1. Given $x = -2$ and $y = 3$, find the value of the expression x^2y^3.

2. On an algebra test taken by all but one student in a class, the mean score is 82 and the median score is 81.5. All test scores were integers between 0 and 100 inclusive. The student who was absent takes the test later and scores 86. Find the absolute value of the difference between the mean and the least possible median for all of the 25 students in the class who took the test. Express your answer as an exact decimal.

3. A $20 item has a price increase of 35%. A second item has a price decrease of 20%. After these changes, both items are now the same price. Find the original price of the second item. Express your answer as a decimal rounded to the nearest hundredth.

4. Find the *y* coordinate of the point of intersection of the graphs of $2x + 3y = 13$ and $2(2x+1) = y$.

5. Find the value(s) of k so that the graph of the equation $y = |x+k| - 2$ has an *x*-intercept of 4.

6. The average of two values is $2x + y$. If a third value of $5x + 4y$ is included, find the average of all three of the values. Express your answer as a simplified polynomial expression in terms of x and y.

7. When Mr. Worth died, he left an estate worth $325,000. His will specified that 35% of his estate should be divided equally among his 5 children, and $\frac{3}{5}$ of his estate should be divided equally among his 12 grandchildren. The remainder was to be divided equally among his 8 great-grandchildren. Find how much more, in dollars, one of his grandchildren will receive than one of his great-grandchildren. Express your answer as a decimal rounded to the nearest hundredth.

8. Given that $x \neq 0$, $y \neq 0$, and $\frac{4x+2y}{3y-2x} = \frac{10}{3}$, find the ratio of x to y. Express your answer in the form $\frac{a}{b}$ where *a* and *b* are integers and have no common factors.

9. A line m goes through the point $(4, -3)$ and is perpendicular to the line $3x+2y=9$. The point $(k, k+3)$ lies on this perpendicular line m. Find the value of k.

10. If $(2x-1)(3x+2) = ax^2+bx+c$ for all values of x, find the smallest solution for y for the equation $-cy^2+(a-1)y-(b+2)=0$.

11. Point A is located on a number line at $2\frac{5}{8}$ and point B is located on a number line at $5\frac{1}{3}$. Point C is located on the number line so that it is halfway between points A and B. Find the number line coordinate of point C. Express your answer as a mixed number.

12. If the ratio of a to b is 4 to 5, find the value of the expression $\frac{2a+3b}{4b-3a}$. Express your answer as a common or improper fraction reduced to lowest terms.

13. There are 12 elements in the intersection of sets A and B, and there are 20 elements in the union of sets A and B. If set A has 4 fewer elements than set B, find the number of elements that are in set B, but not set A.

14. Given $f(x) = \frac{2x+1}{3x-2}$. Find the value(s) of k for which $f(2k-4)$ is undefined.

15. Given $y = \dfrac{2x+1}{3}$, find the numeric value of the expression $\dfrac{10}{6x-9y+1}$.

16. Given that $x^{2n} = 8$, find the value of the expression $3x^{4n} - x^{2n} - 2$.

17. Given that $\dfrac{a-3b}{a+2b} = 3$, find the value of $\dfrac{a+3b}{a-2b}$. Express your answer as an integer or common or improper fraction reduced to lowest terms. Do **<u>not</u>** use decimals.

18. Given the function $f\left(x\right) = 2x^2 + 3x - 1$, find the value(s) of k so that $f\left(k+2\right) = -2$. Express your answer(s) as an integer or decimal. Do **not** use fractions.

19. If $x \neq y$ and $\left(x^2y^3\right)^3\left(x^a y^2\right) = x^8 y^{(4c-1)}$, find the value of $2^{(a+c)}$.

20. When solved for x, the solution set for $\left|7x-3\right| \geq \left|2x+5\right|$ is $\left\{x : x \leq k \text{ or } x \geq w\right\}$. Find the sum $\left(k+w\right)$. Express your answer as a common or improper fraction reduced to lowest terms.

2014 RAA
Algebra I

Name ANSWERS

School ______________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 108

2. 0.16 OR $.16$ (Must be this exact decimal.)

3. 33.75 (Must be this decimal, $ optional.)

4. 4

5. $-2, -6$ (Must have both values in any order.)

6. $3x+2y$ OR $2y+3x$

7. 14218.75 (Must be this decimal, $ optional.)

8. $\frac{3}{4}$ (Must be this reduced common fraction.)

9. -26

10. -3

11. $3\frac{47}{48}$ (Must be this mixed number.)

12. $\frac{23}{8}$ (Must be this reduced improper fraction.)

13. 6 (Elements optional.)

14. $\frac{7}{3}$ OR $2\frac{1}{3}$ OR $2.\overline{3}$

15. -5

16. 182

17. $\frac{3}{13}$ (Must be this reduced common fraction.)

18. $-3, -2.5$ (Must be these exact answers in any order.)

19. 32

20. $\frac{62}{45}$ (Must be this reduced improper fraction.

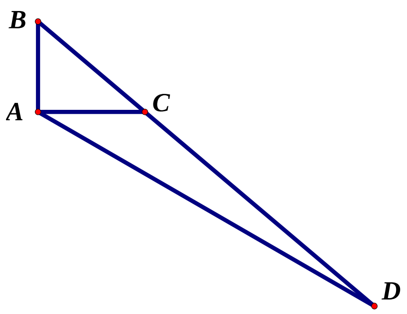
1. The area of a square is 36. If a side of the square is increased by 2, find the area of the resulting square.

2. The degree measure of angle A is $2x+3$ and the degree measure of the complement of angle A is $5x-4$. Find the numeric degree measure of angle A.

3. A rectangle with perimeter 16 has a diagonal of $2\sqrt{10}$. Find the length of the longer side of the rectangle.

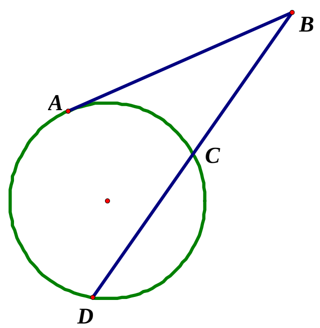
4. In a regular 15-gon, each interior angle measures $k°$, and w total diagonals can be drawn. Find the value of $(k+w)$.

5. If $\overline{AB} \perp \overline{AC}$, $\angle DAC = 30°$, $AB = 3$ and $AC = 4$, the length of $\overline{AD}$ can be written, in simplified form, as $\frac{a\sqrt{b}+c}{d}$. Determine the value of $(a+b+c-d)$.



6. Find the number of times each day that the smaller angle between the minute hand and the hour hand of a clock is equal to $30°$.

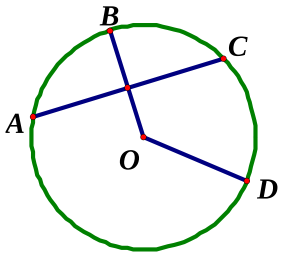
7. $\overline{AB}$ is tangent to the given circle at point A and $\overline{BD}$ is a secant line to the circle. Find the length of $\overline{AB}$ if $BC = 1$ and $CD = 8$.



8. Points A and B are located on a cube. If the length of a diagonal of a face of this cube is 12, find the exact maximum possible distance between A and B.

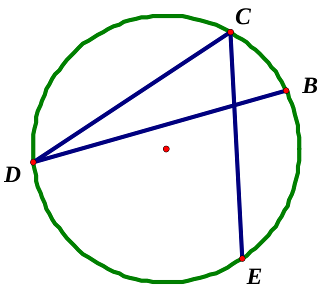
9. The interior of rectangle $ABCD$ is located entirely inside a circle where $AB = 24$, $BC = 7$. $\overline{AB}$ lies on the diameter of the circle and points C and D lie on the circle. If a point is selected at random from inside the circle, find the probability that the point is **NOT** inside the rectangle. Express your answer as a decimal rounded to the nearest hundredth.

10. In circle O, $\overline{AC}$ is the perpendicular bisector of $\overline{OB}$. If $OD = 14$, find the exact length of $\overline{AC}$.



11. A triangle has vertices $A(-5,4)$, $B(4,4)$ and $C(4,13)$. Find the coordinates of the point of intersection of the median from A to $\overline{BC}$ and the altitude from B to $\overline{AC}$. Express your answer as an ordered pair (x, y).

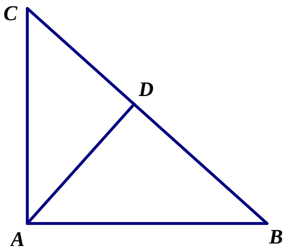
12. In the given circle, $\overset{\frown}{BC} = 42°$ and $\angle DCE = 60°$. The measure of $\overset{\frown}{BE}$ is $46°$ less the measure of $\overset{\frown}{CD}$. Find the degree measure of $\overset{\frown}{BE}$.



13. $ICTM$ is a convex quadrilateral with $IC = 60$, $CT = 7$, $TM = 24$ and $MI = 65$. $\angle CTM$ is a right angle. Find the area of this quadrilateral.

14. Two quadrilaterals $ABCD$ with coordinates $A(5,-2)$, $B(1,-4)$, $C(-2,-3)$, and $D(-1,k)$ exist with numerical area 18.5. One quadrilateral is convex and one is concave. Find the sum of the possible values of k. Express your answer as a common or improper fraction reduced to lowest terms.

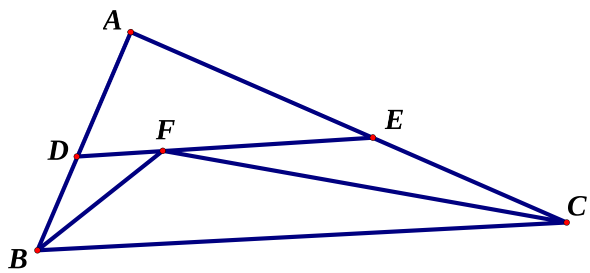
15. In the given diagram, ΔABD and ΔACD are isosceles triangles, and $\overline{AD} \perp \overline{BC}$. The length of $\overline{BC}$ is k times the length of $\overline{AB}$. Find the exact value of k.



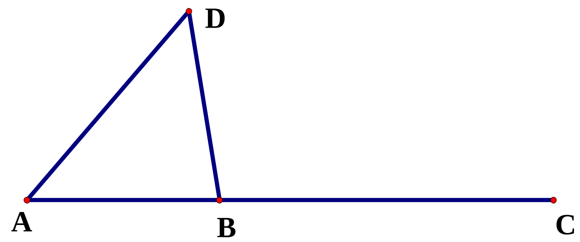
16. The length of the hypotenuse of a right triangle is 20. The radius of an inscribed circle is 4, and the ratio of the area of the circle to the area of the right triangle is $k\pi$. Find the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

17. A 10 cm length stick is broken in one place. Find the probability that the longer piece is at least twice as long as the shorter piece and no more than 5 cm longer than the shorter piece. Express your answer as a common fraction reduced to lowest terms.

18. In ΔABC, $\overline{DE} \parallel \overline{BC}$, $\overline{BF}$ bisects $\angle ABC$ and $\overline{CF}$ bisects $\angle ACB$. If $AB = 26$, $AC = 34$, and $BC = 40$, find the length of $\overline{DE}$.



19. In the diagram shown, triangle ABD is isosceles with $\overline{AB} \cong \overline{BD}$. The degree measure of $\angle \text{CBD} = 96°$. Find the degree measure of $\angle \text{A}$.



20. An inverted cone-shaped paper cup is filled to the top with water. The slant height of the cone is 6 cm and the radius of the top (base) of the cone is 4 cm. There is a small hole in the bottom of the cone, and the water is slowly dripping out of the cup. After 5 minutes, the water in the cone has lost 25% of its volume. Find the distance, in cm, from the top (base) of the cone to the water at that time. Express your answer as a decimal rounded to the nearest hundredth of a cm.

2014 RAA
Geometry

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 64

2. 29 (Degrees optional.)

3. 6

4. 246

5. 160

6. 44

7. 3

8. $6\sqrt{6}$ (Must be this exact answer.)

9. 0.72 OR .72 (Must be this decimal.)

10. $14\sqrt{3}$ (Must be this exact answer.)

11. $(1,7)$ (Must be this (ordered pair.)

12. 76 (Degrees optional.)

13. 834

14. $-\frac{60}{7}$ (Must be this reduced improper fraction.)

15. $\sqrt{2}$ (Must be this exact answer.)

16. $\frac{1}{6}$ (Must be this reduced common fraction.)

17. $\frac{1}{6}$ (Must be this reduced common fraction.)

18. 24

19. 48 (Degrees optional.)

20. 0.41 OR .41 (Must be this decimal, cm optional.)

1. For all real numbers x, $2f(x)+f(6-x)=x^2$. Find the value of $f(3)$.

2. Find the values of x for which $\frac{x+1}{x}>1$ is true. Write your answer as an inequality with x as one side of the inequality.

3. In right ΔABC with right angle at C, $AB=2\sqrt{3}$, $BC=\log x$, and $AC=2\sqrt{\log x}$. Given that $x>1$, find the exact value of $\cos A$.

4. Let $f(x)=\ln(x^2)$ and $g(x)=2\ln(x)$. Choose the best response and give as your answer the **capital letter** of your choice.

   A. The graphs of f and g are identical.
   B. The graphs of f and g have no points in common.
   C. The graphs of f and g have a finite number of common points.
   D. The graphs of f and g have infinitely many points in common.

5. Determine the value of w if $\frac{\log k}{\log w}=\frac{w}{k}=\frac{1}{2}$.

6. The point $(7,5)$ is on the graph of $y=f(x)$. Find the corresponding point that must be on the transformed graph $y=3f(2x-1)$. Write your answer as an ordered pair (x,y).

7. A parabola has an equation of the form $y=ax^2+bx+c$. The parabola passes through the points $(0,0)$, $(4,0)$ and $(2,8)$. Find the value of the coefficient a.

8. Find the area of the region bounded by the graph of $|x|+|y|\le 20$.

9. Find the y-intercept of the line that is tangent to the circle $x^2+(y-2)^2=25$ at the point $(3,6)$. Write your answer as an exact decimal.

10. Point P is located in the first quadrant and lies on the line $y=8$. Point P is also 10 units away from the point $(-4,14)$. Find the coordinates of point P. Write your answer as an ordered pair (x,y).

11. Find the minimum value of $f(x)=\dfrac{x^2+\frac{1}{x}}{x^2-\dfrac{1-\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}}}$ over the interval $1\le x\le 2$. Write your answer as an exact decimal.

12. The face diagonals of a box in the shape of a rectangular prism have lengths 3, 5 and 6. Find the exact length of the diagonal of this rectangular prism.

13. Several married couples attend a party. Everyone shakes hands once with each person at the party except themselves and their spouse. If 40 handshakes take place, how many couples are at the party?

14. Given a, b, and c are real numbers. The average of a and b is c. The average of a and c is one more than b. The average of b and c is two more than twice a. Find the value of c. Write your answer as a common or improper fraction reduced to lowest terms.

15. Suppose $b > c > a > 0$ and $b = a + c$. Find the largest solution for x in the equation $ax^2 + bx + c = 0$.

16. Given the function $f(x) = |3x - 1|$, find all value(s) of x where $x = f(x)$. Write your answer(s) as common or improper fraction(s) reduced to lowest terms.

17. Carly has 369 markers. Out of these, 130 are black, 100 are blue, and the others are neither blue nor black. She chooses two markers at random. Find the probability that one of the two is black and the other is neither blue nor black. Write your answer as a decimal rounded to the nearest thousandth.

18. The letters a and b represent integer digits in the six-digit number $322a4b$. This number is divisible by 99. Find the value of $(a+b)$.

19. Find the ordered pair of exact positive numbers (x, y) that satisfies both $x^2 - y^2 = x - y$ and $xy = x - y$. Write your answer as an ordered pair (x, y).

20. Evaluate $\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + ... + \frac{n}{3^n} + ...$. Write your answer as a common or improper fraction reduced to lowest terms.

2014 RAA
Algebra II

Name ANSWERS

School ____________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 3

2. $x > 0$ OR $0 < x$

3. $\frac{\sqrt{6}}{3}$ OR $\frac{1}{3}\sqrt{6}$ (Must be this exact answer.)

4. D (Must have this capital letter.)

5. $\frac{1}{4}$ OR 0.25 OR .25

6. $(4, 15)$ (Must be this ordered pair.)

7. -2

8. 800 (Square units optional.)

9. 8.25 (Must be this decimal but accept $(0, 8.25)$.)

10. $(4, 8)$ (Must be this ordered pair.)

11. 1.5 (Must be this exact decimal.)

12. Not possible (Note: original answer of $\sqrt{35}$ was also accepted)

13. 5 (Couples optional.)

14. $-\frac{11}{3}$ OR $\frac{-11}{3}$

15. -1

16. $\frac{1}{2}, \frac{1}{4}$ (Must have both reduced common fractions in any order.)

17. 0.266 OR .266 (Must be this exact decimal.)

18. 7

19. $\left(\frac{-1+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right)$ (Must be this ordered pair with these or equivalent exact entries.)

20. $\frac{3}{4}$ (Must be this reduced common fraction.)

1. One of the solutions for x in the equation $x^2+bx+c=0$ where b and c are integers is $x=1+\sqrt{2}$. Find the value of the sum $(b+c)$.

2. $i=\sqrt{-1}$. Solve for b if $1-\dfrac{b}{4+i\sqrt{3}}+\dfrac{b}{4-i\sqrt{3}}=0$.

3. The graph of $4x^2-y^2-8x+2y=1$ is rotated $90°$ clockwise about the origin. Find the coordinates of the center of the new conic. Express your answer as the ordered pair (x,y).

4. Find **<u>the number</u>** of polar points below that are coincident (occupy the same position) as the polar point $\left(5,\dfrac{\pi}{6}\right)$?

   I. $\left(-5,\dfrac{7\pi}{6}\right)$ II. $\left(5,\dfrac{5\pi}{6}\right)$ III. $\left(5,\dfrac{13\pi}{6}\right)$ IV. $\left(5,\dfrac{11\pi}{6}\right)$ V. $\left(-5,\dfrac{-\pi}{6}\right)$

5. Find the value(s) of x for which $4+3\sqrt{7-x}=x+7$.

6. Simplify the expression $\left(\cos^6 x+3\left(\cos^4 x\right)\left(\sin^2 x\right)+3\left(\cos^2 x\right)\left(\sin^4 x\right)+\sin^6 x\right)$.

7. Let $P(x)=0$ be a third degree polynomial equation with **integer** coefficients and leading coefficient of 1. Let the polynomial equation $P(x)=0$ have at least one root that is an **integer**. If $P(6)=-11$, $P(4)=3$, and $P(-4)=-341$, find $P(10)$.

8. A bag contains red and blue marbles. When two marbles are drawn, the probability that they are both red is equal to the probability they are both blue. The probability that one of each color is drawn is $\frac{4}{7}$. Find the least number of marbles that are in the bag.

9. The distance between the lines $y=\frac{3}{4}x+b$ and $3x-4y=7$ is 2. If $b>0$, find the value of b.

10. If $i=\sqrt{-1}$, find the sum of an infinite geometric series whose first term is $2-i$ and common ratio is $\frac{1}{2}i$.

11. Determine the numerical coefficient of $x^2y^2z^2$ in the simplified expansion of $(1+x)^5(1+y)^4(1+z)^3$

12. The solutions for x in the equation $2(\ln x)^2=\ln x^5+3$ are of the form e^a where e is the base of a natural logarithm. Find the sum of all possible values of a. Write your answer as a common or improper fraction reduced to lowest terms.

13. Given $f(4x+3)=8x-1$, $f^{-1}(x)=ax+b$. Find the sum $(a+b)$.

14. $u = 2\sin x + \cfrac{\cos^2 x}{2\sin x + \cfrac{\cos^2 x}{2\sin x + \cfrac{\cos^2 x}{2\sin x + \cdots}}}$, where $0 < \cos x < \sin x$ and $\cos x = \frac{1}{3}$. Find the exact value of *u*. Write your answer as a single reduced fractional expression.

15. Find **<u>the number</u>** of distinct real values for x so that $\begin{bmatrix} x-2 & x+5 \\ 2x & x-3 \end{bmatrix}$ will **<u>not</u>** have an inverse.

16. When David and Jim play pool, the probability that David wins exactly two games out of four is equal to the probability that he wins exactly three games out of four. Find the probability that Jim wins any given game. Write your answer as a common fraction reduced to lowest terms. Assume that the outcome of a game cannot affect the probabilities in other games, and that both players have a non-zero chance of winning a game.

17. A parabola is described by $\begin{cases} x = 2+t \\ y = 1-t^2 \end{cases}$. Find the exact distance between the vertex and the *y*-intercept of this parabola.

18. Determine the exact sum of all values of θ, $0 \le \theta < 2\pi$ for which $\cos^2 \theta = \frac{3}{5}$.

19. The function $y = f(x)$ is periodic with period 3. If $f(6) = 4$, $f(8) = 2$ and $f(10) = 9$, find the value of $2f(1) + 3f(2) + 4f(3)$.

20. In a triangle with sides a, b, and c, $(a+b+c)(a+b-c) = 3ab$. Find $\sec C$, where angle C is the angle opposite side c.

2014 RAA
PreCalculus

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -3

2. $\frac{19i\sqrt{3}}{6}$ (Must be this exact answer only or exact algebraic equivalent.)

3. $(1, -1)$ (Must be this ordered pair.)

4. 2 OR TWO

5. 3

6. 1

7. 177

8. 8

9. $\frac{3}{4}$ OR 0.75 OR .75

10. 2

11. 180

12. $\frac{5}{2}$ (Must be this reduced improper fraction.)

13. 4

14. $\frac{2\sqrt{2}+3}{3}$ OR $\frac{3+2\sqrt{2}}{3}$ (Must be this exact single fract'l expr.)

15. 2

16. $\frac{2}{5}$ (Must be this reduced common fraction.)

17. $2\sqrt{5}$ (Must be this exact answer.)

18. 4π (Must be this exact answer.)

19. 40

20. 2

NO CALCULATORS

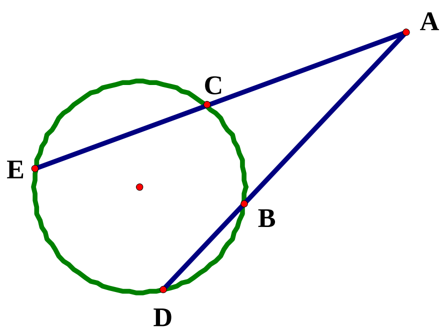
1. The point $(x, 5)$ is the midpoint of the segment connecting the points $(8,2)$ and $(10, y)$. Find the sum $(x+y)$.

2. The length of one leg of a right triangle is 4 and the length of one side of a square is 6. The area of the right triangle is half the area of the square. Find the exact numerical length of the hypotenuse of the right triangle.

3. Given that $xy \neq 0$ and $\dfrac{8x^4\left(2x^2y^3\right)^3}{\left(4x^3y^5\right)^2\left(xy^3\right)} = ax^by^c$, find the sum $(a+b+c)$.

4. The angles of a pentagon have degree measures that are in a ratio of 2:3:3:5:5. If one of the angles is selected at random, find the probability that the angle is obtuse. Write your answer as a common fraction reduced to lowest terms.

5. In the given diagram, $AC = CE = 6$ and $AB = 2(BD)$. Find the exact length of AB.



6. Set A has 5 elements and set B has 4 elements. The elements of set S are all of the possible numbers of elements that could be in $A \cup B$. Find the sum of the elements in set S.

7. ΔABC has vertices at $A(2,-4)$, $B(6,2)$ and $C(-8,4)$. The line containing the median from A to $\overline{BC}$ has a y-intercept of b. Find the value of b. Write your answer as the y-intercept and not the coordinates for the point that is the y-intercept on a graph.)

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

NO CALCULATORS

8. If $1.\overline{5}+1.\overline{45}=\frac{k}{w}$ where the fraction $\frac{k}{w}$ is an improper fraction reduced to lowest terms, find the value of k.

9. Evaluate the expression $\dfrac{\left(\frac{3}{4}\right)^2-3\left(\frac{3}{4}\right)\left(\frac{3}{8}\right)-10\left(\frac{3}{8}\right)^2}{2\left(\frac{3}{8}\right)^2+3\left(\frac{3}{4}\right)\left(\frac{3}{8}\right)+\left(\frac{3}{4}\right)^2}$.

10. A circle is inscribed in a triangle that has sides of lengths 5, 6 and 9. The point of tangency of the circle with the side of length 9 divides that side into two segments of lengths k and w. If $k>w$, find the ratio $k:w$. Write your answer in the form of the ratio $k:w$.

11. $x\odot y$ is defined as $ax+by$. If $7\odot 2=4$ and $3\odot 2=-4$, find the value of $9\odot 2$.

12. If the product $a^0\left(a^1\right)\left(a^2\right)\left(a^3\right)\left(a^4\right)\cdots\left(a^{2014}\right)$ is equal to a^{2014k}, find the value of k.

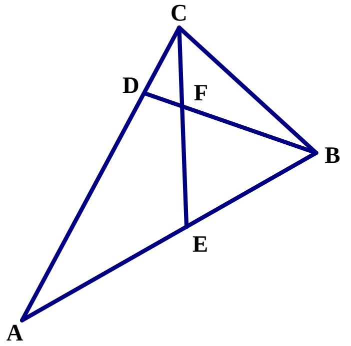
13. Let a and b be the lengths of legs of a right triangle with hypotenuse of length 2. Determine the value of $a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8$.

14. Find all integral value(s) of x so that the expression $\frac{6}{2x+1}$ represents an integer.

 NO CALCULATORS **NO CALCULATORS**

NO CALCULATORS

15. The graphs of $y = x^2 + 2x + 1$ and $y = x + 3$ intersect at a point with coordinates (x, y) in the first quadrant. Find the (x, y) coordinates of this point. Write your answer as an ordered pair (x, y).

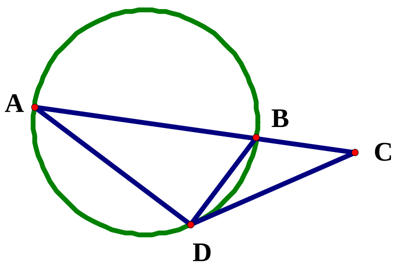


16. ΔABC is isosceles with side $\overline{AC}$ congruent to side $\overline{AB}$. The measure of $\angle ACB$ is twice the measure of $\angle CAB$. $\overline{CE}$ bisects $\angle C$ and $\overline{BD}$ trisects $\angle B$ so that the measure of $\angle CBD$ is less than the measure of $\angle DBA$. Find the degree measure of $\angle CFD$.

17. If $xy^2 = \sqrt{24}$ and $(xy)^3 = 60$, find the ratio of x to y. Write your answer in the form $x : y$ where x and y are positive integers with no common factors.

18. Find the simplified value of the expression $\dfrac{\left(6\times10^4\right)^3}{\left(2\times10^3\right)\left(9\times10^5\right)}$. Write your answer in scientific notation.

19. In the given figure, $AB = 2$, $BC = 1$, $\overline{AD} \perp \overline{BD}$ and $\overline{CD}$ is tangent to the circle at D. Find the exact length of $\overline{AD}$.



20. If $a < 0$ and $8x^2 + 3ax + 3a^2 = 2x^2 + 9a^2 - 2ax$, find the largest solution for x as an expression in terms of a.

2014 RAA

School ANSWERS

Fr/So 8 Person Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 17

2. $\sqrt{97}$ (Must be this exact answer.)

3. 3

4. $\frac{2}{5}$ (Must be this reduced common fraction.)

5. $4\sqrt{3}$

6. 35

7. $\frac{2}{3}$ OR $0.\overline{6}$ OR $.\overline{6}$

8. 298

9. -1

10. $5:4$ (Must be this ratio and in this form.)

11. 8

12. $\frac{2015}{2}$ OR $1007\frac{1}{2}$ OR 1007.5

13. 256

14. $-2, -1, 0, 1$ (Must be these four integers only, in any order.)

15. $(1, 4)$ (Must be this ordered pair.)

16. 60 (Degrees optional.)

17. $5:2$ (Must be this ratio and in this form.)

18. 1.2×10^5 (Must be this answer in scientific notation.)

19. $\sqrt{3}$ (Must be this exact answer.)

20. $-\frac{3a}{2}$ OR $\frac{-3}{2}a$ OR $-1.5a$ (Or exact simplified equivalent.)

NO CALCULATORS

1. Find the $x-$coordinate of the point that is the solution to the system $\begin{cases} \frac{1}{x} + 3y = 14 \\ \frac{4}{x} - 5y = -12 \end{cases}$.

2. Let $i = \sqrt{-1}$. Find the polynomial function $P(x)$ with real coefficients, of smallest degree, and with leading coefficient of 1 that has roots $2-i$ and 5. Write as your answer the **sum** of the numerical coefficients of $P(x)$.

3. Let $i = \sqrt{-1}$. A point x_0 is called a fixed point of function f if $f(x_0) = x_0$. Find all complex numbers which are fixed points of $f(x) = \frac{1}{3}\left(2x + \frac{8}{x^2}\right)$. Express your answer(s) in standard $a+bi$ form.

4. If $f(x+1) = x^2 + 7x + 2$ and $f(x-1) = ax^2 + bx + c$, find the ordered triple (a,b,c).

5. Four distinct numbers are chosen from the first nine positive integers. Find the probability that 3 is the smallest of the numbers chosen. Express your answer as a common fraction reduced to lowest terms.

6. Find the least integer that is a solution for x in $|2x-5| < |x+8|$.

7. Four children were playing in a back yard when one of them broke a window. When confronted, they said:

   "John did it," said Ann.
   John said, "It was Gail who broke it."
   "Anyway, it wasn't me," Sally declared.
   Gail said "John's a liar when he says I did it."

   If only one of the children told the truth, determine who broke the window. Write the **<u>full name</u>** of the person who broke the window for your answer.

8. One of the foci of the ellipse $4(x-1)^2+(y-2)^2=16$ lies in the first quadrant. Find the exact coordinates of this focus. Express your answer as an ordered pair (x, y).

9. The arithmetic mean of two numbers is 20. Find the maximum value of the geometric mean of the two numbers.

10. Find the sum of all x on the interval $0 \le x \le 2\pi$ for which $\sin(3x)=\cos(6x)$. Express your answer as a common or improper fraction reduced to lowest terms.

11. Find the largest possible value of the product (xy) if $|2x-1| \le 9$ any $|2y+9| \le 1$.

12. Find the value of $\sin(2\alpha)$ given that $\sin\alpha+\cos\alpha=\frac{5}{4}$. Express your answer as a common fraction reduced to lowest terms.

13. Find all value(s) of x for which $3\log_2(x+1)^2=12$.

14. In ΔABC with angles measured in degrees, $\angle C=2(\angle B)$, $AB=12$ and $AC=8$. Find $\cos(\angle C)$. Express your answer as a common fraction reduced to lowest terms.

15. $ABCDEF$ is a regular hexagon of side length 3. Find the ratio of the area common to ΔACE and ΔBDF to the area of hexagon $ABCDEF$. Express your answer in the form $k:w$ where k and w are integers with no common factors.

16. Let $i=\sqrt{-1}$. Find the value of $\sum_{n=0}^{8}(1+i)^n$.

17. Find the value of $\log_4\left(\binom{1024}{0}+\binom{1024}{1}+...+\binom{1024}{1023}+\binom{1024}{1024}\right)$. (Note: $\binom{k}{w}={}_kC_w=C(k,w)$ is combination or "choose" notation.)

18. A coin is to be flipped five times resulting in heads or tails. Find the probability that within the five flips, the longest sequence of heads will be exactly two heads in length. Express your answer as a common fraction reduced to lowest terms.

19. The solutions for x of the equation $x^3-3x^2-13x+k=0$ form an arithmetic sequence. Find the value of k.

20. When a polynomial $P(x)$ of degree $n>2$ is divided by $x-3$, the remainder is 5. When $P(x)$ is divided by $x-1$, the remainder is 1. Find the remainder when $P(x)$ is divided by x^2-4x+3.

2014 RAA

School ANSWERS

Jr/Sr 8 Person Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{2}$ OR 0.5 OR .5

2. -8

3. $2, -1+i\sqrt{3}, -1-i\sqrt{3}$ OR $2+0i, -1+i\sqrt{3}, -1-i\sqrt{3}$ (Must have all three values in any order.)

4. $(1, 3, -8)$ (Must be this ordered triple.)

5. $\frac{10}{63}$ (Must be this reduced common fraction.)

6. 0 OR zero

7. Sally (Must be this full name.)

8. $\left(1, 2+2\sqrt{3}\right)$ (Must be this ordered pair or with exact algebraic equivalent entries.)

9. 20

10. $\frac{17\pi}{2}$ OR $\frac{17}{2}\pi$ (Must be this reduced improper fraction.)

11. 20

12. $\frac{9}{16}$ (Must be this reduced common fraction.)

13. $-5, 3$ (Must have both values in either order.)

14. $\frac{1}{8}$ (Must be this reduced common fraction.)

15. $1:3$ (Must be this ratio in this form.)

16. $16-15i$ OR $-15i+16$

17. 512

18. $\frac{11}{32}$ (Must be this reduced common fraction.)

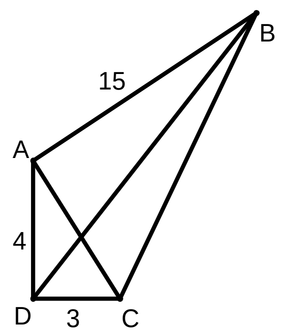
19. 15

20. $2x-1$ OR $-1+2x$

Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. Assume the cost of gasoline is 359.9 cents per gallon and assume that a car averages 27.83 miles per gallon. Find the number of **cents** the gasoline cost to drive this car 148.1 miles.

2. In the diagram (not necessarily drawn to scale), $\overline{AD} \perp \overline{CD}$ and $\overline{AB} \perp \overline{AC}$. $AB = 15$, $AD = 4$, and $CD = 3$. Find the numeric length BD.



3. Find all positive root(s) (or zero(s)) for the equation $y = x^3 + 0.682x^2 - 2.767x - 2.449$.

4. Point $P(3.433, y)$ lies on a line whose equation is $y = 1.244x + 2.113$. At point P, a line is drawn perpendicular to the line with the given equation. If this perpendicular passes through $(5.664, k)$, find the value of k.

5. Let A and B be the foci of the ellipse whose equation is $\frac{(x+1)^2}{100} + \frac{(y-2)^2}{k} = 1$. Let M be one of the endpoints of the minor axis of the ellipse. Let the **minor** axis be parallel to the x-axis. If $\angle AMB = 113.26°$, find the value of k.

6. The diagonals of a parallelogram have lengths 89.19 and 26.19 and intersect at an angle of $39.7°$. Find the area of this parallelogram.

7. The measures of an interior angle of a regular $k-sided$ polygon is $156°$. The measure of an interior angle of a regular $(k+3)-sided$ polygon is $w\%$ more than $156°$. Find the value of w. Write the value of w only as your answer. Do **not** use the % sign.

8. A frustum of a solid right circular cone has a bottom base whose area is twice the area of its upper base. A hemisphere whose flat portion is the upper base of the frustum is carved into the frustum and has its nearest point to the bottom base located 1 cm from the center of the bottom base. The frustum has the hemispherical portion removed from the frustum, and the remaining portion of the frustum has a volume of 10 cm^3. Find the total numerical surface area in cm^2 for this remaining portion of this solid frustum.

9. If $x > 2$, find the value of x such that $(x-2)^2 = \sqrt{5} + \sqrt{\sqrt{3}}$.

10. Bob ran from point A to point B at a constant rate of 7.243 feet per second. After arriving at point B, Bob immediately turned around and ran from point B to point A at a constant rate of x feet per second. In regard to his total distance and his total time, Bob's average rate was 6.964 feet per second. Find the value of x.

11. Find the slope of the line whose equation is $798.3x - 13.25y = 76.51$.

12. In ΔABC, $\angle CAB = 26.35°$, $\angle CBA = 81.43°$, and the area of ΔABC is 3526.37. Find the numeric length AB.

13. If $x > 1$ solve for x when $\log_5 x = \log_x 7$.

14. If the vector $(p + 3.483, w + 6.815, k - 14.34)$ is equal to the vector $(3p - 14.66, 2w - 16.03, 5\text{k} + 2.444)$, find the value of $(p + k + w)$.

15. Let x represent the length of the third side of a right triangle whose other two sides have lengths of 145 and 408, in some order. Find the absolute value of the difference between the smallest possible value of x and the largest possible value of x.

16. Find the real value of x such that $\frac{20^{(3x-7)}}{14^{(2x+5)}} = 1$

17. A circle, represented by the equation $x^2 - 6x + y^2 = 8y + 144$, has its center at P. Point A lies in Quadrant I, lies on the circle, and has an x-coordinate of 15. Point B lies on the positive x-axis and also lies on the circle. Find the area of the segment of the circle that is bounded by the chord from A to B and the minor arc from A to B.

18. If $k > 0$, find the smallest possible value of k such that $\cos(k°) = -0.4399$.

19. If a dealer could purchase his goods for 7.684% less while keeping his selling price fixed, his profit, based on the cost of his purchased goods, would be increased from his present profit of $k\%$ to $(k + 9.843)\%$. Find the value of k. Write your answer as the value of k only. Do **not** use the % sign.

20. Susan is playing Theresa in a best of 7 games in a table tennis match. The first player to win 4 games will be the winner of the match. The probability that Theresa will win any particular game after the first game is constant. Susan now leads the match by a score of 1 game to none. If the probability that Theresa will win the match is now $\frac{2}{3}$, find the probability that Theresa will win any particular game from now on.

2014 RAA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 1915 OR 1.915×10^3 (Cents optional.)

2. 17.69 OR 1.769×10 OR 1.769×10^1

3. 1.732 OR 1.732×10^0

4. 4.590 OR 4.590×10^0 (Trailing zero necessary.)

5. 330.5 OR 3.305×10^2

6. 746.0 OR 7.460×10^2 (Trailing zero necessary.)

7. 2.564 OR 2.564×10^0 (Must be this value only, without % sign.)

8. 35.94 OR 3.594×10 OR 3.594×10^1 (cm^2 optional.)

9. 3.885 OR 3.885×10^0

10. 6.706 OR 6.706×10^0 (Feet per second optional.)

11. 60.25 OR 6.025×10 OR 6.025×10^1

12. 123.7 OR 1.237×10^2

13. 5.869 OR 5.869×10^0

14. 27.72 OR 2.772×10 OR 2.772×10^1

15. 51.64 OR 5.164×10 OR 5.164×10^1

16. 9.211 OR 9.211×10^0

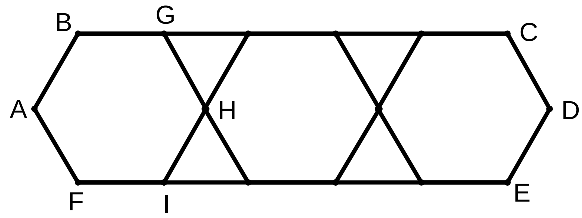
17. 4.865 OR 4.865×10^0

18. 116.1 OR 1.161×10^2 (Degrees optional.)

19. 18.25 OR 1.825×10 OR 1.825×10^1 (Must be this value only, without the % sign.)

20. 0.6598 OR $.6598$ OR 6.598×10^{-1}

1. The given diagram shows 3 congruent regular hexagons between parallel lines containing sides and pairwise sharing a vertex. $ABGHIF$ has numerical area 36. Find the area of hexagon $ABCDEF$.



2. Find the sum of all real solution(s) for the equation $\sqrt{2x+5}-1=x$

3. Let k be the sum of the roots of the equation $2x^2-7x+15=0$. Let w be the slope of the line having an equation $2x-5y=6$. Find $(k+w)$. Express your answer as a common or improper fraction reduced to lowest terms.

4. k is a two-digit integer in which the ten's digit exceeds the unit's digit by 2 and the sum of the ten's digit and twice the unit's digit is 17. w is Jack's age now if five years ago Benny was 5 times as old as Jack was at that time and four years from now Benny will be twice as old as Jack will be. Find $(k+w)$.

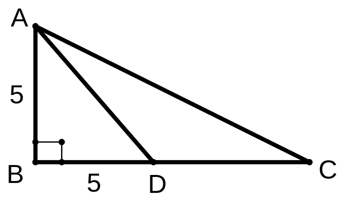
5. Let $k+k\sqrt{3}$ represent the numerical area of a triangle with sides length 12, $6+6\sqrt{3}$, and $6\sqrt{2}$. Let $w\sqrt{2}$ represent the length of the diagonal of a square with perimeter 12. Find $(k+w)$.

6. Let x and y be positive integers such that $x>y$. Several distinct ordered pairs of the form (x,y) are solutions for $(2x+3y)^2=6084$. Let k be the sum of the x coordinates and let w be the sum of the y coordinates of these solutions. Find $(k+w)$.

7. Let $k=\sqrt{\dfrac{7^5+7^5+7^5+7^5+7^5+7^5}{6^5+6^5+6^5+6^5+6^5+6^5+6^5}}$. A 5 gallon jar with 1.5 gallons of antifreeze and the rest water and a 4 gallon jar with 0.5 gallons of antifreeze and the rest water are combined in a large 15-gallon empty tank. Let w be the fractional part of this new mixture that is antifreeze. Find (kw). Express your answer as a common or improper fraction reduced to lowest terms.

8. k is the number of distinct integral solutions for x if $|x+2|\le 1$. Equilateral triangle ΔABC is inscribed in a circle and has area $64\sqrt{3}$. Let w be the exact area of the circumscribing circle. Find the exact value of (kw).

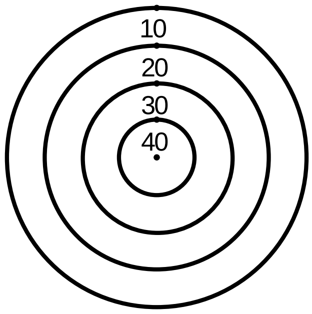
9. In ΔABC, $\overline{AB} \perp \overline{BC}$ and $AB = BD = 5$. The perimeter of ΔABD equals the perimeter of ΔACD. Find the length of $\overline{AC}$. Express your answer as an improper fraction reduced to lowest terms.



10. Let $x^2 - 4kx = -4k^2$ where k represents the length of the altitude drawn to the longest side of a triangle with sides of length 3, 4, and 5. Find the positive solution for x. Express your answer as a common or improper fraction reduced to lowest terms.

11. Set A contains 15 elements, set B contains 9 elements, and set C contains 13 elements. $A \cap B$ (A and B) contains 6 elements, $A \cap C$ 8 elements, $B \cap C$ 5 elements, and $A \cap B \cap C$ contains 2 elements. The Universal set containing A, B, and C contains 50 elements. How many elements are in the universal set but not in any of A, B, or C?

12. A dart board is a set of concentric circles as shown with each annulus (circular band) having the same width as the radius of the innermost circle. Find the probability that, when three darts are thrown, the score achieved is exactly 70. Answer as a common fraction reduced to lowest terms. (Assume all three darts hit the board, stick in one of the bands, and the order in which the darts hit the board does not matter.)



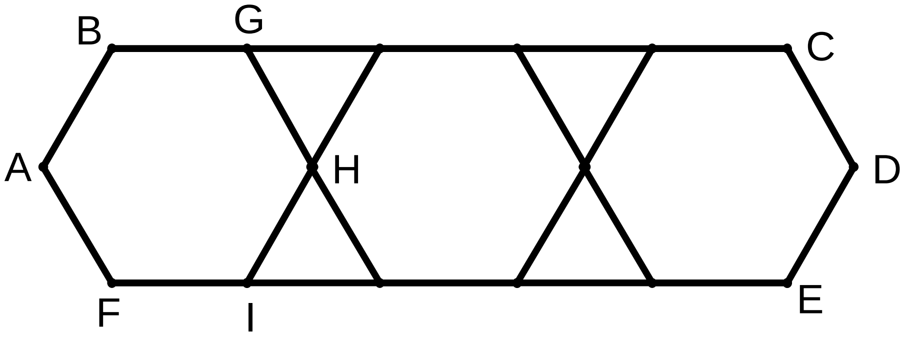
ICTM Math Contest

Freshman – Sophomore

2 Person Team

Division AA

1. The given diagram



shows 3 congruent regular hexagons between parallel lines containing sides and pairwise sharing a vertex. $ABGHIF$ has numerical area 36. Find the area of hexagon $ABCDEF$.

2. Find the sum of all real solution(s) for the equation $\sqrt{2x+5}-1=x$

3. Let k be the sum of the roots of the equation $2x^2 - 7x + 15 = 0$. Let w be the slope of the line having the equation $2x - 5y = 6$. Find $(k + w)$. Express your answer as a common or improper fraction reduced to lowest terms.

4. k is a two-digit integer in which the ten's digit exceeds the unit's digit by 2 and the sum of the ten's digit and twice the unit's digit is 17. w is Jack's age now if five years ago Benny was 5 times as old as Jack was at that time and four years from now Benny will be twice as old as Jack will be. Find $\left(k+w\right)$.

5. Let $k+k\sqrt{3}$ represent the numerical area of a triangle with side lengths 12, $6+6\sqrt{3}$, and $6\sqrt{2}$. Let $w\sqrt{2}$ represent the length of the diagonal of a square with perimeter 12. Find $(k+w)$.

6. Let x and y be positive integers such that $x > y$. Several distinct ordered pairs of the form (x, y) are solutions for $(2x+3y)^2 = 6084$. Let k be the sum of the x coordinates and let w be the sum of the y coordinates of these solutions. Find $(k+w)$.

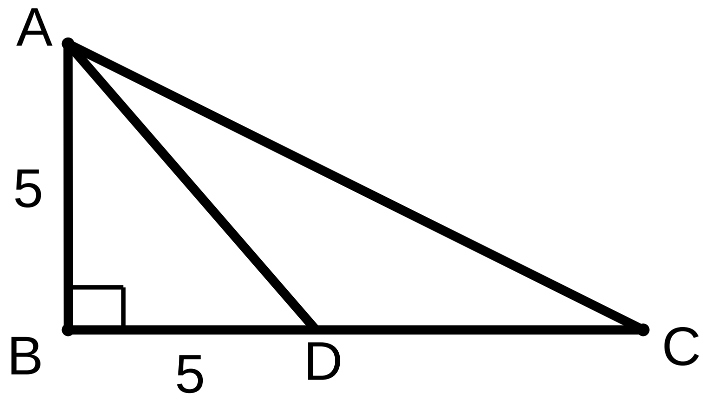
7. Let

$$k = \sqrt{\frac{7^5 + 7^5 + 7^5 + 7^5 + 7^5 + 7^5}{6^5 + 6^5 + 6^5 + 6^5 + 6^5 + 6^5 + 6^5}}.$$

A 5 gallon jar with 1.5 gallons of antifreeze and the rest water and a 4 gallon jar with 0.5 gallons of antifreeze and the rest water are combined in a large 15-gallon empty tank. Let w be the fractional part of this new mixture that is antifreeze. Find (kw).

Express your answer as a common or improper fraction reduced to lowest terms.

8. k is the number of distinct integral solutions for x if $|x+2| \le 1$. Equilateral triangle ΔABC is inscribed in a circle and has area $64\sqrt{3}$. Let w be the exact area of the circumscribing circle. Find the exact value of (kw).

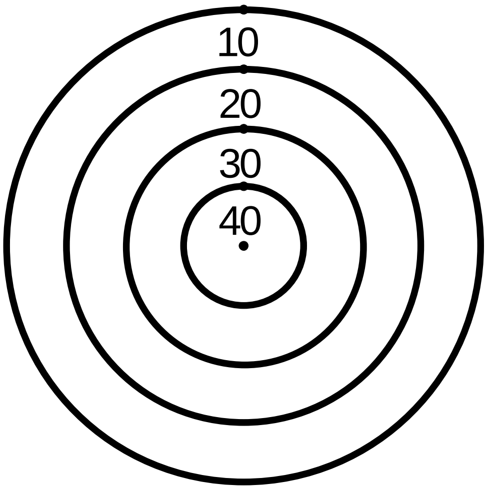


9. In ΔABC, $\overline{AB} \perp \overline{BC}$ and $AB = BD = 5$. The perimeter of ΔABD equals the perimeter of ΔACD. Find the length of $\overline{AC}$. Express your answer as an improper fraction reduced to lowest terms.

10. Let $x^2 - 4kx = -4k^2$ where k represents the length of the altitude drawn to the longest side of a triangle with sides of length 3, 4, and 5. Find the positive solution for x. Express your answer as a common or improper fraction reduced to lowest terms.

11. Set A contains 15 elements, set B contains 9 elements, and set C contains 13 elements. $A \cap B$ (A and B) contains 6 elements, $A \cap C$ contains 8 elements, $B \cap C$ contains 5 elements, and $A \cap B \cap C$ contains 2 elements. The Universal set containing A, B, and C contains 50 elements. How many elements are in the universal set but not in any of A, B, or C?

12. A dart board is a set of concentric circles as shown with each annulus (circular band) having the same width as the radius of the innermost circle. Find the probability that, when three darts are thrown, the score achieved is exactly 70. Answer as a common fraction reduced to lowest terms. (Assume all three darts hit the board, stick in one of the bands, and the order in which the darts hit the board does not matter.)



2014 RAA

School **ANSWERS**

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer	**Score** (to be filled in by proctor)
1.	132	
2.	2	
3.	$\frac{39}{10}$ (Must be this reduced improper fraction.)	
4.	83	
5.	21	
6.	245	
7.	$\frac{49}{162}$ (Must be this reduced common fraction.)	
8.	256π (Must be this exact answer.)	
9.	$\frac{25}{3}$ (Must be this reduced improper fraction.)	
10.	$\frac{24}{5}$ (Must be this reduced improper fraction.)	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 30

12. $\frac{39}{256}$ (Must be this reduced improper fraction.)

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. The inequality $|x-1|<7$ has the solution $\{x : k < x < w\}$. Find the sum $(k+w)$.

2. In right ΔABC with right angle at C, $AC=4$ and $AB=8$. The expression $\tan A+\cos B+2\sin C$ has a value that can be written in simplified and reduced form as $\frac{k+w\sqrt{p}}{q}$. Find the sum $(k+w+p+q)$.

3. Let $\log_3(k)+\log_3(4)=2$. Let S be the sum of the arithmetic sequence consisting of the first 23 positive integers. Find the value of the product (kS).

4. The solutions of the equation $x^3-x^2-22x+40=0$ are k, w, and p. Find the value of $(k^2+w^2+p^2)$.

5. Points A, B, and C lie on circle O with A and B endpoints of a diameter. The area of circle O is 27π and the altitude of ΔABC to side $\overline{AC}$ is 9. $k=\cos(\angle ABC)$. If $f^{-1}(x)=\sqrt{2x+\sqrt{3}}$ then $w=f(6)$. Find the sum $(k+w)$.

6. Assume Matt Garza pitches exactly 200 innings in a season. By August of 2013, he had allowed 8 home runs in 71 innings in the National League before being traded and allowing 7 home runs in 45 innings in the American League. At these rates, find the difference in home runs, rounded to the nearest whole number, he would have allowed if he had pitched the entire season in one league or the other.

7. Let k be the sum of the solutions for $3^{2x^2+5x-3}=1$. Let w be the sum of the distinct rational zeros of the function $f(x)=3x^4+16x^3-35x^2$. Find the sum $(k+w)$. Express your answer as a common or improper fraction reduced to lowest terms.

8. Find the value of x if $314_{nine}+213_{eleven}=x_{six}$.

9. Find the coefficient of the term containing $x^7y^6z^5$ in the expansion of $(x+y+z)^{18}$.

10. The graph of a parabola opens upward and has endpoints of the latus rectum at $A(-5,5)$ and $B(11,5)$. The equation of the circle passing through the endpoints of the latus rectum and the vertex of the parabola can be written as $(x-h)^2+(y-k)^2=r^2$. Find the sum $(h+k+r)$.

11. Find the exact sum of the solution(s) for x if $\frac{x}{x^2+3x-4}+\frac{x+1}{x^2+6x+8}=\frac{2x}{x^2+x-2}$. (Assume non-zero denominators.)

12. If a, b, and c are the lengths of the sides of right ΔABC with $a \le b \le c$. The area of the right ΔABC is $2\sqrt{3}$ and $c = 2a$. Find the numerical length of the shorter leg.

ICTM Math Contest

Junior – Senior

2 Person Team

Division AA

1. The inequality $|x-1|<7$ has the solution $\{x : k < x < w\}$.

Find the sum $(k+w)$.

2. In right ΔABC with right angle at C, $AC = 4$ and $AB = 8$. The expression $\tan A + \cos B + 2\sin C$ has a value that can be written in simplified and reduced form as $\frac{k + w\sqrt{p}}{q}$. Find the sum $(k + w + p + q)$.

3. Let $\log_3(k)+\log_3(4)=2$. Let S be the sum of the arithmetic sequence consisting of the first 23 positive integers. Find the value of the product (kS).

4. The solutions of the equation
$x^3 - x^2 - 22x + 40 = 0$
are k, w, and p.

Find the value of
$\left(k^2 + w^2 + p^2\right)$.

5. Points A, B, and C lie on circle O with A and B endpoints of a diameter. The area of circle O is 27π and the altitude of ΔABC to side $\overline{AC}$ is 9. $k = \cos\left(\angle ABC\right)$.

If $f^{-1}\left(x\right) = \sqrt{2x + \sqrt{3}}$ then $w = f\left(6\right)$.

Find the sum $\left(k + w\right)$.

6. Assume Matt Garza pitches exactly 200 innings in a season. By August of 2013, he had allowed 8 home runs in 71 innings in the National League before being traded and allowing 7 home runs in 45 innings in the American League. At these rates, find the difference in home runs, rounded to the nearest whole number, he would have allowed if he had pitched the entire season in one league or the other.

7. Let k be the sum of the solutions for $3^{2x^2+5x-3}=1$. Let w be the sum of the distinct rational zeros of the function $f\left(x\right)=3x^4+16x^3-35x^2$. Find the sum $\left(k+w\right)$. Express your answer as a common or improper fraction reduced to lowest terms.

8. Find the value of x if

$$314_{nine} + 213_{eleven} = x_{six}.$$

9. Find the coefficient of the term containing $x^7 y^6 z^5$ in the expansion of $\left(x+y+z\right)^{18}$.

10. The graph of a parabola opens upward and has endpoints of the latus rectum at $A(-5,5)$ and $B(11,5)$. The equation of the circle passing through the endpoints of the latus rectum and the vertex of the parabola can be written as $(x-h)^2+(y-k)^2=r^2$. Find the sum $(h+k+r)$.

11. Find the exact sum of the solution(s) for x if

$$\frac{x}{x^2+3x-4}+\frac{x+1}{x^2+6x+8}=\frac{2x}{x^2+x-2}$$

(Assume non-zero denominators.)

12. If a, b, and c are the lengths of the sides of right ΔABC with $a \le b \le c$ and $c = 2a$. The area of right ΔABC is $2\sqrt{3}$. Find the numerical length of the shorter leg.

2014 RAA

Jr/Sr 2 Person Team

School **ANSWERS**

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. 2	
2. 12	
3. 621	
4. 45	
5. 18	
6. 9 (Must be this integer.)	
7. $-\frac{47}{6}$ OR $\frac{-47}{6}$ (Must be this reduced improper fraction.)	
8. 2212	
9. 14,702,688 OR 14702688	
10. 24	

TOTAL SCORE: ________

(*enter in box above)

Extra Questions:

11. $-\frac{1}{6}$ OR $\frac{-1}{6}$ OR $-0.1\overline{6}$ OR $-.1\overline{6}$

12. 2

13. ________

14. ________

15. ________

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION AA

Problem 1:

The mail typically arrives at Jane's house at a random time between 3PM and 5PM. Jane always arrives home from work at 4PM and then leaves to takes her dog out for an hour long walk at 4:30PM. What is the probability that Jane will be at her home when her mail arrives?

Problem 2:

The telephone company has notified their customers that there will be a 1 hour service outage sometime after 1PM which will be over sometime before 5PM today. Billy and his grandmother plan to be on the phone for 30 minutes from 2:30PM to 3:00PM today. What is the probability that their conversation will be affected by the outage?

Problem 3:

Suppose a point with coordinates (A, B) is chosen randomly from within a square whose vertices have coordinates (0,0), (10,0), (10,10) and (0,10).

a) What is the probability that $B < 8$?

b) What is the probability that $B + 2 < A$?

c) What is the probability that $|A - 3| \leq 2$?

d) What is the probability that $A^2 + B^2 \leq 4$?

e) What is the probability that $(A-1)^2 + B^2 \leq 2$?

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION AA

EXTEMPORANEOUS QUESTIONS

<u>Give this sheet to the students at the beginning of the extemporaneous question period.</u>

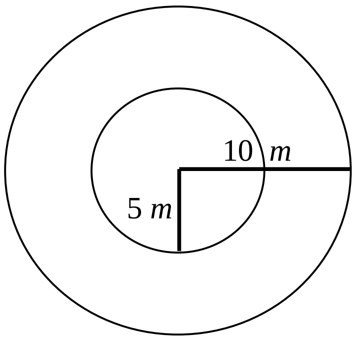
<u>STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.</u>

Extemporaneous Problem 1:

The diagram below consists of two concentric circles, a larger (outer) circle of radius 10 meters and an smaller (inner) circle of radius 5 meters. Suppose a point is randomly chosen somewhere on the interior of the larger circle.

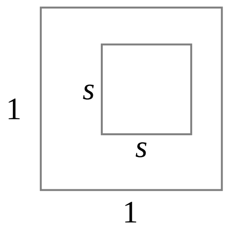
a) What is the probability that the point chosen is also inside of the smaller circle ?

b) What is the probability that the point chosen is **not** inside of the smaller circle ?



Extemporaneous Problem 2:

A random point is chosen from within a square with sides of length 1. A smaller square with sides of length *s* lies inside this square. For what value of s does the point have a probability of 0.75 of being **outside** of the smaller square?



Extemporaneous Problem 3:

Dan and Tom have decided to meet for lunch. They are approaching the restaurant from opposite directions and are 10 miles apart. The restaurant is somewhere between them but neither knows where. If Dan is biking 20 miles per hour and Tom is biking 10 miles per hour what is the probability that Tom will reach the restaurant first?

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION AA – Judges Solutions

Problem 1:

The mail typically arrives at Jane's house at a random time between 3PM and 5PM. Jane always arrives home from work at 4PM and then leaves to takes her dog out for an hour long walk at 4:30PM. What is the probability that Jane will be at her home when her mail arrives ?

Solution:
The probability that Jane will be home is the probability that the mail will arrive between 4PM and 4:30PM which is: $\frac{30 \text{ minutes}}{2 \text{ hours}} = \frac{1}{4}$**.**

Problem 2:

The telephone company has notified their customers that there will be a 1 hour service outage sometime after 1PM which will be over sometime before 5PM today. Billy and his grandmother plan to be on the phone for 30 minutes from 2:30PM to 3:00PM today. What is the probability that their conversation will be affected by the outage ?

Solution:
The crucial idea for this problem is to base the analysis on when the outage begins. Billy's conversation with his grandmother will only be affected by the outage if it begins sometime between 1:30PM and 3:00PM. Furthermore, the outage will begin sometime after 1PM, but because the outage has to end before 5PM, it must begin before 4PM. Hence the probability we are looking for is:

$$\frac{\text{Span of start time which will affect Billy}}{\text{Span of all possible start time}} = \frac{1.5 \text{ hours}}{3 \text{ hours}} = \frac{1}{2}$$

Problem 3:

Suppose a point with coordinates (A,B) is chosen randomly from within a square whose vertices have coordinates (0,0), (10,0), (10,10) and (0,10).

a) What is the probability that $B < 8$?

b) What is the probability that $B + 2 < A$?

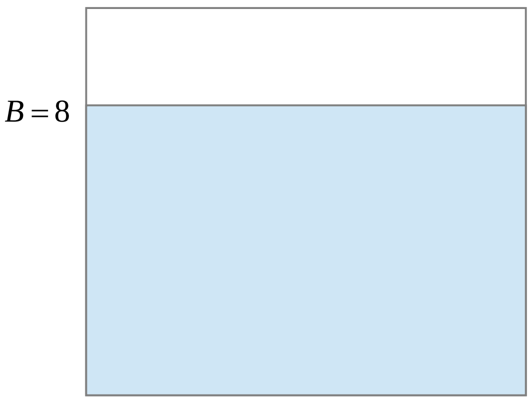
c) What is the probability that $|A - 3| \le 2$?

d) What is the probability that $A^2 + B^2 \le 4$?

e) What is the probability that $(A-1)^2 + B^2 \le 2$?

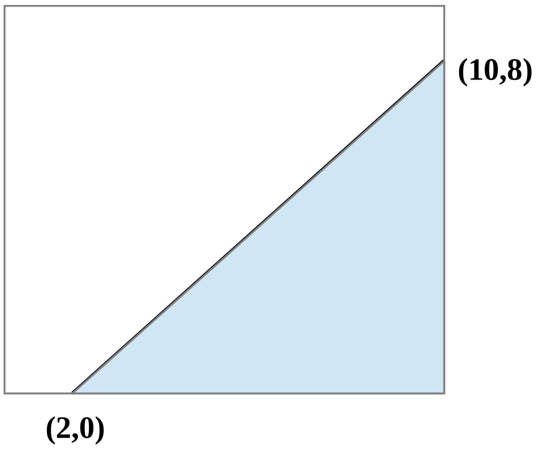
Solution:

a) The region $B < 8$ looks like:



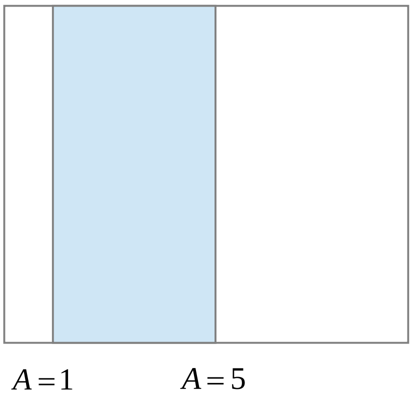
The probability of a point falling in the shaded region is $\frac{8(10)}{10(10)} = \frac{4}{5}$.

b) The region $B+2<A$ **cuts the square as illustrated:**



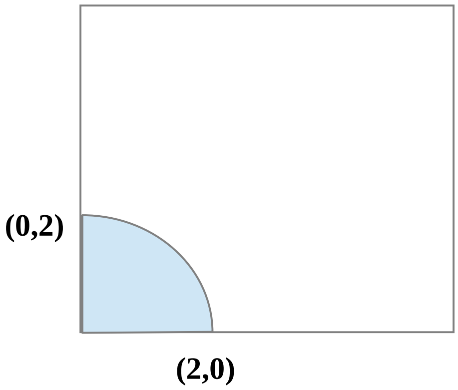
So the probability is: $\frac{area\ of\ triangle}{area\ of\ square}=\frac{.5(8)(8)}{100}=\frac{32}{100}=\frac{8}{25}$.

c) The region $|A-3|\le 2$ **means that** $1\le A\le 5$ **which looks like:**



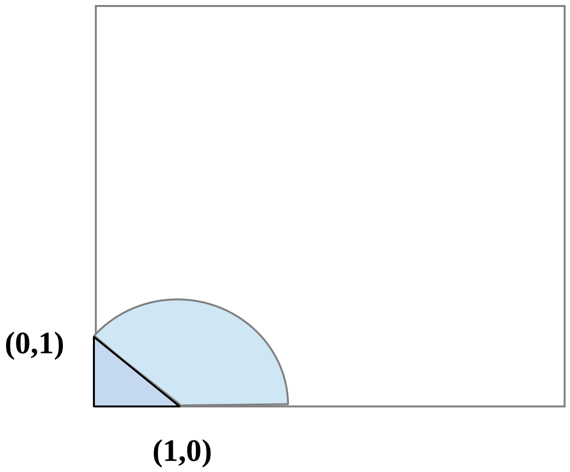
The probability of a point falling in the shaded region is $\frac{4(10)}{10(10)}=\frac{2}{5}$**.**

d) The region $A^2 + B^2 \le 4$ is a quarter circle centered at (0,0) with radius 2:



So the probability is $\dfrac{\textit{area of quarter circle}}{\textit{area of square}} = \dfrac{\left(\frac{1}{4}\right)\pi\left(2^2\right)}{100} = \dfrac{\pi}{100}$ **.**

e) The region $(A-1)^2 + B^2 \le 2$ is a sector of a circle with center at $(1,0)$ and radius $\sqrt{2}$ and the triangle with vertices at (0,0), (0,1) and (1,0) which appears as follows:



The circle sector has a radius $\sqrt{2}$ and sweeps $\dfrac{3\pi}{4}$ radians or $135°$. This area can be computed by finding the area of the sector. The sector is $\dfrac{135°}{360°} = \dfrac{3}{8}$ of the circle and has area $\dfrac{3}{8}(\textit{Area of circle}) = \dfrac{3}{8}\left(\pi\left(\sqrt{2}\right)^2\right) = \dfrac{3}{4}\pi$**. The triangle has area $\dfrac{1}{2}(1)(1) = \dfrac{1}{2}$.**

The probability is then $\dfrac{\textit{area of sector} + \textit{area of triangle}}{\textit{area of square}} = \dfrac{\left(\frac{3}{4}\right)\pi + \frac{1}{2}}{100} = \dfrac{3\pi + 2}{400}$ (or exact equivalent).

ORAL COMPETITION
ICTM REGIONAL 2014 DIVISION AA – JUDGES SOLUTIONS

EXTEMPORANEOUS QUESTIONS

Give this sheet to the students at the beginning of the extemporaneous question period.

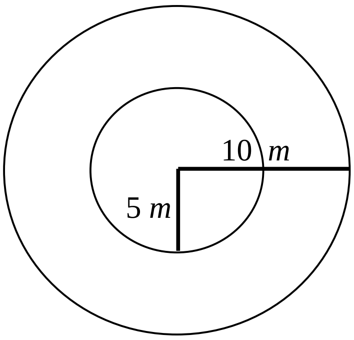
STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

Extemporaneous Problem 1:

The annular region illustrated below consists of two concentric circles, a larger (outer) circle of radius 10 meters and an smaller (inner) circle of radius 5 meters. Suppose a point is randomly chosen somewhere on the interior of the larger circle.

a) What is the probability that the point chosen is also inside of the smaller circle ?

b) What is the probability that the point chosen is **not** inside of the smaller circle ?



Solution:

a) The probability that the point is inside the smaller circle is:

$$\frac{\textit{area of inner circle}}{\textit{area of outer circle}} = \frac{\pi\left(5^2\right)}{\pi\left(10^2\right)} = \frac{1}{4} \ .$$

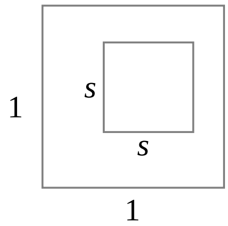
b) The probability that the point is outside the smaller circle is:

1 – the probability from (a), so the answer is $\frac{3}{4}$.

Note that an alternate solution would be to find the difference in the areas of the larger and smaller circles and then divide by the area of the larger circle: $\frac{100\pi - 25\pi}{100\pi} = \frac{75\pi}{100\pi} = \frac{3}{4}$**.**

Extemporaneous Problem 2:

A random point is chosen from within a square with sides of length 1. A smaller square with sides of length *s* lies inside this square. For what value of s does the point have a probability of 0.75 of being **outside** of the smaller square?



Solution:

The probability that the point is outside the smaller square is $1-s^2$**. Thus, the value of *s* that meets the conditions is the solution to** $1-s^2=0.75$ **which is** $s=0.5$ **.**

Extemporaneous Problem 3:

Dan and Tom have decided to meet for lunch. They are approaching the restaurant from opposite directions and are 10 miles apart. The restaurant is somewhere between them but neither knows where. If Dan is biking 20 miles per hour and Tom is biking 10 miles per hour what is the probability that Tom will reach the restaurant first ?

Solution:

The specific distance of 10 miles between Dan and Tom is not used in the solution. By the time Dan and Tom would hypothetically reach each other Dan will have covered twice as much ground as Tom. Therefore, Tom has a 1 in 3 chance of reaching the restaurant first. So the probability that Tom will reach the restaurant first is $\frac{1}{3}$**.**