A(8,6)$, $B(2,5)$, and $C(4,-2)$. Find the fourth vertex. Express your answer as an **ordered pair**.

3. The lengths of the sides of a triangle are in the ratio of $3:4:5$. If the perimeter of the triangle is 48, find the length of the shortest side of the triangle.

4. In the diagram, points A, B, C, D, and P lie on a circle. $\overline{AD}$ is a diameter, and $m\angle BPC = 45°$. The measures of minor arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are in the ratio of $7:8$ respectively. Find the degree measure of $\angle APC$.



5. A square has a perimeter of 80. Find the length of a diagonal of that square.

6. A right circular cone has a slant height whose length is 6.3. If the **diameter** of the circular base is 11.2, then the **lateral area** of the cone is $k\pi$. Find the value of k. Express your answer as an **exact decimal**.

7. ΔABC has vertices at $A(4,8)$, $B(12,7)$, and $C(x,y)$. The midpoint of one of the sides of ΔABC is $(7,5)$. Find the area of ΔABC.

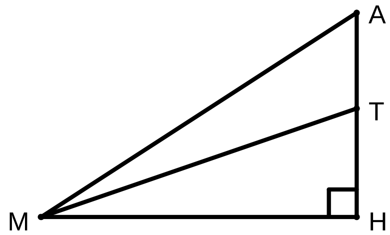
8. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   Which of the following is an equation for the locus of points in a plane that are equidistant from $(1,0)$ and $(7,2)$?

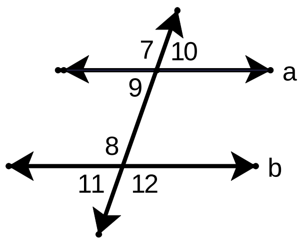
   A) $y=-3x+13$ B) $y=3x-3$ C) $y=\frac{1}{3}x-\frac{1}{3}$ D) $y=-\frac{1}{3}x+\frac{1}{3}$ E) $y=-\frac{1}{3}x-\frac{1}{3}$

   Note: Be certain to write the correct capital letter as your answer.

9. $\overline{MH} \perp \overline{HA}$ and A, T, H are collinear points. If $MT = 8\sqrt{3}$, $m\angle TMA = 15°$ and $m\angle HTM = 60°$, then the length of $\overline{AT} = k + w\sqrt{p}$. Find the sum $(k + w + p)$.



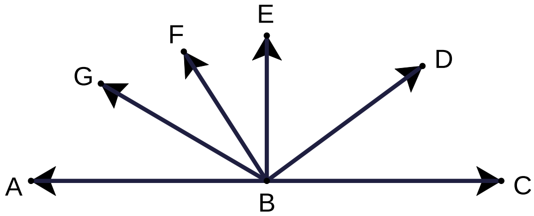
10. In the diagram, parallel lines a and b are cut by a transversal. If the degree measure of $\angle 8$ is $15x - 60$, and the degree measure of $\angle 9$ is $3x + 6$, find the numerical degree measure of $\angle 9$.



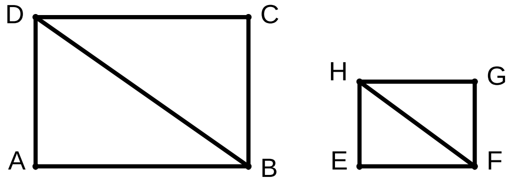
11. The length of one side of a regular hexagon is $2x$ and the length of the side that is opposite the first side is $3y$. If the area of the regular hexagon is $1944\sqrt{3}$, find the value of $(x + y)$.

12. The numerical total surface area of a sphere is $k\pi$ square units, and the numerical volume of the same sphere is $w\pi$ cubic units. If k and w are both integers that contain the same number of digits, find the sum of all distinct possibilities for the number of units in the length of a radius of the sphere.

13. In the coplanar figure shown, A, B, and C are collinear, $\overline{EB} \perp \overline{AC}$, $\overrightarrow{BD}$ bisects $\angle EBC$, $\overrightarrow{BF}$ bisects $\angle GBE$, and $\angle FBD = 74°$. Find the degree measure of $\angle ABG$.

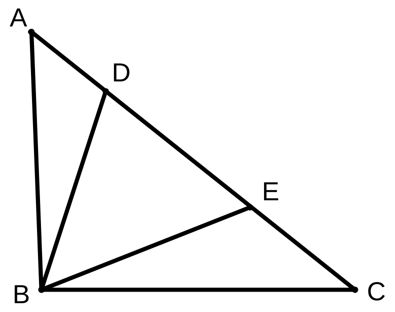


14. In the diagram, $ABCD$ and $EFGH$ are rectangles such that the lengths of all sides and diagonals are integers. The area of rectangle $ABCD$ is 144.375% of the area of rectangle $EFGH$. If $BD = 85$ find HF.



15. A square is inscribed in a regular octagon, with each vertex of the square at the midpoint of a side of the regular octagon. Each side of the regular octagon has a length of 6. The perimeter of the square can be expressed in the form $k + w\sqrt{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k + w + p)$.

16. In ΔABC, points D and E lie on $\overline{AC}$ as shown. $\angle ABD = 17°$ and $\angle BAD = 37°$. If $\overline{BD} \cong \overline{BE}$, find the degree measure of $\angle DBE$.



17. A line is tangent at the point $(6,10)$ to a circle whose equation is $(x-2)^2 + (y-7)^2 = 25$. If the equation of this tangent line is written in the form $y = mx + b$, find the value of b.

18. Find the slope of a line perpendicular to the line whose equation is $y - \frac{1}{3}x = 18$.

19. The sum of the lengths of the 3 distinct altitudes of a triangle whose side-lengths are 3, 7, and 8 can be expressed as $\frac{k\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k + w + p)$.

20. Two circles have radii of 8 and 12 with centers that are 24 units apart. Find the length of a common internal tangent between these two circles.

2013 RAA
Geometry

Name ANSWERS

School ____________________
(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 16
2. $(10,-1)$ (Must be this ordered pair.)
3. 12
4. 66 (Degrees optional.)
5. $20\sqrt{2}$ (Must be this exact answer.)
6. 35.28 (Must be this exact decimal.)
7. 21
8. A (Must be this capital letter.)
9. 11
10. 45 (Degrees optional.)
11. 30
12. 36 (Units optional.)
13. 32 (Degrees optional.)
14. 68
15. 38
16. 72 (Degrees optional.)
17. 18
18. -3
19. 118
20. $4\sqrt{11}$ (Must be this exact answer.)

1. For the rational function, $f(x)=\frac{x}{x^2-4}+\frac{2x-1}{x^2+x-6}$, certain values are **NOT** included in the domain. Find the sum of these distinct restricted values.

2. **(Yes or No)** Is $(3x+1)$ a factor of the polynomial $-18x^3+27x^5-8x^2-x$? Be sure to write the whole word for your answer,

3. Let $f(x)=x+3$ and $g(x)=2x-1$. If $f(x)=g(x)-3$, find the value of x.

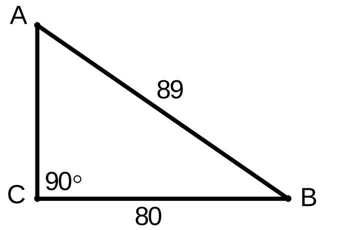
4. The major and minor axes of an ellipse have lengths respectively of 14 and 10. Using each focus as a center, the smallest possible circle that is tangent to the ellipse is drawn. The absolute value of the difference between the area of the ellipse and the sum of the areas of the two circles can be expressed as $k\pi$. Find the value of k.

5. The second term of a geometric sequence is $\frac{2}{3}$ and the fourth term is $\frac{1}{3}$. Find the sum of the eighth and tenth terms of this geometric sequence. Express your answer as a reduced common or improper fraction.

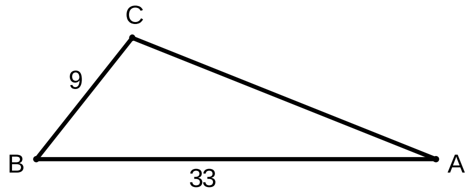
6. Let $f(x)=k\left(\log_7(x)\right)$ where k represents a non-zero constant. Then $f(343)-f(7)=wk$. Find the value of w.

7. The sum of the squares of two original positive numbers is 10. The product of the squares of these two original numbers is 2. Find the square of the sum of these two original numbers.

8. In the diagram of ΔABC with measures as shown, $AC = k$. Let $i = \sqrt{-1}$. Find $|80 + ki|$.



9. In the diagram, AB = 33 and BC = 9. If $\dfrac{\sin\left(0.5\left(\angle CAB - \angle CBA\right)\right)}{\cos\left(0.5\left(\angle ACB\right)\right)} = -0.\overline{57}$, find AC.



10. Let k be a prime integer such that $100 < k < 225$. How many distinct values for k exist such that $k = a^3 + b^3$ where both a and b are positive integers?

11. If $f(x) = x^2 + 8x - 19$, find the minimum value of $f(x)$.

12. Let $C(x, y)$ be a symbol for combinations of x distinct things taken y at a time. If $C(k,4) - C(w,4) = 425$, find the value of $(k + w)$.

13. Let a, b, and c be positive integers such that a, b, and c are the distinct roots for x in $x^3 - 2ax^2 + (15b+4)x - 40c = 0$. Find the **ordered triple** (a,b,c).

14. Each of 25 persons writes down 5 different integers at random from the 25 integers from 1 to 25 inclusive. Each of the 25 integers is then called off one at a time in a random order. As soon as all 5 of a person's numbers have been called off, the person yells: "Bingo." Find the probability that at least 1 of the 25 persons will have yelled "Bingo" before the 18^{th} number has been called. Express your answer as a decimal rounded to 4 significant digits.

15. Let $i=\sqrt{-1}$. If z represents a complex number, then $\overline{z}$ represents its complex conjugate. If $z=4-5i$, then $3\overline{z}=k+wi$ where k and w represent real numbers. Find the value of $(k+w)$.

16. In a three-dimensional rectangular coordinate system, ΔABC has vertices $A(18,0,0)$, $B(0,13,2)$, and $C(0,0,8)$. The distance from the centroid of ΔABC to the y-axis can be expressed as $\frac{k\sqrt{w}}{P}$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.

17. Let k and w be positive integers such that $k>1$ and $w<71$. Karen travels from A to B at an average rate of 40 mph. She immediately turns around and travels from B to A via the same route at an average rate of k mph. If her average rate for the total distance traveled is w mph, find the sum of all possible distinct values of k.

18. Let $f(x)=9x-256$ and $f\left(g(x)\right)=g\left(f(x)\right)=x$. Find all points that the graphs of $f(x)$ and $g(x)$ have in common. Express your answer for any and all distinct points as **ordered pair(s)**.

19. Find the focus of the parabola whose equation is $y-3=\frac{1}{12}(x+2)^2$. Express your answer as an **ordered pair** of the form (x,y).

20. Let a and b be real numbers. Let a be an even integer greater than -53 and less than 59. The largest possible value of the sum of the squares of the roots for x in the cubic equation $x^3-ax^2+ax\sqrt{74}+b=0$ can be written in simplest radical form as $k+w\sqrt{p}$. Find the value of $(k+w+p)$.

2013 RAA
Algebra II

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -3
2. YES (Must be this whole word.)
3. 7
4. $56\sqrt{6}-111$ OR $-111+56\sqrt{6}$ (Must be this exact answer.)
5. $\frac{1}{8}$ (Must be this reduced common fraction.)
6. 2
7. $10+2\sqrt{2}$ OR $2\left(5+\sqrt{2}\right)$ (Must be this exact answer or commutative equivalent.)
8. 89
9. 28
10. 0 OR zero OR none
11. -35
12. 20
13. $(8,5,3)$ (Must be this ordered triple.)
14. 0.9548 OR .9548 (Must be this decimal.)
15. 27
16. 111
17. 782
18. $(32,32)$ (Must be this orderd pair only.)
19. $(-2,6)$ (Must be this ordered pair.)
20. 2882 (Must be this integer.)

1. Given the relation $\{(2,3),(4,k),(4,2),(5,1)\}$. The **product** of all the distinct numbers in the **range** of the relation shown is 60. Find the value of k.

2. Let $i=\sqrt{-1}$. Let k and w represent positive integers with $k<w$. If $(3i)(ki)(wi)=-12i$, find the value of k.

3. The vector $(5.1,\ 3.4)$ is perpendicular to the vector $(-6.55,\ k)$. Find the value of k. Express your answer as an **exact decimal**.

4. The sum of the infinite geometric series of real terms whose first term is k is $\frac{2k}{k+1}$. If $k>0$ and the second term is k^2, find the value of k. Express your answer as a common fraction reduced to lowest terms.

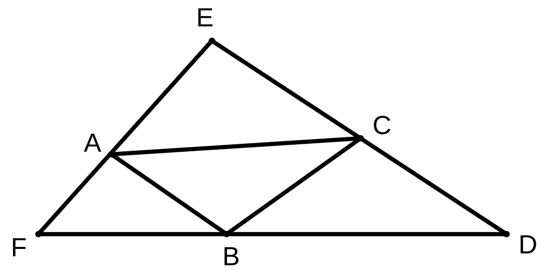
5. Find the length of the minor axis of the conic section whose equation is $5x^2-10x+9y^2-72y+104=0$.

6. Let $i=\sqrt{-1}$ and let k and w represent real numbers. If $(k+wi)(9-2i)=x+7i$ is solved for x, then $x=74-14i$. Find the value of $(k+w)$.

7. Bob, Christie, and Judy repeatedly take turns tossing a fair, standard cubical die. Bob begins, Christie always follows Bob, and Judy always follows Christie. Find the probability that Christie will be the first to toss a die that lands so that its uppermost face displays an odd number of spots. Express your answer as a common fraction reduced to lowest terms.

8. Let f be a real-numbered function. If $f(x)=\dfrac{\log(x-2)}{x^2-40}+\sqrt{8\sqrt{3}-x}$, then the domain of f, given in simplified radical form in interval notation, is $(k, a\sqrt{b})\cup(c\sqrt{d}, g\sqrt{h}]$ where k, a, b, c, d, g, and h are positive integers. Find the value of $3k+2a+b+c+d+7g+h$.

9. In the diagram, points A, B, and C lie on $\overline{FE}$, $\overline{FD}$, and $\overline{ED}$ respectively. If $EA=10$, $AF=5$, $BF=12$, $BD=22$, $CD=16$, and $CE=19$, find the area of ΔABC. Express your answer as an improper fraction reduced to lowest terms.



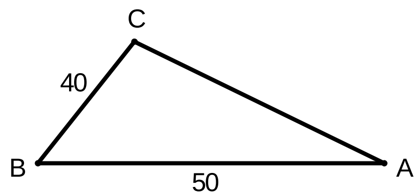
10. Find the degree of the following polynomial: $3x(x^3+5)+6(x^2(x-7)^3)^2+34x-9$.

11. There are 12 teams in a league. Each team in the league must play every other team in the league exactly 10 times in one season. Find the number of games that will be played among the teams in this league during one season.

12. Find the sum of all distinct positive values of y that satisfy the system: $\begin{cases}\log_9 x=\log_3 y\\ x^2-5y^2=-6\end{cases}$.

13. The first term of an arithmetic sequence is 8, and the thirteenth term of this arithmetic sequence is -28. Find the **sum** of the other eleven terms of this arithmetic sequence.

14. In the diagram, $AB=50$ and $BC=40$. If $\dfrac{\sin\left(0.5\left(\angle CAB-\angle CBA\right)\right)}{\cos\left(0.5\left(\angle ACB\right)\right)}=\dfrac{12}{25}$, find the exact length AC.



15. Let $i = \sqrt{-1}$ and let k represent a real number. If $7 + ki^3 + x = 4 + 6i^2 + 3i$ is solved for x, then $x = -9 - 7i$. Find the value of k.

16. Let n represent an integer such that $0 < n < 6$. Two of the roots for x of the cubic equation $x^3 + nx^2 - (n+1)x + k = 0$ are additive inverses. Find the sum of all possible distinct values of k.

17. Katie randomly places 3 identical math, 2 identical science, and 2 identical English books on a shelf. Find the probability that the 3 math books are together on the shelf. Express your answer as a common fraction reduced to lowest terms.

18. Given that $\sin(A) = -\dfrac{2}{3}$ and $\pi < A < \dfrac{3\pi}{2}$ (where A is measured in radians). Then $\sin(2A)$ can be expressed in simplest radical form as $\dfrac{k\sqrt{w}}{p}$ where k and p are relatively prime integers and $p > 0$. Find the value of $(k + w + p)$.

19. Let $i = \sqrt{-1}$ and let $(x - 1 - 2i)$, $(x - 1 + 2i)$, and $(x - 5)$ be factors of the polynomial $x^3 + kx^2 + wx + p$ where k, w, and p represent integers. Find the value of $(k + p)$.

20. In ΔABC, let $AB = c$, $AC = b$, and $BC = a$. $\angle BAC = 60°$, and $(a + b + c)(a + b - c) = \dfrac{49}{19}ab$. If the lengths of all sides of the triangle are integers, find the smallest possible value of c.

2013 RAA

Pre-Calculus

Name ___**ANSWERS**___

School ______________________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 10

2. 1

3. 9.825 (Must be this decimal.)

4. $\frac{1}{3}$ (Must be this reduced common fraction.)

5. $2\sqrt{5}$ (Must be this exact answer.)

6. 9

7. $\frac{2}{7}$ (Must be this reduced common fraction.)

8. 91

9. $\frac{4812}{85}$ (Must be this reduced improper fraction.)

10. 10

11. 660 (Games optional.)

12. $\sqrt{2}+\sqrt{3}$ OR $\sqrt{3}+\sqrt{2}$ (Must be this exact answer.)

13. -110

14. 16

15. -10

16. -70

17. $\frac{1}{7}$ (Must be this reduced common fraction.)

18. 18

19. -32

20. 21

NO CALCULATORS

1. If the sum of two consecutive **odd** integers is 624, find the smaller of the two integers.

2. The number of bacteria in a certain culture doubles by the **end** of each day. At the **start** of the first day, there were 4 bacteria in the culture. How many bacteria would be in the culture at the **start** of the fourth day?

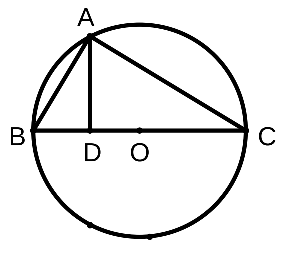
3. How many distinct negative integers greater than -182 are equal to 6 times an **even** integer?

4. The lengths of all sides of a right triangle are whole numbers. If the length of one side of the triangle is 12, find the smallest possible perimeter of the right triangle.

5. Find the length of a **diameter** of a circle whose equation is $x^2 - 14x + y^2 + 2y = 71$.

6. $\{x : x \leq k\}$ is the set of all real number values of x such that $2x + 5x + \sqrt{16 - x}$ is a real number. Find the value of k.

7. In the diagram, points A, B, and C lie on the circle, points B, D, O, and C are collinear, point O is the center of the circle, $AC = 17$, $AD = 8$, and $\overline{AD} \perp \overline{BC}$. Find the perimeter of ΔABC. Express your answer as an improper fraction reduced to lowest terms.



NO CALCULATORS

NO CALCULATORS

8. Find the numeric value of the expression $\frac{8x^2+6xy-9y^2}{16x^2+24xy-27y^2}$ given that $x=2.5$ and $y=\frac{2}{3}$. Give your final answer as a simplified fraction.

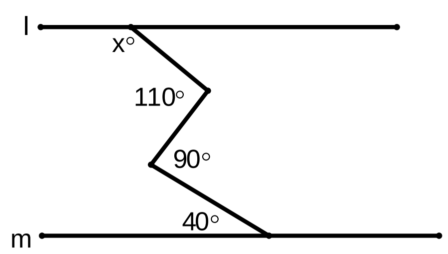
9. The degree measure of one of two complementary angles is 24 degrees less than twice the degree measure of the other. Find the degree measure of the supplement of the larger of the two angles.

10. The equation of the circle passing through the point $(3,-1)$ and tangent to the line $3x+2y=5$ at the point $(1,1)$ can be written in the form $(x-h)^2+(y-k)^2=r^2$ where $r>0$. Find the value of $(h+k+r)$.

11. Two of the vertices of an equilateral triangle are $(0,0)$ and $(14,0)$. If the third vertex of this equilateral triangle is in the fourth quadrant, find the **ordered pair** of coordinates for the third vertex.

12. Let a, b, c, and d represent 4 positive integers, all greater than 4, such that the sum of the 4 integers is 245. If 6 were added to the first, 6 subtracted from the second, 6 multiplied by the third, and 6 divided into the fourth, all the resulting integers would be equal. Find the product $(abcd)$.

13. In the diagram, lines *l* and *m* are parallel. The angles have degree measures as shown. Find the value of x.



14. For what value(s) of k will the points $(6, k-1)$, $(k+1, k+2)$ and $(k+5, -1)$ be collinear?

NO CALCULATORS

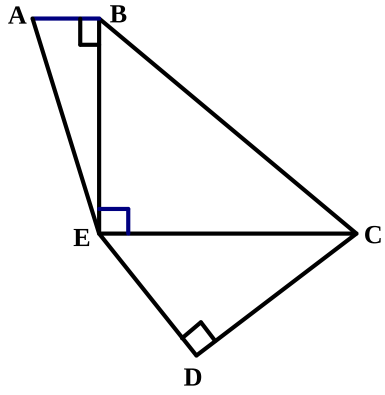
15. The ratio of the hypotenuse of a given right triangle to one of the legs is $2:1$. If one of the angles of this triangle is selected at random, what is the probability that its measure is greater than or equal to $45°$? Express your answer as a reduced common fraction.

16. Find the largest integer that divides 169, 419, and 793 with remainders R_1, R_2, and R_3, respectively such that $R_2 = R_1 + 2$ and $R_3 = R_2 + 2$.

17. Assume that the trains between Chicago and St. Louis leave each city for the other every hour on the hour. On its run from Chicago to St. Louis, a train will meet exactly k trains that have left St. Louis and are headed to Chicago. If the one-way trip in either direction takes exactly 6 hours and ten minutes, find the value of k.

18. Find the smallest possible area of a right triangle with one side whose length is 8 if one of angles of the triangle has a measure of 30°.

19. In the diagram, if $\text{AE} = 28\sqrt{3}$, $\text{ED} = 24$, $\text{CD} = 32$ and $m\angle\text{AEC} = 120°$, find the length of $\overline{\text{BC}}$?



20. Find the exact value of $\sqrt{97 + 56\sqrt{3}}$.

2013 RAA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 311

2. 32

3. 15

4. 30

5. 22

6. 16

7. $\frac{136}{3}$ (Must be this reduced improper fraction.)

8. $\frac{7}{16}$ (Must be this reduced common fraction.)

9. 128 (Degrees optional.)

10. $12+2\sqrt{13}$ OR $2\left(6+\sqrt{13}\right)$ (Must be this exact answer or commutative equivalent.)

11. $\left(7,-7\sqrt{3}\right)$ (Must be this ordered pair.)

12. 777,600 OR 777600

13. 120

14. $3, -1$ (Must have both values, either order.)

15. $\frac{2}{3}$ (Must be this reduced common fraction.)

16. 124

17. 13

18. $8\sqrt{3}$ (Must be this exact answer.)

19. 58

20. $7+4\sqrt{3}$ OR $4\sqrt{3}+7$ (Must be this exact answer.)

NO CALCULATORS

1. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never—**whichever is correct.

   If $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$ represent 4 vectors such that $\vec{a} = \vec{c}$ and $\vec{b} = \vec{d}$, then $\vec{a} + \vec{b} \neq \vec{c} + \vec{d}$.

2. Let $C(n,k) = \dfrac{n!}{k!(n-k)!}$. Find the value of $C(9,4)$.

3. When $(x-3y)^5$ is expanded and completely simplified, the coefficient of one of the terms is 90. Find the **exponent** of x for that term.

4. On a trip to Alaska, Judy asked a person in Fairbanks what the temperature was. The person replied: "$x°$." "Is that Fahrenheit or Celsius?" Judy asked. "Doesn't matter," was the reply. Find the value of x.

5. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never—**whichever is correct.

   Is the conclusion drawn from the two given statements valid?

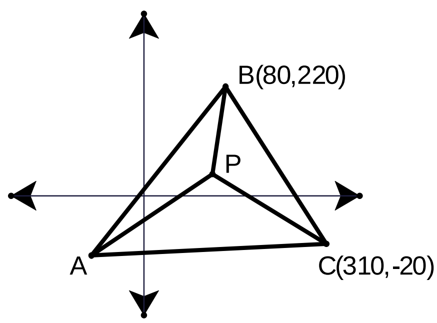
   (1) The number k is even if and only if k is divisible by 2.
   (2) k is divisible by 2.

   Conclusion: The number k is even.

6. When $\left(x+\dfrac{1}{x}\right)^4\left(x-\dfrac{1}{x}\right)^4$ is expanded and completely simplified, find the coefficient of the degree 4 (x^4th) term.

7. Let $i=\sqrt{-1}$. Let k and w represent positive integers with $k<w$. If $(3i)(ki)(wi)=-36i$, find the **sum** of all possible values of k.

8. In the diagram (not necessarily drawn to scale) with ΔABC and coordinates $B(80,220)$, $C(310,-20)$, and point P in the interior of the triangle as shown, the ratio of the area of ΔPAB to the area of ΔPBC to the area of ΔPAC is $1:4:5$. Find the **ordered pair** that represents point A if the **ordered pair** that represents point P is $(37,66)$.



NO CALCULATORS

9. Let $\oplus n$ be the sum of the integers from 1 to n inclusive. For example $\oplus 3$ would be $1+2+3=6$. If $(\oplus 10)-(\oplus 9)=(\oplus k)$, find the value of k.

10. Let p represent a single-digit **positive** integer. If k represents the smallest positive integer and w represents the largest possible integer for all possible values of p such that the statement $w<\log_2(p)<k$ is always true, find the value of $(k+w)$.

11. A tennis ball dropped straight down from a height of 30 feet bounces straight up 40% of the height from which it fell on each bounce. Find the number of feet in the vertical distance that the ball travels before coming to rest.

12. In ΔABC, $AB=16$, $BC=5$, and $AC=19$. $\overrightarrow{AD}$ bisects $\angle CAB$, and $\overrightarrow{CD}$ bisects $\angle ACB$. Expressed in simplest radical form, $CD=k\sqrt{w}$ where k and w are positive integers. Find the value of $(k+w)$.

13. Two infinite geometric series have the same finite sum. One infinite geometric series has $\frac{1}{4}$ as its common ratio. The second infinite geometric series has $\left(-\frac{1}{3}\right)$ as its common ratio. If the first term of one of these two series is 12, find the sum of all possible first terms for the other series. Write your answer as an integer if possible or as a reduced common or improper fraction if your answer is not an integer.

14. The sum of the first and fourth terms of an arithmetic progression is 70. If the third term of this arithmetic progression is subtracted from the tenth term of the progression, the result is 84. Find the sum of the first eight terms of this arithmetic progression.

15. A parabola has a directrix whose equation is $y=0$ and has its vertex at $(4,2)$. The equation of this parabola can be expressed in the form $ky+w=(x-4)^2$. Find the value of $(7k+2w)$.

NO CALCULATORS

16. Each school day Linda and her swimming instructor each report to the swimming pool at a random time between 5:00 A. M. and 6:00 A. M. If the instructor arrives first, then if Linda arrives before or at 5:30 A. M., Linda swims 20 laps; otherwise, when Linda arrives after 5:30 A. M., Linda swims 40 laps. If Linda arrives first, then if the instructor arrives before or at 5:30 A. M., Linda swims 16 laps; otherwise, when the instructor arrives after 5:30 A. M., Linda swims 12 laps. Find the expected value of the number of laps that Linda will swim early in the morning on the next school day.

17. Let $i = \sqrt{-1}$. Which quadrant contains the most 25th roots of $(-1+i)$? Express your answer as a **Roman numeral**.

18. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

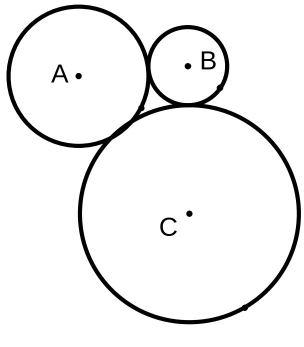
    Eight students took a statistics test, and each received an integral score from 0 to 100 inclusive. The eight scores were: 75, 74, 73, 77, 79, 78, 83, x. Rounded to the nearest hundredth, the total population standard deviation (σ_X) of the 8 scores was 16.15. Was x an outlier?

    A) Yes.
    B) No.
    C) Cannot tell without knowing the median of the 8 scores.
    D) Cannot tell without knowing the mean of the 8 scores.
    E) Cannot tell without knowing the sample population standard deviation of the 8 scores.

    Note: Be certain to write the correct capital letter as your answer.

19. Let a, b, and c be integers and let $x^3 + ax^2 + bx + c = 0$ have $\sqrt[3]{5} + \sqrt[3]{25}$ as one of the roots for x. Find the value of $(a+b+c)$.

20. In the diagram, (not necessarily drawn to scale), each circle is tangent to the other two circles. The centers of the circles are A, B, and C. $BC = 10$, and the lengths of the radii (in some order) of circles A and B are in the ratio of $1:4$. If the lengths of the radii of the three circles are all integers, and if the area of ΔABC is an integer, find the largest possible area of ΔABC.



2013 RAA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. Never (Must be this whole word.)
2. 126
3. 3
4. -40
5. Always (Must be this whole word.)
6. -4
7. 6
8. $(-85, -105)$ (Must be this ordered pair.)
9. 4
10. 3
11. 70 (Feet optional.)
12. 20
13. $\frac{337}{12}$ (Must be this reduced improper fraction.)
14. 472
15. 24
16. 24
17. II (Must be this Roman numeral.)
18. A (Must be this capital letter.)
19. -45
20. 48

Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

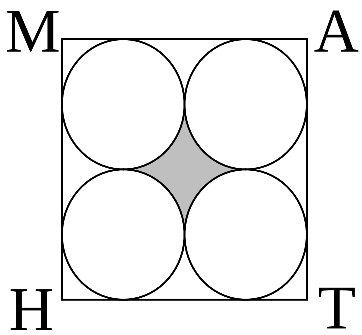
1. $\sqrt[5]{x} = 3.777$. Find the value of x.

2. The perimeter of a regular hexagon is 723.5. Find the length of an apothem of the regular hexagon.

3. If $x = -5.125$ and $y = 1.668$, find the value of $\left(x - 3.145y\right)^2$.

4. Polly was admitted to the hospital at 13 minutes and 27 seconds after 9:00 in the morning of April 3, 2009, and was released from the hospital at 11 minutes and 16 seconds after 4:00 in the afternoon of April 15, 2009. Find the number of **seconds** that elapsed between Polly's admission to the hospital and her release from the hospital. Express your answer as an **exact integer**. Do **not** use scientific notation.

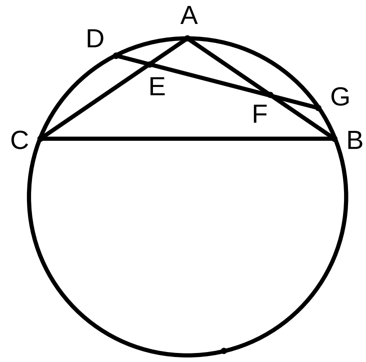
5. In the diagram, $MATH$ is a square. $AT = 6.000$. The 4 circles are congruent. The circles are tangent to other circles and to the square as shown. Find the area of the shaded region enclosed solely by minor arcs of all 4 circles.



6. Express the value of 2013^{2013} in scientific notation. Be certain to express your answer in **scientific notation**.

7. Find the area of a circle inscribed in a triangle with sides whose lengths are 4.610, 8.750, and 11.17.

8. In the diagram (not necessarily drawn to scale), points A, B, C, D, and G lie on the circle. $\overline{AC} \cong \overline{AB}$, points D, E, F and G are collinear, points A, E, and C are collinear, and points A, F, and B are collinear. $\overline{AC} \cong \overline{DG}$, $\angle CAB = 150°$, and $\overset{\frown}{BG} = 10°$. If $CE = 143.4$, find EF.



9. Find the value of $\tan(15°22')$.

10. In ΔABC, $AB = 4.000$, $BC = 7.000$, and $AC = 9.000$. The bisectors of $\angle BAC$ and $\angle BCA$ meet at P. Find AP.

11. Hilda deposits \$975 in an account that pays 5.5% annual percentage rate interest compounded continuously. Find the balance in this account after 6 years. Answer in standard dollars and cents decimal notation, rounded to the nearest penny.

12. In Illinois, birth weights of newborn babies are normally distributed with mean 7.2 pounds and standard deviation 1.1 pounds. In Ohio, birth weights of newborn babies are normally distributed with mean 7.0 pounds and standard deviation 0.9 pounds. Baby William of Illinois had a birth weight corresponding to a z-score of 1.3. Baby Chloe of Ohio had a birth weight corresponding to a z-score of (-0.6). By how many pounds did William outweigh Chloe at birth? Answer as a decimal rounded to the nearest tenth of a pound.

13. The fraction $\frac{179}{57}$ is a close approximation of π. Let k and w be relatively prime positive integers such that $(k+w) < 523$. Find the value of $(k+w)$ such that $\left|\frac{k}{w} - \pi\right|$ is a minimum. Express your answer for $(k+w)$ as an **integer**.

14. The number of square units in the area of a circle is 203.58948 more than the number of units in the circumference of this circle. Find the length of a radius of this circle.

15. Given the following sequence of eight numbers: 1, 12, 123, 1234, 12345, 123456, 1234567, 123456789. Find the largest prime that is a factor of at least one member of the sequence. Express your answer as a **whole number**.

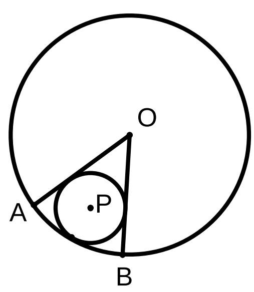
16. In a certain state's lottery there are 56 balls. Each ball has been painted with a unique positive integer from 1 through 56. Six balls are chosen at random without repetition. Let k be the number of different six number **combinations** possible in this certain state's lottery. If k coffee cups each with three inch outside diameters are placed side by side in a straight line, how many **miles** would the line of coffee cups stretch? Round your answer to the nearest mile, and then express your answer as that **integer**.

17. Find the number of distinct non-similar triangles for which the degree measure of the first angle is an integral multiple of 3, the degree measure of the second angle is an integral multiple of 7, and the degree measure of the third angle is an integral multiple of 11. Express your answer as an **exact whole number**.

18. Find the value of $\log(0.3279)$.

19. The length of a side of a regular pentagon is k units. If the number of square units in the area of the regular pentagon is equal to the number of units in the perimeter of the regular pentagon, find the value of k.

20. $\overline{OA}$ and $\overline{OB}$ are radii of the circle with center at O and are each tangent to the smaller circle with center at P. The smaller circle is tangent to the larger circle. The area of the larger circle is 546.284, and $\angle AOB = 68.24°$. Find the area of the region that is in the exterior of the smaller circle but in the interior of the 68.24° sector of the larger circle.



2013 RAA

Calculator Team

School **ANSWERS**

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 768.7 OR 7.687×10^2

2. 104.4 OR 1.044×10^2

3. 107.6 OR 1.076×10^2

4. 1061869 (Must be this integerk seconds optional.)

5. 1.931 OR 1.931×10^0

6. 4.340×10^{6650} (Must be in scientific notation, trailing zero necessary.)

7. 7.547 OR 7.547×10^0

8. 85.15 OR 8.515×10 OR 8.515×10^1

9. 0.2748 OR $.2748$ OR 2.748×10^{-1}

10. 3.286 OR 3.286×10^0

11. 1356.19 (Must be this decimal, $ optional.)

12. 2.2 (Must be this decimal, pounds optional.)

13. 468 (Must be this integer.)

14. 9.112 OR 9.112×10^0

15. 9721 (Must be this whole number.)

16. 1537 (Must be this integer.)

17. 59 (Must be this integer.)

18. -0.4843 OR $-.4843$ OR -4.843×10^{-1}

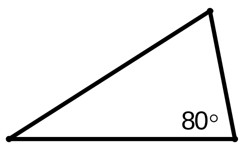
19. 2.906 OR 2.906×10^0

20. 33.01 OR 3.301×10 OR 3.301×10^1

1. If $7x+78=1002-7y$, then $k=x+y$. Let w be the **number** of distinct positive integral factors of 2013. Find the value of $(k+w)$.

2. A rectangle has an area of 72, and the length of one side of the rectangle is twice the length of a second side. Let k be the length of a diagonal of the rectangle. Let $w=\sqrt{80}+3\sqrt{125}+4\sqrt{2000}$. Find the value of $(k+w)$.

3. Wanda finished $\frac{5}{8}$ of a job in $\frac{3}{4}$ of a day. At this constant rate, she will need k days for the entire job. The other two angles of the triangle shown have degree measures in the ratio of $7:18$. Let w be the degree measure of the smallest angle of the triangle. Find the value of $(k+w)$. Express your answer as a simplified improper fraction.

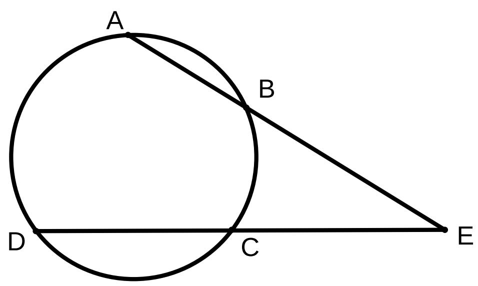


4. The 4 midpoints, taken in order, of the consecutive sides of a rectangle whose side-lengths are 32 and 60 are joined to form a quadrilateral whose perimeter is p. Let $i=\sqrt{-1}$ and let $3a+bi = 27i-33$. Find the value of $(p+a+b)$.

5. Let k be the area of the region bounded by the graphs of $y=|x|-6$ and $y=-|x|+6$. Two chords of a circle intersect in the interior of the circle and have lengths of 22 and w. The divisions of one chord have lengths of 16 and 6; one division of the other chord has a length of 4. Find the value of $(k+w)$.

6. The lengths of all sides of a triangle are integers. Two of the sides have lengths of 7 and 14. Let k be the possible number of non-congruent **scalene acute** triangles. Let x represent a positive integer. Let S be the sum of all distinct values of x such that $4-2(x-7)\geq 3x+1$. Find the value of $(k+S)$.

7. Let k be the least possible sum of 4 consecutive positive integers if k is an integral multiple of 9. In the diagram, points A, B, C, and D lie on the circle. $AB=7.8$, $BE=5$, $EC=4$, and $CD=w$. Points A, B, and E are collinear. Points D, C, and E are collinear . Find the value of $(k+w)$.



8. S. T. Atistic gave a test to 20 students. The first 12 students had an average score of 80, while the overall average for the 20 students was 78. Let k be the average of the other 8 students. Let r be the radius of the circumscribed circle of a right triangle whose legs have lengths of 32 and 60. Find the value of $(k+r)$.

9. Four circles of equal radius are situated around the inside of a larger circle of radius 20 so each of the smaller circles is tangent to the larger circle and the two smaller circles on either side. Find the radius of one of these smaller circles. Report your answer as a decimal rounded to 4 significant digits.

10. If a, b, and c are positive integers, how many distinct ordered triples of the form (a,b,c) exist such that $a+2b+3c=15$?

11. If $x+17=y$, find the value of $|x-y| + |y-x|$.

12. The total surface area of a rectangular solid is 1378. Two of the edges of the rectangular solid have lengths of 19 and 16. It is known that the length of a third edge of the rectangular solid is less than 16. Find the length of this third edge.

13. Let k and w be positive integers such that $2^k = 512$ and $3^w = 729$. Find the value of $(k+w)$.

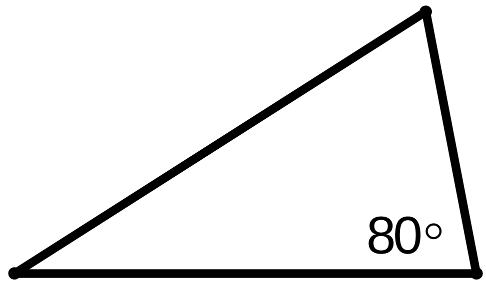
14. The line whose equation is $y = mx + b$ has a slope of 5 and contains the point $(17,15)$. In a right triangle with legs of respective lengths of 32 and 60, A and B are the respective midpoints of the two legs. Let k be the distance from A to B. Find the value of $(b+k)$.

15. Let k represent the cost to a merchant of one item of merchandise. The merchant set the selling price at 50% above his cost. When it did not sell, he reduced the price by 20% to a final price of \$12.60. Find k in dollars and cents form, rounded to the nearest penny. Using the rounded value of k, let w be the positive difference between the numeric values for area and circumference for a circle with diameter k. As your answer, write the value of w only, as a decimal rounded to the nearest hundredth.

1. If $7x+78=1002-7y$, then $k=x+y$. Let w be the **number** of distinct positive integral factors of 2013. Find the value of $(k+w)$.

2. A rectangle has an area of 72, and the length of one side of the rectangle is twice the length of a second side. Let k be the length of a diagonal of the rectangle. Let $w = \sqrt{80} + 3\sqrt{125} + 4\sqrt{2000}$. Find the value of $(k + w)$.

3. Wanda finished $\frac{5}{8}$ of a job in $\frac{3}{4}$ of a day. At this constant rate, she will need k days for the entire job. The other two angles of the triangle shown have degree measures in the ratio of $7:18$. Let w be the degree measure of the smallest angle of the triangle. Find the value of $\left(k+w\right)$. Express your answer as a simplified improper fraction.



4. The 4 midpoints, taken in order, of the consecutive sides of a rectangle whose side-lengths are 32 and 60 are joined to form a quadrilateral whose perimeter is p.
Let $i=\sqrt{-1}$ and let
$3a+bi = 27i-33$.
Find the value of $\left(p+a+b\right)$.

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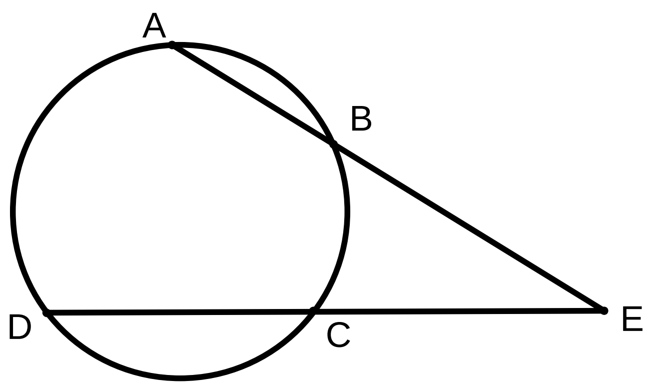

8. S. T. Atistic gave a test to 20 students. The first 12 students had an average score of 80, while the overall average for the 20 students was 78. Let k be the average of the other 8 students. Let r be the radius of the circumscribed circle of a right triangle whose legs have lengths of 32 and 60. Find the value of $\left(k+r\right)$.

9. Four circles of equal radius are situated around the inside of a larger circle of radius 20 so each of the smaller circles is tangent to the larger circle and the two smaller circles on either side. Find the radius of one of these smaller circles. Report your answer as a decimal rounded to 4 significant digits.

10. If a, b, and c are positive integers, how many distinct ordered triples of the form (a,b,c) exist such that $a+2b+3c=15$?

11. If $x+17=y$, find the value of $|x-y|+|y-x|$.

12. The total surface area of a rectangular solid is 1378. Two of the edges of the rectangular solid have lengths of 19 and 16. It is known that the length of a third edge of the rectangular solid is less than 16. Find the length of this third edge.

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15. Let k represent the cost to a merchant of one item of merchandise. The merchant set the selling price at 50% above his cost. When it did not sell, he reduced the price by 20% to a final price of $12.60. Find k in dollars and cents form, rounded to the nearest penny. Using the rounded value of k, let w be the positive difference between the numeric values for area and circumference for a circle with diameter k. As your answer, write the value of w only, as a decimal rounded to the nearest hundredth.

2013 RAA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		**Score** (to be filled in by proctor)
1. 140		______
2. $105\sqrt{5}$	(Must be this exact simplified answer.)	______
3. $\frac{146}{5}$	(Must be this reduced improper fraction.)	______
4. 152		______
5. 100		______
6. 8		______
7. 30		______
8. 109		______
9. 8.284	(Must be this exact decimal.)	______
10. 12	(Ordered triples optional.)	______

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 34
12. 11
13. 15
14. –36
15. 53.60 (Must be this decimal, trailing zero necessary.)

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let $\log_3(k)+\log_3(4)=2$. Let S be the sum of the arithmetic sequence consisting of the first 23 positive integers. Find the product (kS).

2. Of a group of 26 mathletes, 11 like both algebra and geometry problems, 12 like both geometry and trig problems, and 9 like both algebra and trig problems. Find the absolute value of the difference between the **greatest number** possible and the **least number** possible of the 26 who like all three types of problems.

3. Let n be a positive integer such that $1 \le n \le 16$. Find the sum of all distinct values of n such that $\left(1+\sum_{k=1}^{n}(k!)\right)$ will have a units digit of 4.

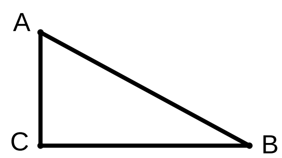
4. How many more distinct ordered quadruples of positive integers are solutions to $a+b+c+d \le 7$ than are solutions to $a+b+c+d=7$?

5. The first term of a geometric sequence is equal to the first term of an arithmetic sequence, and the fourth terms of each sequence are also equal. The common ratio of the geometric sequence is 2. If the first term is an integer, and the common difference d of the arithmetic sequence is an integer such that $0<d<54$, find the largest possible acceptable value of d.

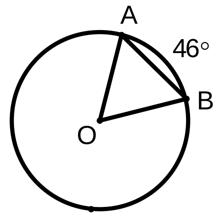
6. Let k, b, c, and d be integers such that the graph of $y=\dfrac{x+k}{x^3+bx^2+cx+d}$ has a "hole" discontinuity (is undefined) at $x=6$ and vertical asymptotes of $x=-2$ and $x=7$. Let w be the numerical coefficient of the term involving m^5p^2 of $(m-2p)^7$. Find the value of $(k+b+c+d+w)$.

7. Let k be the number of distinct positive integers that leave a remainder of 23 when divided into 1904. Let S be the sum of the first eleven terms of the geometric progression whose first term is 1 and whose fourth term is 8. Find the value of $(k+S)$.

8. In the diagram, $AB=67.24$, $BC=96.45$, the area of ΔABC is 2193.2, and $\angle ABC$ is an **acute** angle of $k°$. A hyperbola has the equation $xy=582.468$. Let (a,b) be the coordinates of the vertex that lies in Quadrant I of the hyperbola. Expressed as a **decimal** rounded to the nearest tenth, find the value of $(k+a+b)$.



9. Let r be the numerical remainder when dividing $\left(x^3 - 25x^2 + 13x - 2\right)$ by $\left(x - 3\right)$. O is the center of the circle shown, the area of the segment of the circle bounded by $\overline{AB}$ and its $46°$ minor arc is 6.317239, and the circumference of the circle is $k\pi$. Expressed as a **decimal** rounded to the nearest tenth, find the value of $\left(r + k\right)$.



10. If $(3a - b)^{10}$ is expanded and completely simplified, find the sum of the numerical coefficients.

11. Find the absolute value of the distance from the point $(2,-1,7)$ to the plane whose equation is $4y-3z=2x$.

12. The equation of the directrix of the parabola whose equation is $y^2-10y-16x=7$ is $x=k$. Find the value of k.

13. Let B represent the largest value of y such that $y=-|x+6|+3$ where x is a real number. Let A represent the smallest product of two real numbers whose difference is 6. Find the value of $(A+B)$.

14. In base x, $123_x=227_{ten}$; in base y, $147_y=628_{ten}$. Express the value of $(x+y)$ as a base ten numeral if both x and y are positive integers.

15. $\sum_{n=1}^{100}\left[(\sin n^\circ)(2\cos n^\circ)\right]=k\left(\sum_{n=1}^{100}\left[(\sin n^\circ)(\cos n^\circ)\right]\right)$. Find the value of k.

1. Let $\log_3(k)+\log_3(4)=2$. Let S be the sum of the arithmetic sequence consisting of the first 23 positive integers. Find the product (kS).

2. Of a group of 26 mathletes, 11 like both algebra and geometry problems, 12 like both geometry and trig problems, and 9 like both algebra and trig problems. Find the absolute value of the difference between the **greatest number** possible and the **least number** possible of the 26 who like all three types of problems.

3. Let n be a positive integer such that $1 \leq n \leq 16$. Find the sum of all distinct values of n such that $\left(1+\sum_{k=1}^{n}(k!)\right)$ will have a units digit of 4.

4. How many more distinct ordered quadruples of positive integers are solutions to $a+b+c+d \leq 7$ than are solutions to $a+b+c+d=7$?

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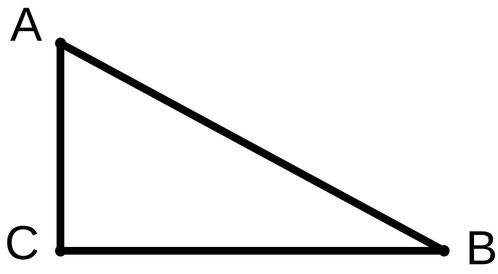
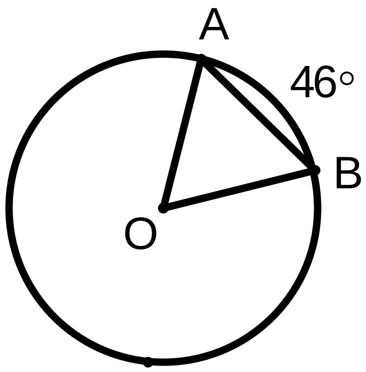


8. In the diagram, $AB = 67.24$, $BC = 96.45$, the area of ΔABC is 2193.2, and $\angle ABC$ is an **acute** angle of $k°$. A hyperbola has the equation $xy = 582.468$. Let (a, b) be the coordinates of the vertex that lies in Quadrant I of the hyperbola. Expressed as a **decimal** rounded to the nearest tenth, find the value of $(k + a + b)$.

9. Let r be the numerical remainder when dividing $\left(x^3 - 25x^2 + 13x - 2\right)$ by $\left(x - 3\right)$.

O is the center of the circle shown, the area of the segment of the circle bounded by $\overline{AB}$ and its 46° minor arc is 6.317239, and the circumference of the circle is $k\pi$. Expressed as a **decimal** rounded to the nearest tenth, find the value of $\left(r + k\right)$.



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13. Let B represent the largest value of y such that $y = -|x+6| + 3$ where x is a real number. Let A represent the smallest product of two real numbers whose difference is 6. Find the value of $(A + B)$.

14. In base x, $123_x = 227_{ten}$; in base y, $147_y = 628_{ten}$. Express the value of $(x+y)$ as a base ten numeral if both x and y are positive integers.

15. Given the equation below:

$$\sum_{n=1}^{100}\left[\left(\sin n^\circ\right)\left(2\cos n^\circ\right)\right]$$

$$=k\left(\sum_{n=1}^{100}\left[\left(\sin n^\circ\right)\left(\cos n^\circ\right)\right]\right).$$

Find the value of k.

2013 RAA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. 621	
2. 6	
3. 132	
4. 15	
5. 49	
6. 167	
7. 2054	
8. 90.8 (Must be this decimal.)	
9. –136.4 (Must be this decimal.)	
10. 1024	

TOTAL SCORE: ________
(*enter in box above)

Extra Questions:

11. $\sqrt{29}$ (Must be this exact answer.)

12. –6

13. –6

14. 37

15. 2

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

ORAL COMPETITION
ICTM REGIONAL 2013 DIVISION AA

1. In ordinary algebra the Cancellation Law for multiplication says:
Let *a*, *b* and *c* be real numbers, with $a \neq 0$. If $ab = ac$, then *b* = *c*.

Write the corresponding "law" for vectors when the operation is the dot product and determine whether it is in fact a law in vector algebra.

2. Let $\vec{P} = \vec{i} + \vec{j} + 2\vec{k}$, $\vec{Q} = \vec{i} - \vec{j} + \vec{k}$, $\vec{R} = -\vec{i} + \vec{j} + 3\vec{k}$, $\vec{S} = 2\vec{i} - \vec{j} + \vec{k}$.
Use the definition to prove that the set $\{\vec{P}, \vec{Q}, \vec{R}, \vec{S}\}$ is linearly dependent.

3. Let $\left|\overrightarrow{PQ}\right| = 12$ and let $\overrightarrow{PC} = t\,\overrightarrow{CQ}$ and $\overrightarrow{PD} = -t\,\overrightarrow{DQ}$, where *t* is a scalar.

(*i*) Find $\left|\overrightarrow{CD}\right|$ when *t* = 2.

(*ii*) Find $\left|\overrightarrow{CD}\right|$ when *t* = - 4.

(*iii*) There is no solution to the problem "Find $\left|\overrightarrow{CD}\right|$ when *t* = 1."
Explain why this is so.

ORAL COMPETITION
ICTM REGIONAL 2013 DIVISION AA

EXTEMPORANEOUS QUESTIONS

Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. In two-space, two non-zero vectors which are both perpendicular to the same non-zero third vector are parallel to each other.
   Explain why this is so and determine whether the same result holds in three-space.

2. Given the vectors $\vec{v} = \vec{i} + \sqrt{3}\,\vec{j}$ and $\vec{w} = 4\vec{i}$.
   Find the projection of $\vec{v}$ on $\vec{w}$ and the projection of $\vec{w}$ on $\vec{v}$.

ORAL COMPETITION
ICTM REGIONAL 2013 DIVISION AA

1. In ordinary algebra the Cancellation Law for multiplication says:
Let *a*, *b* and *c* be real numbers, with $a \neq 0$. If $ab = ac$, then $b = c$.

Write the corresponding "law" for vectors when the operation is the dot product and determine whether it is in fact a law in vector algebra.

SOLUTION

The vector statement reads:
Let $\vec{A}, \vec{B}$ and $\vec{C}$ be vectors, with $\vec{A} \neq \vec{0}$. If $\vec{A} \bullet \vec{B} = \vec{A} \bullet \vec{C}$, then $\vec{B} = \vec{C}$.

The conclusion does not follow.

What *does* follow from $\vec{A} \bullet \vec{B} = \vec{A} \bullet \vec{C}$, using the distributive law for dot product over addition/subtraction, is $\vec{A} \bullet (\vec{B} - \vec{C}) = 0$. This means that $\vec{A}$ and $\vec{B} - \vec{C}$ are perpendicular.

The vectors $\vec{A} = \vec{i} + 2\vec{j}$, $\vec{B} = 5\vec{i} + 4\vec{j}$ and $\vec{C} = 3\vec{i} + 5\vec{j}$ provide a counter-example to the "law" since $\vec{A} \bullet \vec{B} = 5 + 8 = 13$ and $\vec{A} \bullet \vec{C} = 3 + 10 = 13$, but $\vec{B} \neq \vec{C}$.

2. Let $\vec{P}=\vec{i}+\vec{j}+2\vec{k}, \quad \vec{Q}=\vec{i}-\vec{j}+\vec{k}, \quad \vec{R}=-\vec{i}+\vec{j}+3\vec{k}, \quad \vec{S}=2\vec{i}-\vec{j}+\vec{k}.$
Use the definition to prove that the set $\{\vec{P}, \vec{Q}, \vec{R}, \vec{S}\}$ is linearly dependent.

SOLUTION

We must show that there are scalars (numbers) x_1, x_2, x_3, x_4, not all of which are 0, so that $x_1\vec{P}+x_2\vec{Q}+x_3\vec{R}+x_4\vec{S}=\vec{0}.$
Substituting, we get $x_1(\vec{i}+\vec{j}+2\vec{k})+x_2(\vec{i}-\vec{j}+\vec{k})+x_3(-\vec{i}+\vec{j}+3\vec{k})+x_4(2\vec{i}-\vec{j}+\vec{k})=\vec{0}.$
Rewrite as $(x_1+x_2-x_3+2x_4)\vec{i}+(x_1-x_2+x_3-x_4)\vec{j}+(2x_1+x_2+3x_3+x_4)\vec{k}=\vec{0}.$
Since the set $\{\vec{i}, \vec{j}, \vec{k}\}$ is linearly independent (Reference, page 48) we have

$$x_1+x_2-x_3+2x_4=0$$
$$x_1-x_2+x_3-x_4=0$$
$$2x_1+x_2+3x_3+x_4=0$$

This is a system of three linear equations in four unknowns, which always has a solution in which at least one unknown is non-zero. One such solution here is $x_1=4, x_2=9, x_3=-3, x_4=-8.$ There are infinitely many others. A similar argument shows that any set of four vectors in 3-space is linearly dependent.

Note: Giving a single such solution (set of coefficients) and explaining that the existence of that one solution shows, based on the definition, that the set is linearly dependent is a sufficient solution for full credit.

ORAL COMPETITION
ICTM REGIONAL 2013 DIVISION AA

3. Let $\left|\overrightarrow{PQ}\right| = 12$ and let $\overrightarrow{PC} = t\,\overrightarrow{CQ}$ and $\overrightarrow{PD} = -t\,\overrightarrow{DQ}$, where t is a scalar.

(*i*) Find $\left|\overrightarrow{CD}\right|$ when $t = 2$.

(*ii*) Find $\left|\overrightarrow{CD}\right|$ when $t = -4$.

(*iii*) There is no solution to the problem "Find $\left|\overrightarrow{CD}\right|$ when $t = 1$."
Explain why this is so.

SOLUTION

First note that $\overrightarrow{PC} = t\,\overrightarrow{CQ}$ and $\overrightarrow{PD} = -t\,\overrightarrow{DQ}$ imply that C and D lie on line $\overleftrightarrow{PQ}$.

Use line $\overleftrightarrow{PQ}$ as the x-axis and make $P = (0, 0)$, $Q = (12, 0)$, $C = (c, 0)$ and $D = (d, 0)$.

(*i*) With $t = 2$, point C is between P and Q and a drawing shows that $C = (8, 0)$ and $D = (24, 0)$.
Use $c - 0 = 2(12 - c)$ to get c = 8. And use $d - 0 = -2(12 - d)$ to get $d = 24$.
Thus $\left|\overrightarrow{CD}\right| = 16$.

(*ii*) With $t = -4$, Q is between P and C and $C = (16, 0)$ and $D = (48/5, 0)$.
Use $c - 0 = -4(12 - c)$ to get $c = 16$. And use $d - 0 = 4(12 - d)$ to get $d = 48/5$.
Thus $\left|\overrightarrow{CD}\right| = 16 - 48/5 = 32/5$, or 6.4.

(*iii*) There is no location for D for which t = 1.
A drawing shows that as C approaches the midpoint of segment $\overline{PQ}$, the value of t approaches 1. Conversely, as t approaches 1, C approaches the midpoint of $\overline{PQ}$. But when t is near 1, D is forced to run far to the left if t is a bit smaller than 1 and far to the right if t is a bit larger than 1. In particular, if $t = .99$, $C \approx (5.97, 0)$ and $D = (-1188, 0)$; if $t = 1.01$, $C \approx (6.03, 0)$ and $D = (1212, 0)$.
Putting $t = 1$ leads to solving the equation $\frac{PD}{DQ} = -1$, or $\frac{d-0}{12-d} = -1$, which leads to $d = -12 + d$, which has no solution.

ORAL COMPETITION
ICTM REGIONAL 2013 DIVISION AA

EXTEMPORANEOUS QUESTIONS

1. In two-space, two non-zero vectors which are both perpendicular to the same non-zero third vector are parallel to each other.
Explain why this is so and determine whether the same result holds in three-space.

SOLUTION

In a plane there is just one direction perpendicular to the direction of the third vector. Think vertical vs. horizontal or slope *m* vs. slope -1/*m*.
In three-space, however, there are infinitely many directions perpendicular to the direction of the third vector. Think of $\vec{i}$ and $\vec{j}$ for the first two and $\vec{k}$ for the third. Since $\vec{i}$ and $\vec{j}$ are not parallel the result does not hold in three-space.

Students may also reference simple geometry.

2. Given the vectors $\vec{v} = \vec{i} + \sqrt{3}\vec{j}$ and $\vec{w} = 4\vec{i}$.
Find the projection of $\vec{v}$ on $\vec{w}$ and the projection of $\vec{w}$ on $\vec{v}$.

SOLUTION

Both of these projections are scalars (Reference, page 65.)
The projection of one vector on a second vector is the length of the first vector multiplied by the cosine of the angle between the vectors placed tail-to-tail.
Here that angle is 60 degrees and the cosine is one-half. The lengths of the given vectors are 2 and 4, so the projections are 1 and 2, respectively.

Note that the projection *vectors* are $\vec{i}$ (for $\vec{v}$ on $\vec{w}$) and $\vec{i} + \sqrt{3}\vec{j}$ (for $\vec{w}$ on $\vec{v}$), whose respective lengths are 1 and 2