1. If $x=3$ and $y=-1$, find the value of the expression x^4y^6.

2. When solved for x, the equation $2(3x-5)-2=6(x+k)$ has infinite solutions. Find the value of k.

3. The points $(2,a)$ and $(b,7)$ lie on the line with equation $x-3y+13=0$. Find the sum $(a+b)$.

4. Let A be a positive two-digit integer. The integer B is the same as A when its digits are reversed. Find the largest value of A such that $A=3B-2$?

5. Barbara bought a bracelet on QPC. She later decided to sell it on D-bay. The bracelet had lost $\frac{2}{3}$ of its value by then. Anna bought it at the price equal to its new value. She paid \$56, which included \$20 for shipping and handling. Find the original cost, in dollars, of the bracelet on QPC?

6. If $3x^2+8=56$, find the smallest possible value of $4x-5$.

7. The average of three values is $3x+2y$. If two of the values are $4x+2y$ and $3x-y$, find the third value. Give your answer as an expression in terms of x and/or y.

8. For what value(s) of k does the graph of the equation $y = |x + k| - 2$ have a y-intercept of 4?

9. The expression $\sqrt{40x}$ represents an integer. Find the smallest possible positive integer value for x.

10. If $\begin{bmatrix} 3 & 2 \\ d & 4 \end{bmatrix} + 2\begin{bmatrix} 4 & 3 \\ e & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ 4 & c \end{bmatrix}$, find the value of the expression $(a + b + c + 2d + 4e)$.

11. For what value of x does $\left(a^4\right)^3\left(a^5\right) = a^x$?

12. A first line has the equation $4x + 3y = 20$ and a second line has the equation $2x - 3y = 12$. A third line has the same x-intercept as the first line and a y-intercept that is $\frac{1}{2}$ the y-intercept of the second line. Find the slope of the third line. Give your answer as a common or improper fraction reduced to lowest terms.

13. The sum of the smallest and largest of three consecutive even integers is 52. The integer k is 10 less than the middle integer of the three consecutive even integers. If $p^2 = k$ where p is a positive integer, find the value of p.

14. The numeric value of the expression $2014 + k$ is a multiple of 9, where k is a positive integer less than 50. How **many** possible values are there for k?

15. How many of the first 200 natural numbers are divisible by all of the values 3, 4, 5, and 6?

16. Find ***the number*** of integer values for which $8x^2 - 6x - 25 \le 2x^2 + 5x + 10$.

17. Given the function $f(x) = 3x^2 + 2x + 3$, find the sum of the value(s) of k such that $f(k+4) = 11$. Write your answer as a common or improper fraction reduced to lowest terms.

18. When eight coins are flipped, find the probability that tails occurs exactly three times. Write your answer as a common fraction reduced to lowest terms.

19. If $b \ne 0$ and the ratio of a to b is 2 to 1, find the value of the expression $\frac{2a+b}{4b+3a}$.

20. 120 students were provided a choice of breakfast. They had three choices, fruit, cereal and/or yogurt. 53 students had cereal, 47 students had yogurt, 47 students had fruit. Also, 38 students had only cereal, 12 students had yogurt and fruit but no cereal, 27 students had only fruit, and 25 students had only yogurt. Find the number of students that chose not to have breakfast.

2014 RA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

_____ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 81

2. -2

3. 13

4. 82

5. 108 ($ or dollars optional.)

6. -21

7. $2x+5y$ OR $5y+2x$

8. $6, -6$ (Must have both values in either order.)

9. 10

10. 41

11. 17

12. $\frac{2}{5}$ (Must be this reduced common fraction.)

13. 4

14. 6

15. 3

16. 5

17. $-\frac{26}{3}$ OR $\frac{-26}{3}$ (Must be this reduced improper fraction.)

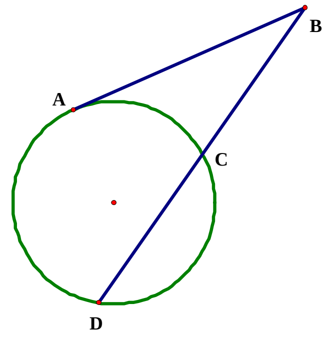
18. $\frac{7}{32}$ (Must be this reduced common fraction.)

19. $\frac{1}{2}$ OR 0.5 OR .5

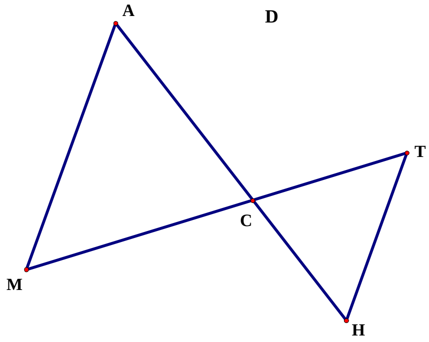
20. 3 (Students optional.)

1. The ratio of two supplementary angles is $1:5$. Find the degree measure of the larger of the two angles.

2. In the given diagram (not necessarily drawn to scale), the length of tangent $\overline{AB}$ is four less than the length of secant $\overline{BD}$. If the length of $\overline{BC}$ is two more than the length of $\overline{CD}$, find the length of $\overline{AB}$.

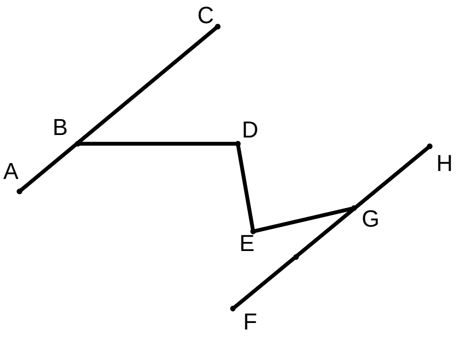


3. In the given diagram, $\overline{MA} \parallel \overline{TH}$, $MA = 4x$, $AC = 5x + 2$, $MC = 3x + 4$, $CT = x + 3$ and $TH = x + 2$. Find the length of $\overline{CH}$.



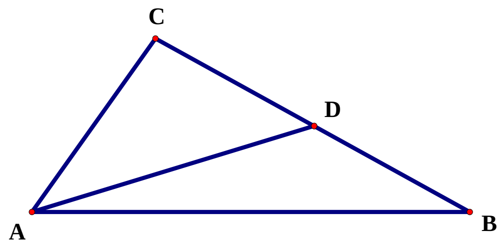
4. $\overline{AB}$ is a chord in circle O such that the degree measure of minor arc $\overset{\frown}{AB}$ is $\frac{1}{4}$ the degree measure of major arc $\overset{\frown}{AB}$. Find the degree measure of $\angle OAB$.

5. In the given diagram, $\overline{AC} \parallel \overline{HF}$, $\angle CBD \cong \angle GED$, the measure of $\angle E$ is $18°$ less than $\angle D$ and the measure of $\angle FGE$ is $44°$. Find the degree measure of $\angle D$.



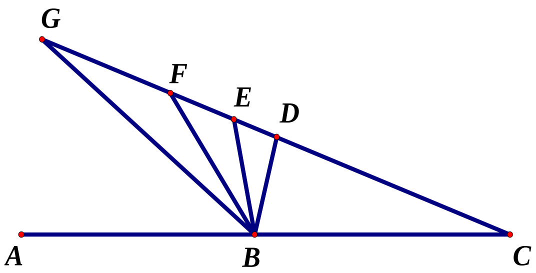
6. $\Delta ABC \sim \Delta DEF$ with $\angle B = \angle E = 90°$, $AB = 4$, and $DE = \sqrt{2}$. The length of the hypotenuse of ΔABC is $\sqrt{x}$ and the length of the hypotenuse of ΔDEF is $\sqrt{y}$. If $x = ky$, find the numerical value of k.

7. In the given diagram, angle C is a right angle, $BC = 120$, the length of $\overline{AB}$ is 100 more than the length of $\overline{AC}$, and the length of $\overline{CD}$ is 4 more than the length of $\overline{BD}$. Find the length of $\overline{AD}$. Express your answer as a decimal, rounded to the nearest tenth.



8. In ΔABC, A is the origin, $B(40,0)$, and $C(x,y)$ lies in the first quadrant. $AC = 32$ and $BC = 24$. Find the exact coordinates for point $C(x,y)$. Express your answer as an ordered pair with coordinates written as common or improper fractions reduced to lowest terms.

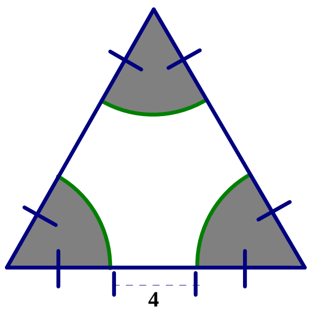
9. In the given diagram, the measure of $\angle CBF$ is $73°$ more than the measure of $\angle ABG$. $\overline{BD}$ bisects $\angle CBG$ and $\overline{BE}$ and $\overline{BF}$ trisect $\angle DBG$. Find the degree measure of $\angle EBC$.



10. A right triangle has acute angles such that the degree measure of one is half of the degree measure of the other. The area of this triangle is $18\sqrt{3}$. Find the length of the hypotenuse of this triangle.

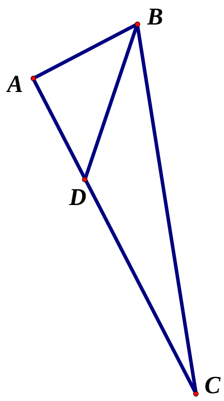
11. A rectangle has a diagonal of $2\sqrt{13}$ and one side that is 2 more than another. Find the length of the shortest side of this rectangle.

12. The perimeter of the equilateral triangle shown in the diagram is 48. The three shaded regions are each sectors of congruent circles centered at the vertices of the triangle. Find the exact numeric **unshaded** area.



13. Find the degree measure of the smaller angle formed by the hour and minute hands of a standard analog clock at precisely 10:14.

14. In the given diagram, point A has coordinates $(1,3)$, point B has coordinates $(3,4)$ and point D has coordinates $(2,1)$. Points A, D, and C are collinear in the order shown. If the area of ΔABD is $\frac{1}{3}$ the area of triangle ABC, find the coordinates of point C. Express your answer in the form of an ordered pair (x, y).

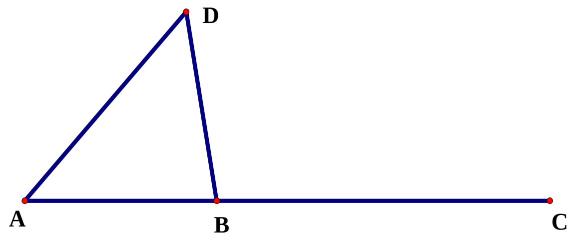


15. A triangle has sides with lengths of 4 and 9. If a number is selected at random from the set $\{1, 2, 3, 4, 5, ...19, 20\}$, find the probability that the number selected could be the length of the third side of the triangle. Express your answer as a common fraction reduced to lowest terms.

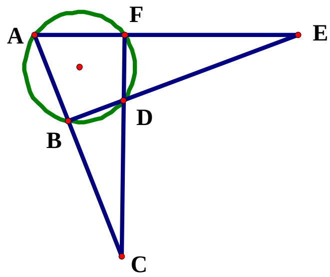
16. A circle with area 81π is inscribed in an equilateral triangle. Find the exact area of the equilateral triangle.

17. Find the number of square units in the area of the region bounded by the lines $y = 2$, $y = 8$, $2x + y = 10$ and $x - y = 6$.

18. In the diagram shown, triangle ABD is isosceles with $\overline{AB} \cong \overline{BD}$. The degree measure of $\angle CBD = 96°$. Find the degree measure of $\angle A$.



19. Quadrilateral $ABDF$ is inscribed in a circle with the sides extended to meet at points C and E as shown. The measure of $\angle C$ is $30°$ and the measure of $\angle E$ is $40°$. Find the degree measure of $\angle A$.



20. The degree measure of the smallest interior angle of an *n*-sided polygon is $120°$. The other angles each have an integral degree measure that is $5°$ more than the previous angle. Find all possible values of *n*.

2014 RA

Geometry

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 150 (Degrees optional.)

2. 12

3. 6

4. 54 (Degrees optional.)

5. 80 (Degrees optional.)

6. 8

7. 65.8 (Must be this decimal.)

8. $\left(\frac{128}{5},\frac{96}{5}\right)$ (Must be this ordered pair with reduced improper fraction entries.)

9. 92 (Degrees optional.)

10. 12

11. 4

12. $64\sqrt{3}-18\pi$ (Or exact equivalent.)

13. 137 (Degrees optional.)

14. $(4,-3)$ (Must be this ordered pair.)

15. $\frac{7}{20}$ (Must be this reduced common fraction.)

16. $243\sqrt{3}$ (Must be this exact answer.)

17. 51

18. 48 (Degrees optional.)

19. 55 (Degrees optional.)

20. $9, 16$ (Must have both answers in either order.)

1. Solve for x: $\frac{1}{2}x+\frac{1}{3}=\frac{1}{4}x+\frac{1}{5}$. Express your answer as a common or improper fraction reduced to lowest terms.

2. Find all value(s) for x such that $\left|\,|x+1|-4\,\right|=3$.

3. Find the largest number of pigeonholes which 200 pigeons can occupy, given that there must be at least one pigeon in each hole and that no two holes can contain the same number of pigeons.

4. Point P is located on the positive x axis, 10 units away from the point $(6,-8)$. Find the (x,y) coordinates of point P. Express your answer as an ordered pair (x,y).

5. Given a recursive sequence $a_{n+1}=a_n(2-2a_n)$ with $a_1=2$, find the value of a_4.

6. Given that $i-2i^2+3i^3-4i^4=a+bi$ where $i=\sqrt{-1}$ and a and b are real numbers, find the value of a^2+b^2.

7. Find the **<u>number</u>** of distinct integers that are solutions for $x^2-5<4$.

8. Simplify the rational expression $\frac{x^4+x^3-9x^2-9x}{4x^2-8x-12}$, assuming no denominator is 0. Express your answer as a single rational expression (fraction).

9. List all (x, y) pairs that solve the system $\begin{cases} 9x+18y=54 \\ 18x+9y=27 \end{cases}$. Express your answer(s) as ordered pair(s) (x, y).

10. The point $(3,7)$ is on the graph of $y = f(x)$. Find the corresponding point that must be on the graph of the translation $y = f(x-4)+1$. Express your answer as an ordered pair (x, y).

11. If $\frac{2+\sqrt{3}}{4-\sqrt{3}} = \frac{a+b\sqrt{3}}{c}$ where a, b and c are integers with no common factors and $c > 0$, find the value of a.

12. Find the sum of the sequence $2, -4, 6, -8, \ldots, -448, 450$.

13. The sum of two numbers is 30. A third number is 9 more than the smaller of the two numbers. The larger of the two original numbers is twice a fourth number. The sum of the smaller of the two original numbers and the fourth number is equal to the third number. Find the value of the third number.

14. A parabola has an equation of the form $y = ax^2 + bx + c$. It passes through the points $(0,0)$, $(4,0)$ and $(2,8)$. Find the value of the coefficient a.

15. Given that a varies directly with c^2, and $a = 12$ when $c = 2$, find the value of a when $c = -3$.

16. Find the area of the region bounded by the graph of $|x| + |y| \le 20$.

17. If $p = \log_x 7$, $q = \log_x 3$ and $r = \log_x 5$ for $x > 1$, find the value of $\log_x 4.2$. Express your answer as an expression in terms of p, q, and/or r.

18. If the ordered pair of positive numbers (x, y) satisfies both $x^2 - y^2 = x - y$ and $y = 3x$, find the value of $(2x + 6y)$.

19. The face diagonals of a box in the shape of a rectangular prism have lengths 3, 5 and 6. Find the exact length of a diagonal of the rectangular prism.

20. Evaluate the expression $100^2 - 99^2 + 98^2 - 97^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$.

2014 RA

Algebra II

Name **ANSWERS**

School ______________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-\frac{8}{15}$ (Must be this reduced common fraction.)

2. $-8, -2, 0, 6$ (Must have all four values in any order.)

3. 19 (Pigeonholes or holes optional.)

4. $(12, 0)$ (Must be this ordered pair.)

5. -3280

6. 8

7. 5 (Must be this integer.)

8. $\frac{x^2+3x}{4}$ OR $\frac{1}{4}x(x+3)$ (Or exact simplified equivalent.)

9. $(0, 3)$ (Must be this ordered pair only.)

10. $(7, 8)$ (Must be this ordered pair.)

11. 11

12. 226

13. 21

14. -2

15. 27

16. 800

17. $p+q-r$ (Must be this expression or equivalent with terms in any order.)

18. 5

19. Not Possible (Note: original answer of $\sqrt{35}$ was also accepted)

20. 5050

1. Find the value of the expression $\log_4\left(16^2\right)$.

2. Find the value of the expression $\left(\sin\frac{\pi}{3}+\tan\frac{\pi}{4}+\cos\frac{7\pi}{6}+\sec\frac{4\pi}{3}+\csc\frac{\pi}{6}+\cot\frac{3\pi}{2}\right)$.

3. The function $f\left(x\right)=-x^2-4x+12$ has a maximum value of m when $x=k$. Find the sum $\left(m+k\right)$.

4. Find the sum of an infinite geometric series whose first term is 6 and common ratio is $\frac{1}{4}$.

5. Find **<u>the number</u>** of distinct real values for x so that $\begin{bmatrix} x-2 & x+5 \\ 2x & x-3 \end{bmatrix}$ will **<u>not</u>** have an inverse.

6. Find the smallest integer in the solution set for $5\le\left|\,2x+3\,\right|<11$.

7. The composite function $f\left(g\left(x\right)\right)$ is undefined for $x=\frac{k}{12}$. If $f\left(x\right)=\frac{x-1}{2x+3}$ and $g\left(x\right)=3x-1$, find the value of k.

8. Find the number of distinct arrangements of the letters in the word BOOKKEEPER.

9. The inner product (dot product) of the vectors $(4,\ k-10)$ and $(2,\ 6-k)$ is 12. Find the value of k.

10. Determine the sum of all values of θ, $0 \le \theta < 2\pi$ for which $\cos^2\theta = \frac{3}{5}$.

11. The graph of $4x^2 - y^2 - 8x + 2y = 1$ is rotated $90°$ clockwise about the origin. Find the coordinates of the center of the new rotated conic. Write your answer as the ordered pair (x, y).

12. A bag contains red and blue marbles. When two marbles are drawn, the probability that they are both red is equal to the probability they are both blue. The probability that one of each color is drawn is $\frac{4}{7}$. Find the total number of marbles in the bag.

13. The first term of an arithmetic sequence is 32 and the last term is 74. There are 38 terms in the sequence. Find the sum of the 38 terms in this sequence.

14. The graph of $y = \frac{14}{6x+k}$ has a vertical asymptote at $x = \frac{7}{3}$. Find the value of k.

15. $i = \sqrt{-1}$. Find all exact complex value(s) (including pure real numbers) for b such that $\frac{b}{4+i\sqrt{3}} - \frac{b}{4-i\sqrt{3}} = 1$.

16. Let $\|(k,w)\|$ represent the norm of the vector represented by (k,w). Find the least value of k such that $\|(k,50)\| = \|(380, 399)\|$. Write your answer as a decimal rounded to the nearest hundredth.

17. Find the exact value of the sine of the largest angle of a triangle if the lengths of the sides are 3, 7, and 8.

18. Find the product of the solutions for the equation $x^3 - 7x^2 - 14x + 67 = 0$.

19. A triangle has vertices at $A(5,3)$, $B(9,2)$ and $C(-3,8)$. Find the slope of the altitude from point B to side $\overline{AC}$. Write your answer as a common or improper fraction reduced to lowest terms.

20. When written in interval notation, the domain of $f(x) = \sqrt{20 - 14x - x^2}$ is $(-\infty, k] \cup [w, \infty)$. Find the sum $(k + w)$.

2014 RA

PreCalculus

Name **ANSWERS**

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 4

2. 1

3. 14

4. 8

5. 2 (Values optional.)

6. -6

7. -2

8. 151200 (Arrangements optional.)

9. 8

10. 4π

11. $(1,-1)$ (Must be this ordered pair.)

12. 8 (Marbles optional.)

13. 2014

14. -14

15. $\dfrac{19i\sqrt{3}}{6}$ (This exact answer only or exact algebraic equivalent.)

16. -548.73 (Must be this decimal.)

17. $\dfrac{4\sqrt{3}}{7}$ OR $\dfrac{4}{7}\sqrt{3}$ (Must be this exact answer.)

18. -67

19. $\dfrac{8}{5}$ (Must be this reduced improper fraction.)

20. -14

NO CALCULATORS

1. Two integers have a sum of 30 and a difference of 14. Find the larger of the two integers.

2. Find the value of $\left(2014^2 - 2013^2\right)$

3. The ratio of the degree measure of an angle A to the degree measure of its complement is 5:13. Find the positive difference between the degree measure of the supplement of angle A and the degree measure of angle A.

4. The angles of a triangle have degree measures that are in a ratio of 2:5:8. If one of the angles is selected at random, find the probability that the degree measure of the angle is less than $65°$. Write your answer as a common fraction reduced to lowest terms.

5. Megan runs into her friends Zack and Andrew on a particular day of the week. She knows that Zack lies on Mondays, Tuesdays and Wednesdays, and tells the truth on the other days of the week. Andrew, on the other hand, lies on Thursdays, Fridays and Saturdays, but tells the truth on the other days of the week. Zack says "Yesterday was one of my lying days." Andrew responds "Yesterday was one of my lying days." What day of the week is this particular day? Write the whole word for this day of the week.

6. If $1.\overline{45} = \frac{k}{w}$ where the fraction $\frac{k}{w}$ is a reduced improper fraction, find the value of k.

7. Given that $\frac{4x+2y}{3y} = \frac{5}{3}$, find the ratio of x to y. Express your answer in the form $k : w$ where k and w have no common factors.

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

NO CALCULATORS

8. Find the exact area of a circle inscribed in a triangle with sides of 6, 8 and 10.

9. One base of an isosceles trapezoid is 6 cm longer than the other base. The legs are each 5 cm. If the area of the trapezoid is 72 cm^2, find the numeric value of the sum of the length of the bases.

10. Find the **number** of points, with integral coordinates, that are inside the region satisfying the constraints $y < \frac{4}{3}x$, $y < \frac{-2}{3}x + 6$, and $y > 0$.

11. One of the following statements is selected at random. Find the probability that the selected statement is true for all parallelograms. Write your answer as a common fraction reduced to lowest terms.
 - The opposite angles are supplementary.
 - The opposite angles are congruent.
 - Any pair of consecutive angles are supplementary.
 - The opposite sides are congruent.
 - The diagonals are congruent.
 - The diagonals are perpendicular to each other.
 - The diagonals bisect the vertex angles.
 - The diagonals bisect each other.

12. Evaluate $\frac{a^2 - 3ab - 10b^2}{a^2 + 3ab + 2b^2}$ for $a = \frac{3}{4}$ and $b = \frac{3}{8}$.

13. One leg of a right triangle is two millimeters longer than twice the length of the other leg. The hypotenuse is one millimeter longer than the longer of two legs. Find the number of millimeters in the perimeter of the triangle.

NO CALCULATORS

14. If $x^2y^4 = 24$ and $x^3y^3 = 60$, find the value of $\frac{x}{y}$. Write your answer as a common or improper fraction reduced to lowest terms.

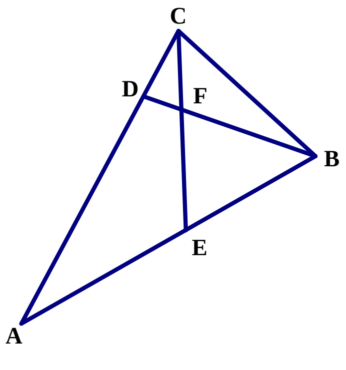
15. Find the number of consecutive zeroes that appear at the end of $2^{19} \cdot 3 \cdot 5^{21}$ when completely multiplied.

16. If $a > 0$ and $6x^2 + 5ax = 6a^2$, find the largest solution when solved for x as an expression in terms of a.

17. In ΔABC, $m\angle B = 60°$, $m\angle C = 75°$ and $BC = 12$. Find the exact length of $\overline{AB}$.

18. A bag contains three blue marbles, three green marbles, and three red marbles. Find the minimum number of marbles you must withdraw (without looking or replacement) to ensure that you will have removed at least two marbles of the same color and at least one marble of a different color from a previously drawn matched pair.

19. ΔABC is isosceles with vertex angle A having a measure of $36°$. $\overline{CE}$ bisects $\angle ACB$ and $\overline{BD}$ is one of the trisectors of $\angle ABC$ so that the measure of $\angle CBD$ is less than the measure of $\angle DBA$. Find the degree measure of $\angle CFB$.



20. Find the integral value(s) of x so that the expression $\frac{6}{x+1}$ represents a positive integer.

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

2014 RA

School ANSWERS

Fr/So 8 Person Team

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 22

2. 4027

3. 130 (Degrees optional.)

4. $\frac{2}{3}$ (Must be this reduced common fraction.)

5. Thursday (Must be this whole word day of the week.)

6. 16

7. $3:4$ (Ratio must be written in this form.)

8. 4π (Must be this exact answer.)

9. 36 (cm. optional.)

10. 13 (Points optional.)

11. $\frac{1}{2}$ (Must be this reduced common fraction.)

12. -1

13. 30 (Millimeters or mm. optional.)

14. $\frac{5}{2}$ (Must be this reduced improper fraction.)

15. 19 (Zeros optional.)

16. $\frac{2a}{3}$ OR $\frac{2}{3}a$ ("x=" ok.)

17. $6+6\sqrt{3}$ OR $6\left(\sqrt{3}+1\right)$ (Or exact equivalent.)

18. 4 (Marbles optional.) (Note: an answer of 6 was also accepted upon appeal)

19. 120 (Degrees optional.)

20. 0, 1, 2, 5 (Must have all 4 values listed, in any order.)

NO CALCULATORS

1. Find the value of k for which $x+3$ is a factor of $2x^2 + kx - 27$.

2. Find the polynomial function $P(x)$ with integer coefficients of smallest degree and leading coefficient of 1 that has zeros $\sqrt{2}$ and 5. Write as your answer the **sum** of the numerical coefficients of $P(x)$.

3. The function $f(x)$ is symmetric with respect to the line $x = -4$. If the point $(-7, 3)$ is on the graph of $f(x)$, determine a second point that must be on the graph of $f(x)$. Express your answer as an ordered pair (x, y).

4. A coin is to be flipped five times, resulting in either heads or tails. Determine the probability that exactly four of the five flips are the same. Write your answer as a common fraction reduced to lowest terms.

5. Find the ordered pair of positive integers (x, y) for which $x^2 - y^2 = 17$.

6. On the reality show "Castaways", one of the three final contestants is a mole. When the final three contestants are questioned, they make the following statements:

   Annie says, "Barbie is the mole."
   Barbie says, "Connie is the mole."
   Connie says, "Barbie is lying."

   Assuming the mole lies and the other contestants tell the truth, determine who is the mole. Write the **full name** of the person who is the mole as your answer.

7. In an arithmetic sequence, the second term is $\frac{3}{4}$ and the fifth term is $\frac{7}{8}$. Find the seventh term of this sequence. Express your answer as a common or improper fraction reduced to lowest terms.

8. Find the value of $\sum_{n=0}^{\infty} \cos^n\left(\frac{x}{2}\right)$ when $x = \frac{2\pi}{3}$.

9. If $f(x) = ax^2 + bx + c$ and $f(x+1) = x^2 + 7x + 4$, find the ordered triple (a, b, c).

10. Determine the **number** of distinct prime numbers that are factors of $30!$.

11. Find the $y-$intercept of the line that is tangent to the circle $(x-1)^2 + (y+2)^2 = 13$ at the point $(3,1)$. Write as your answer the y-intercept only, not the coordinates of the point that is the y-intercept.

12. Find the exact area of an equilateral triangle inscribed in the circle with equation $x^2 + y^2 = 12$.

13. A point x_0 is called a fixed point of a function f if $f(x_0) = x_0$. Find the sum of all values of x which are fixed points of $f(x) = x^2 - x - 3$.

14. If $x \neq -4$ and $x \neq 2$, find the difference $(A - B)$ given that $\frac{A}{x+4} + \frac{B}{x-2} = \frac{8x+2}{x^2+2x-8}$ for all values of x.

15. Find the largest possible value of the product (xy) if $\sqrt{x+10} \le 4$ and $|2y+9| \le 1$.

16. Find the sum of all x on the interval $0 \le x \le \frac{\pi}{2}$ for which $\sin(3x) = \cos(6x)$. Write your answer as a common or improper fraction reduced to lowest terms.

17. Given that $\log 4 = w$, express $\log 20$ in terms of w.

18. Give the coordinates of the point of intersection of the graphs of $3x^2 + 2y^2 = 30$ and $3x^2 - 4y^2 = -24$ that lies in the fourth quadrant. Write your answer as an ordered pair (x, y).

19. The line $y = mx + b$ is represented by the parametric equations $x = 4t - 1$ and $y = 3t + 2$. Find the value of m. Write your answer as a common or improper fraction reduced to lowest terms.

20. Three distinct numbers are chosen from the first nine positive integers. Find the probability that 3 is the smallest of the numbers chosen. Write your answer as a common fraction reduced to lowest terms.

2014 RA

School ANSWERS

Jr/Sr 8 Person Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -3

2. 4

3. $(-1, 3)$ (Must be this ordered pair.)

4. $\frac{5}{16}$ (Must be this reduced common fraction.)

5. $(9,8)$ (Must be this ordered pair.)

6. Barbie (Must be this whole word name.)

7. $\frac{23}{24}$ (Must be this reduced common fraction.)

8. 2

9. $(1, 5, -2)$ (Must be this orederd triple.)

10. 10 (Prime numbers optional.)

11. 3 (Must be this integer only.)

12. $9\sqrt{3}$ (Must be this exact answer.)

13. 2

14. 2

15. 50

16. $\frac{5\pi}{6}$ OR $\frac{5}{6}\pi$ (Must be this exact answer.)

17. $\frac{1}{2}w+1$ OR $\frac{w+2}{2}$ (Or exact equivalent.)

18. $(2, -3)$ (Must be this ordered pair.)

19. $\frac{3}{4}$ (Must be this reduced common fraction.)

20. $\frac{5}{28}$ (Must be this reduced common fraction.)

Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. Find the value of $x^3 - 2x^2 + 5x + 4.831$ when $x = 2.222$.

2. Find the volume of the largest sphere that can be inscribed in a cube that has volume 2014.

3. Find the slope of the line passing through the points $(9.472, 1.215)$ and $(0.425, 7.196)$.

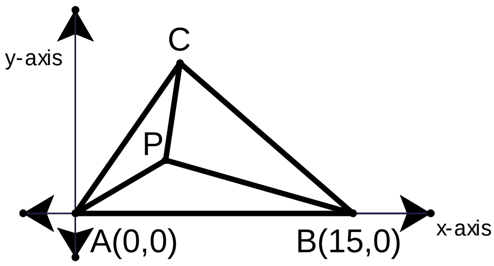
4. During the month of July in Tom's village, the cost of electricity is 12.08 cents per kilowatt hour. Tom used an average of 3423 watts for the equivalent of 4.112 hours per day during the month of July. Find the number of **cents** in Tom's electric bill for that month of July.

5. The line that contains points $A(4.013, -7.061)$ and $B(17.22, k)$ has a y-intercept of 8.885. Find the value of k.

6. Determine the minimum **vertical** distance between the line $y = \frac{20}{3}x - 37$ and the parabola $y = 2x^2 - 5x + 7$.

7. From a standard deck of 52 cards with 4 suits and 13 ranks per suit, 13 cards are dealt at random without replacement. Find the probability that among those 13 cards, at most 3 of the 4 suits are represented.

8. In the diagram (not necessarily drawn to scale) with coordinates as shown, $AC = 34$, and $BC = 35$. The ratio of the area of ΔPAB to the area of ΔPAC to the area of ΔPBC is $2:3:6$. Find the distance from P to C.



9. The first three terms of a geometric sequence are respectively 1220, 1061.4, and 923.418. Find the fourth term of this geometric sequence. Express your answer as an **exact** decimal. Do not round off or use scientific notation.

10. If x is a radian measure, find the **number** of distinct solutions for $\tan(2x) = 2\cot(x)$ that exist in the interval $[-4\pi, 4\pi]$.

11. Find the value of $\log_8(\log_2 64)$.

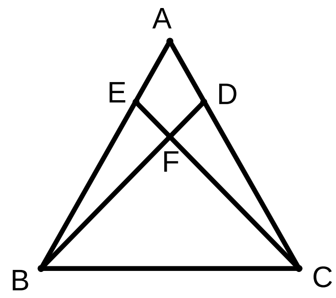
12. A plane is flying due (straight) north at 500.3 mph. but is being blown off course by a 40.14 mph. wind blowing **in the direction** of $N40°W$ ($40°$ west of due north). Because of the wind, the plane is actually flying **in** a direction of $Nk°W$. Find the value of k.

13. Kim just invested $1,000 in a savings account that pays 7.1% annual percentage rate (APR) compounded semi-annually. After the interest is credited one year after Kim's investment, find the number of dollars that will be in Kim's savings account.

14. The lengths of two of the sides of a right triangle are 145 and 408. The length of the third side is also an integer. Find that integer. Write your answer as an **exact integer**.

15. Find the sum of $6^2+7^2+8^2+9^2+10^2+\cdots+101^2$. Express your answer as an **exact integer**.

16. In equilateral ΔABC, $\overline{BD}$ and $\overline{CE}$ meet at F. $BF=CF$, and $\angle BFC$ is a right angle. E lies on $\overline{AB}$, and D lies on $\overline{AC}$. If $BC=24.36$, find the area of quadrilateral $ADFE$.



17. Two candles of the same length are made of different materials so that one burns out completely at a uniform rate in 3.035 hours and the other in 3.897 hours. Both candles are lit k **minutes** before 5:00 pm so that at exactly 5:00 pm, one stub is twice the length of the other. Find the value of k.

18. Given $(14.07^{0.87})(63.11^{0.96})=x$. Find the value of x.

19. $t_n=\sqrt{421\cdot t_{(n-1)}+3210000}$ and $t_1=999$. Find the value of t_{20}.

20. Kathie and Mimi agree to meet at 5:00 pm at the library. Since neither can arrive before 5:00 pm, Mimi agreed to wait 5 minutes if Kathie is not there and Kathie agreed to wait 15 minutes if Mimi is not there. The library closes at exactly 6:00 pm. Find the probability they can meet inside the library. Express your answer as a common fraction reduced to lowest terms.

2014 RA

School **ANSWERS**

Calculator Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = ☐

Note: **All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.**

1. 17.04 OR 1.704×10 OR 1.704×10^{1}

2. 1055 OR 1.055×10^{3}

3. -0.6611 OR $-.6611$ OR -6.611×10^{-1}

4. 5271 OR 5.271×10^{3} (Cents optional.)

5. -59.54 OR -5.954×10 OR -5.954×10^{1}

6. 26.99 OR 2.699×10 OR 2.699×10^{1}

7. 0.05116 OR 5.116×10^{-2}

8. 27.49 OR 2.749×10 OR 2.749×10^{1}

9. 803.37366 (Must be this exact decimal.)

10. 24 (Must be this exact integer, solutions optional.)

11. 0.8617 OR $.8617$ OR 8.617×10^{-1}

12. 2.782 OR 2.782×10^{0} (Degrees optional.)

13. 1072 OR 1.072×10^{3} ($ or dollars optional.)

14. 433 (Must be this exact integer.)

15. 348496 (Must be this exact integer.)

16. 29.10 OR 2.910×10 OR 2.910×10^{1} (Trailing zero necessary.)

17. 149.1 OR 1.491×10^{2} (Minutes optional.)

18. 533.5 OR 5.335×10^{2}

19. 2014 OR 2.014×10^{3}

20. $\dfrac{43}{144}$ (Must be this reduced common fraction.)

1. $k+23=2k+17$ and $3w+7=130$. Find the sum $(k+w)$.

2. When written in simplified radical form, $10-5\sqrt{40}+\sqrt{160}-3\sqrt{250}$ is written in the form $k+w\sqrt{p}$. Find $(k+w+p)$.

3. Find the positive difference between the numerical area and the numerical perimeter of a rectangle with one side of length 12 and one diagonal of length 20.

4. Set A contains 15 elements, set B contains 9 elements, and set C contains 13 elements. $A\cap B$ (A and B) contains 6 elements, $A\cap C$ 8 elements, $B\cap C$ 5 elements, and $A\cap B\cap C$ contains 2 elements. Find the number of elements that are in $B\cap\bar{A}\cap\bar{C}$ (B and not A and not C).

5. One side of an equilateral triangle has endpoints with coordinates $(4,-1)$ and $(7,10)$. Let k be the numerical value of the area of this triangle. A quadratic function $f(x)=2x^2-8x+3$ has a total *number* of w distinct real zeroes. Find the exact value of (kw).

6. Find the value of w so the function $f(x)=x^2-10x+w$ has exactly one x-intercept when graphed. The ratio of the sides of a triangle are $3:4:5$. The numerical perimeter of this triangle is 180. The numerical area of this triangle is (kw). Find the value of k.

7. In ΔABC, $A=(1,5)$, $B=(-2,-3)$ and $C=(5,-7)$. If k represents the slope of the altitude in ΔABC from vertex B and w represents the slope of the median in ΔABC from vertex B, find (kw). Express your answer as a common or improper fraction reduced to lowest terms.

8. Let x and y be positive integers such that $x>y$. For several distinct ordered pairs of the form (x,y), $2x+3y=36$. Let k be the sum of the x coordinates and w the sum of the y coordinates of these solutions. Find $(k+w)$.

9. k represents the degree measure of the smaller angle formed by the hour and minute hands of a clock at precisely $12:45$. w represents the length of the radius of the circumscribed circle about a 5, 12, 13 triangle. Find (kw). Express your answer as an improper fraction reduced to lowest terms.

10. Three surface areas of a right rectangular solid are numerically 20, 45, and 50. Find the exact numerical volume of this solid.

11. Two students make a New Year's resolution to get more exercise. One student decides to go to a health club aerobics class every other day, and the other decides to go every third day. They go together on January 2. ***How many other*** days in January (31 day month) will they be in aerobics class together?

12. An 18-foot street light and a 6-foot stop sign are on level ground. The light from the street light creates a 4-foot shadow in front of the stop sign. Let k be the distance, in feet, from the base of the street light to the base of the stop sign. Two parallel chords of a circle are 12 feet apart and each have length 16 feet. Let w be the radius of the circle. Find the numeric value of $(k+w)$.

ICTM Math Contest

Freshman – Sophomore

2 Person Team

Division A

1. $k+23=2k+17$ and $3w+7=130$.

Find the sum $(k+w)$.

2. When written in simplified radical form, $10-5\sqrt{40}+\sqrt{160}-3\sqrt{250}$ is written in the form $k+w\sqrt{p}$.

Find $\left(k+w+p\right)$.

3. Find the positive difference between the numerical area and the numerical perimeter of a rectangle with one side of length 12 and one diagonal of length 20.

4. Set A contains 15 elements, set B contains 9 elements, and set C contains 13 elements. $A \cap B$ (A and B) contains 6 elements, $A \cap C$ contains 8 elements, $B \cap C$ contains 5 elements, and $A \cap B \cap C$ contains 2 elements. Find the number of elements that are in $B \cap \overline{A} \cap \overline{C}$ (B and not A and not C).

5. One side of an equilateral triangle has endpoints with coordinates $(4,-1)$ and $(7,10)$. Let k be the numerical value of the area of this triangle.
A quadratic function $f(x)=2x^2-8x+3$ has a total <u>*number*</u> of w distinct real zeroes.
Find the exact value of (kw).

6. Find the value of w so that the function $f(x) = x^2 - 10x + w$ has exactly one x-intercept when graphed.
The ratio of the sides of a triangle are $3:4:5$. The numerical perimeter of this triangle is 180. The numerical area of this triangle is (kw). Find the value of k.

7. In ΔABC, $A=(1,5)$, $B=(-2,-3)$ and $C=(5,-7)$. If k represents the slope of the altitude in ΔABC from vertex B and w represents the slope of the median in ΔABC from vertex B, find (kw).

Express your answer as a common or improper fraction reduced to lowest terms.

8. Let x and y be positive integers such that $x > y$. For several distinct ordered pairs of the form (x, y), $2x + 3y = 36$. Let k be the sum of the x coordinates and w the sum of the y coordinates of these solutions. Find $(k + w)$.

9. k represents the degree measure of the smaller angle formed by the hour and minute hands of a clock at precisely $12:45$.
w represents the length of the radius of the circumscribed circle about a 5, 12, 13 triangle. Find (kw). Express your answer as an improper fraction reduced to lowest terms.

10. Three surface areas of a right rectangular solid are numerically 20, 45, and 50. Find the exact numerical volume of this solid.

11. Two students make a New Year’s resolution to get more exercise. One student decides to go to a health club aerobics class every other day, and the other decides to go every third day. They go together on January 2. ***How many other*** days in January (31 day month) will they be in aerobics class together?

12. An 18-foot street light and a 6-foot stop sign are on level ground. The light from the street light creates a 4-foot shadow in front of the stop sign. Let k be the distance, in feet, from the base of the street light to the base of the stop sign. Two parallel chords of a circle are 12 feet apart and each have length 16 feet. Let w be the radius of the circle. Find the numeric value of $\left(k+w\right)$.

2014 RA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. 47	________
2. -1	________
3. 136	________
4. 0 (Elements optional.)	________
5. $65\sqrt{3}$ (Must be this exact value.)	________
6. 54	________
7. $\frac{2}{15}$ (Must be this reduced common fraction.)	________
8. 48	________
9. $\frac{2925}{4}$ (Must be this reduced improper fraction.)	________
10. $150\sqrt{2}$ (Must be this exact answer.)	________

TOTAL SCORE: ________

(*enter in box above)

Extra Questions:

11. 4 (Days optional.)

12. 18 (Feet optional.)

13. ________

14. ________

15. ________

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Given $9x-4y=7$, let k be the value of y when $x=-5$. Let w be the value of p^2-11 when $p=-3$. Find the sum $(k+w)$.

2. The vertex of the parabola $y=x^2-4x+7$ is (k,w). The remainder when $3x^2-2x+7$ is divided by $x-2$ is R. Find the sum $(k+w+R)$.

3. The sum of two numbers is 2 and their product is 1. Let k be the sum of the squares of these two numbers. The first term of a geometric sequence is 2 and the 7th term is 128. Let w be the third term. Find the product (kw).

4. Let k be the sum of the solutions for the equation $2x^2-11x+12=0$. Let $y=w$ be the horizontal asymptote of the function $f(x)=\dfrac{8x^5+7x^2}{2x^5+12x^3+1}$. Find the sum $(k+w)$. Express your answer as a common or improper fraction reduced to lowest terms.

5. Let $k=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$. Let $w=1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\cdots$. Find the sum $(k+w)$. Write your answer as a common or improper fraction reduced to lowest terms.

6. Let p be the greatest common factor of 840 and 900. Let q be the least common multiple of 72 and 270. Find the sum $(p+q)$.

7. Let $\begin{bmatrix} a & 2 \\ 3 & 4 \end{bmatrix}+\begin{bmatrix} 5 & b \\ 7 & 8 \end{bmatrix}-\begin{bmatrix} 9 & 10 \\ c & 12 \end{bmatrix}=\begin{bmatrix} 13 & 14 \\ 15 & d \end{bmatrix}$. Find the determinant of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

8. The four roots for x of $x^4+ax^3-59x^2+bx+c=0$ are r, r, $r-11$, and $r-11$. If $|r|<|r-11|$, find the value of $(a+b+c)$.

9. Tom flipped a fair coin three times while Kay flipped another fair coin four times, each flip resulting in a head or a tail. Find the probability that Tom and Kay each flipped the same number of heads. Express your answer as a common fraction reduced to lowest terms.

10. Given the system $\begin{cases} 3x+2y-4z+a=11 \\ x-4y+8z+2a=7 \\ 2x+4y-2z+4a=-2 \\ 4x-8y+2z-a=4 \end{cases}$, find the value of $(30x-18y+12z+18a)$.

11. The graph of the circle with equation $(x+1)^2+(y-1)^2=16$ intersects the x-axis at $(k,0)$ with $k \geq 0$ and intersects the y-axis at $(0,w)$ with $w \leq 0$. Find the sum $(k+w)$.

12. In the State of Confusion, truck license plates are made with a combination of the digits 0-9 and the normal 26 letters of the alphabet. License plate numbers consist of five digits followed by two letters, except the letters O and I cannot be used. Find the number of possible license plates.

ICTM Math Contest

Junior – Senior

2 Person Team

Division A

1. Given $9x-4y=7$, let k be the value of y when $x=-5$.

Let w be the value of p^2-11 when $p=-3$.

Find the sum $(k+w)$.

2. The vertex of the parabola $y = x^2 - 4x + 7$ is (k, w).
The remainder when $3x^2 - 2x + 7$ is divided by $x - 2$ is R.
Find the sum $(k + w + R)$.

3. The sum of two numbers is 2 and their product is 1. Let k be the sum of the squares of these two numbers. The first term of a geometric sequence is 2 and the 7^{th} term is 128. Let w be the third term. Find the product (kw).

4. Let k be the sum of the solutions for the equation $2x^2 - 11x + 12 = 0$. Let $y = w$ be the horizontal asymptote of the function

$$f(x) = \frac{8x^5 + 7x^2}{2x^5 + 12x^3 + 1}.$$

Find the sum $(k + w)$. Write your answer as a common or improper fraction reduced to lowest terms.

5. Let

$$k = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots.$$

Let

$$w = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots.$$

Find the sum $\left(k + w\right)$.

Express your answer as a common or improper fraction reduced to lowest terms.

6. Let p be the greatest common factor of 840 and 900.

Let q be the least common multiple of 72 and 270.

Find the sum $(p+q)$.

7. Let

$$\begin{bmatrix} a & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & b \\ 7 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 10 \\ c & 12 \end{bmatrix} = \begin{bmatrix} 13 & 14 \\ 15 & d \end{bmatrix}$$

Find the determinant of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

8. The four roots for x of $x^4 + ax^3 - 59x^2 + bx + c = 0$ are r, r, $r-11$, and $r-11$.

If $|r| < |r-11|$, find the value of $(a+b+c)$.

9. Tom flipped a fair coin three times while Kay flipped another fair coin four times, each flip resulting in a head or a tail. Find the probability that Tom and Kay each flipped the same number of heads. Express your answer as a common fraction reduced to lowest terms.

10. Given the system

$$\begin{cases} 3x+2y-4z+a=11 \\ x-4y+8z+2a=7 \\ 2x+4y-2z+4a=-2 \\ 4x-8y+2z-a=4 \end{cases}$$

find the value of

$$\left(30x-18y+12z+18a\right).$$

11. The graph of the circle with equation $(x+1)^2+(y-1)^2=16$ intersects the *x*-axis at $(k,0)$ with $k \geq 0$ and intersects the *y*-axis at $(0,w)$ with $w \leq 0$. Find the sum $(k+w)$.

12. In the State of Confusion, truck license plates are made with a combination of the digits 0-9 and the normal 26 letters of the alphabet. License plate numbers consist of five digits followed by two letters, except the letters O and I cannot be used. Find the number of possible license plates.

2014 RA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. -15	
2. 20	
3. 16	
4. $\frac{19}{2}$ (Must be this reduced improper fraction.)	
5. $\frac{8}{3}$ (Must be this reduced improper fraction.)	
6. 1140	
7. 110	
8. 842	
9. $\frac{35}{128}$ (Must be this reduced common fraction.)	
10. 60	

TOTAL SCORE: ______ **(*enter in box above)**

Extra Questions:

11. 0 OR Zero
12. 57,600,000 OR 57600000
13.
14.
15.

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION A

1. Compute and explain the meaning of $\log_2 8$ as if you were speaking to an Algebra 1 student.

2. Below is Jane's step-by-step solution to a problem on her logarithms unit exam: $\log_7(2x+3)+\log_7(3x-1)=1$. If the problem is worth seven points on the exam, how many points (maximum seven) would you give to Jane? Justify the score given, identifying the error(s) made. Then, solve the problem correctly.

   Copy Problem: $\log_7(2x+3)+\log_7(3x-1)=1$

   Step 1: $\log_7\left(6x^2+7x-3\right)=1$

   Step 2: $6x^2+7x-3=1$

   Step 3: $6x^2+7x-4=0$

   Step 4: $(3x+4)(2x-1)=0$

   Step 5: $x=\frac{-4}{3},\frac{1}{2}$

3. Find the value(s) of x that satisfy the equation $\log x+\log(x-2)=\log\left(x^2-2x\right)$.

4. Given that $a^{2b}=5$, find the exact value of $2a^{6b}-4$, explaining how you arrived to your answer.

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION A

EXTEMPORANEOUS QUESTIONS

<u>Give this sheet to the students at the beginning of the extemporaneous question period.</u>

<u>STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.</u>

1. Assume the value of log 7 is approximately 0.8451. Approximate the value of log 70. Explain how you could arrive at an answer without using a calculator.

2. Arrange the following three logarithmic expressions in order from smallest to largest:

   $\log_3 10$ $\quad\quad$ $\log_2 12$ $\quad\quad$ $\log_7 9$

   Explain how you could arrive at an answer without using a calculator.

3. Given log 3 = a and log 5 = b, express log 6 in terms of a and b. Explain how you arrived at your answer.

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION A – Judges Solutions

1. Compute and explain the meaning of $\log_2 8$ as if you were speaking to an Algebra 1 student.

Solution: Note that the contestant is speaking to an Algebra 1 student, so judges should pay particular attention to the contestant's presentation to such an audience. A good answer will probably involve a contestant who speaks slowly, carefully, and who develops concepts incrementally in small doses. Of course, the value of $\log_2 8$ is 3, and the contestant should be given some credit for that part alone. However, the explanation of that answer is more critical. A contestant who involves at least one logarithm other than $\log_2 8$ should be given more credit than a contestant whose answer deals only with a single logarithm.

2. Below is Jane's step-by-step solution to a problem on her logarithms unit exam: $\log_7(2x+3)+\log_7(3x-1)=1$. If the problem is worth seven points on the exam, how many points (maximum seven) would you give to Jane? Justify the score given, identifying the error(s) made. Then, solve the problem correctly.

Copy Problem: $\log_7(2x+3)+\log_7(3x-1)=1$

Step 1: $\log_7\left(6x^2+7x-3\right)=1$

Step 2: $6x^2+7x-3=1$

Step 3: $6x^2+7x-4=0$

Step 4: $(3x+4)(2x-1)=0$

Step 5: $x=\frac{-4}{3},\frac{1}{2}$

Solution: The score given for the solution should be no more than five. The student should point out most of not all of the following mistakes:

Step 2: The 1 should be a 7 $\left(7^1\right)$.

Step 4: The factoring done is not correct; in fact, the quadratic is not factorable.

Step 5: The solution $\frac{-4}{3}$, while incorrect, should still have been eliminated since it creates the logarithm of a negative number.

The correct solution to the problem is:

Copy Problem:	$\log_7(2x+3)+\log_7(3x-1)=1$
Step 1:	$\log_7\left(6x^2+7x-3\right)=1$
Step 2:	$6x^2+7x-3=7$
Step 3:	$6x^2+7x-10=0$
Step 4:	$(6x-5)(x+2)=0$
Step 5:	$x=\frac{5}{6}, -2$ X

3. Find the value(s) of x that satisfy the equation $\log x+\log(x-2)=\log\left(x^2-2x\right)$.

Solution: While on first glance, logarithmic properties allow the equation to be rewritten as $\log\left(x^2-2x\right)=\log\left(x^2-2x\right)$, one must keep in mind that the domain of the left side of the equation is limited by the two individual logarithms. Therefore, the values of x which satisfy the equation are limited to $x > 2$.

4. Given that $a^{2b}=5$, find the exact value of $2a^{6b}-4$, explaining how you arrived to your answer.

Solution: $a^{2b}=5 \rightarrow \left(a^{2b}\right)^3=(5)^3 \rightarrow a^{6b}=125 \rightarrow 2a^{6b}=250 \rightarrow 2a^{6b}-4=250-4=246$

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION A – JUDGES SOLUTIONS

EXTEMPORANEOUS QUESTIONS

Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. Assume the value of log 7 is approximately 0.8451. Approximate the value of log 70. Explain how you could arrive at an answer without using a calculator.

Solution:
The student should recognize log 70 can be rewritten as $\log(7 \times 10)$ or $\log 7 + \log 10$. Thus, the approximate value of log 70 would be 0.8451 + 1 = 1.8451.

2. Arrange the following three logarithmic expressions in order from smallest to largest:

$$\log_3 10 \qquad \log_2 12 \qquad \log_7 9$$

Explain how you could arrive at an answer without using a calculator.

Solution:
$\log_7 9$, $\log_3 10$, $\log_2 12$ is the order from smallest to largest. Since $7^1 = 7$ and $7^2 = 49$, $1 < \log_7 9 < 2$. Since $3^2 = 9$ and $3^3 = 27$, $2 < \log_3 10 < 3$. Since $2^3 = 8$ and $2^4 = 16$, $3 < \log_2 12 < 4$.

3. Given log 3 = a and log 5 = b, express log 6 in terms of a and b. Explain how you arrived at your answer.

Solution: $\log 6 = \log \frac{3 \cdot 10}{5} = \log 3 + \log 10 - \log 5 = a + 1 - b$