1. Determine the value of the expression $\frac{a^2 - 2b + 3c}{b - a + c}$ when $a = -2$, $b = 3$ and $c = \frac{1}{2}$. Express your answer as a common or improper fraction reduced to lowest terms.

2. For all real values of x, $6x^2 + bx + c = 3\left(ax^2 + 3x + 4\right)$. Determine the sum $\left(a + b + c\right)$.

3. The sum of the mode and median is k larger than the arithmetic mean for the set of numbers $\left\{2, 2, 8, 2, 0, 1, 5\right\}$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

4. Determine the value of $\frac{3x + 2y}{y}$ when $\frac{x}{y} = 12$.

5. A prime number less than 20 is randomly selected. Determine the probability that the number selected is even. Express your answer as a common fraction reduced to lowest terms.

6. The slope of a loading dock ramp has a 5:12 ratio for vertical height to horizontal length measure. The length of the actual ramp is 39 feet. Determine the height of the dock in feet.

7. Let k be ***the number*** of values for x, $5 \le x \le 30$ for which the expression $4x - 5$ is an integral multiple of 3. Determine the value of *k*.

8. Determine the value of $\frac{3x+4}{7}$ when $\frac{9x+4}{4}=3x-5$.

9. A town's population increased by 700 people, and then this new population decreased by 5%. The town now has 20 fewer people than it did before the 700 person increase. Determine the number of people in the town's original population.

10. Determine the value(s) of k for which the equation $4(2x+k)=8(x+3)+12$, when solved for x, will have an infinite number of solutions.

11. $[a]$ is defined to be the largest integer less than or equal to a, Determine the value of $\left[\left(\frac{5}{3}\right)^3\right]$.

12. Determine the smallest integer x such that $\frac{1}{2}$ of $\frac{2}{3}$ of the sum of x and 6 is greater than $\frac{1}{6}$ of the sum of x and 30.

13. x is a positive odd integer. Determine *the sum* of all the distinct value(s) of x such that $-(x-11)=|x-11|$.

14. Let k be a positive integer number base so that $28_k=132_5$. Determine the value of k.

15. Jeffrey drove to a town 25 miles away at an average speed of 50 miles per hour. The return trip along the same route took 20 minutes longer than the trip to town. Determine the average speed in miles per hour for Jeffrey's round trip.

16. The ratio of boys to girls in a room is 4 to 5. If 4 boys left and 5 more girls came in, the ratio of boys to girls in the room would be 2 to 5. Determine the total number of students who were in the room originally.

17. $3^{x-2y} = \frac{3^{2x-y}}{81}$. Determine the sum $(x+y)$.

18. A number x is 3 more than twice a number y. The number y is 3 more than twice the number x. Determine the ordered pair (x, y) for these numbers. Give your answer as an ordered pair (x, y).

19. A rectangular flower bed on the grounds of the Field Museum measures 5 feet more in length than 3 times the width. The sidewalk that surrounds the bed is 4 feet wide. The combined area of the flower bed and sidewalk is 6806.25 square feet. Determine the sum, in feet, of the length and width of the flower bed.

20. Determine the coordinates of the point(s) in the fourth quadrant where the graphs of $x^2 + 2y^2 = 33$ and $x + 2y = 1$ intersect. Express your answer as ordered pair(s) (x, y).

2015 RA
Algebra I

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-\frac{1}{11}$ OR $\frac{-1}{11}$ OR $\frac{1}{-11}$ (Must be this reduced common fraction.)

2. 23

3. $\frac{8}{7}$ (Must be this reduced improper fraction.)

4. 38

5. $\frac{1}{8}$ OR 0.125 OR .125

6. 15 ("feet" optional.)

7. 9 ("values" optional.)

8. 4

9. 13700 ("people" and comma optional.)

10. 9 (This value only.)

11. 4

12. 19 (Must be this integer.)

13. 36

14. 17

15. 37.5 OR $\frac{75}{2}$ ("mph" optional.)

16. 27 ("students" optional.)

17. 4

18. $(-3,-3)$ (Must be this ordered pair.)

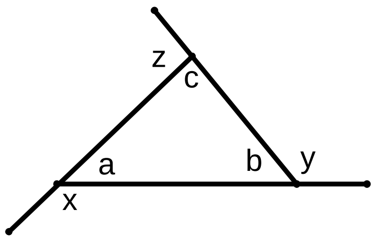
19. 171 ("feet" optional.)

20. $(5,-2)$ (Must be this ordered pair only.)

1. $MN = 105$ in the diagram shown. Determine the value of k.

2. Circle O has radius 7 and Circle P has radius 3 and lies entirely in the interior of Circle O. Determine the ratio of the area in the interior of Circle O but not in the interior of Circle P to the entire area of Circle O. Express your answer as a common fraction reduced to lowest terms.

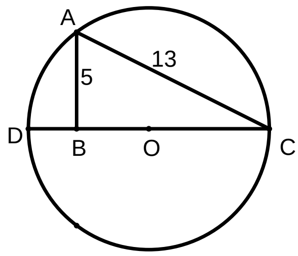
3. A triangle with sides extended is shown (but not necessarily drawn to scale) with angle measures as labeled. $a:b:c = 5:7:8$. Determine the ratio $x:y:z$. Express your answer in the form $x:y:z$.



4. A trapezoid with legs of length 8 and 7 and height of length 6 has numerical perimeter 40. Determine the numerical area of this trapezoid.

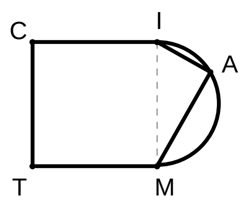
5. Katie jogged 3 miles due north, then 2 miles due east, then 4 miles due north, then 13 miles due east and finally 1 mile due north. Determine Katie's exact direct distance now from the starting point in miles.

6. Points A, D and C lie on circle O with diameter $\overline{CD}$ $\overline{AB}$ is perpendicular to $\overline{CD}$ at point B with $AB = 5$ and $AC = 13$. Determine the length of the diameter of this circle. Express your answer as a common or improper fraction reduced to lowest terms.



7. $\overline{KW}$ is a chord of Circle O. $KW = 1024$ and $OK = 600$. Determine the exact distance from O to chord $\overline{KW}$.

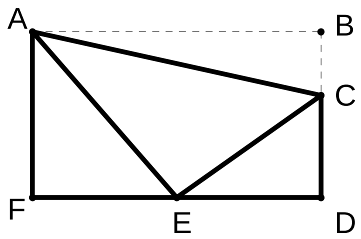
8. $ICTM$ is a square with numeric perimeter of 40 and semi-circle $\widehat{IAM}$ constructed with $\overline{IM}$ as a diameter. $AI = \frac{1}{2}(IM)$. The exact numeric area of pentagon $ICTMA$ can be written in reduced and simplified radical form as $\frac{k + w\sqrt{p}}{q}$ where k, w, p, and $q > 0$ are integers. Determine the sum $(k + w + p + q)$.



9. A circle has circumference 37.64 and a radius $(k+1)$. Determine the value of k. Express your answer as a decimal correct to four significant digits.

10. Determine all ordered pair(s) of positive integers (r,h) for r the radius and h the height of a cone whose numeric volume is equal to the numeric surface area. Express each answer(s) as an ordered pair (r,h).

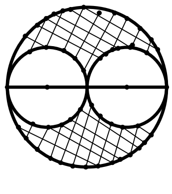
11. A piece broke off rectangle $ABDF$ leaving trapezoid $ACDF$. $BD=16$, $BC=7$, $FD=24$ and E is the midpoint of $\overline{FD}$. Determine the exact perimeter of ΔACE.



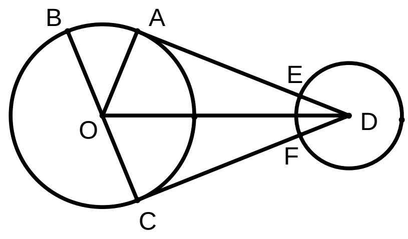
12. A rhombus has diagonals of lengths 18 and 14. Determine the exact perimeter for this rhombus.

13. A right circular cone with base diameter 10 and slant height 13 fits exactly inside a square prism. The base of the cone is inscribed in the base of the prism and both the cone and the prism have the same height. Determine the exact numeric volume interior to the prism but exterior to the cone.

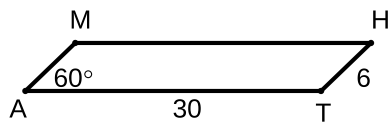
14. Congruent circles are constructed inside a larger circle with the centers of the smaller circles on a diameter of the larger circle. The smaller circles are tangent to each other and each is tangent to the larger circle. The distance between the centers of the smaller circles is 8. Determine the exact numeric area inside the larger circle but exterior to the smaller circles (the cross-hatched region shown in the diagram).



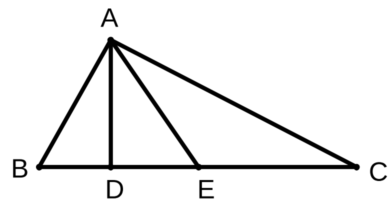
15. $\overline{DA}$ and $\overline{DC}$ are tangent to Circle O. $OA = 4$, $\overset{\frown}{AB} = 60°$ and $CF = 3(FD)$. A, E, and D are collinear as are points C, F, and D and points B, O, and C. Determine the exact area of Circle D.



16. Parallelogram $MATH$ has $AT = 30$, $TH = 6$, and $\angle MAT = 60°$ as shown. Determine the exact numeric area for Parallelogram $MATH$.



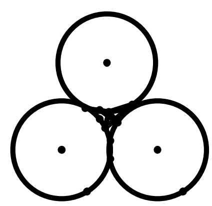
17. Right ΔABC, with hypotenuse $\overline{BC}$, has altitude $\overline{AD}$ and median $\overline{AE}$. $AB = 6$ and $BD = 4$. Determine the exact length of $\overline{DE}$.



18. The measures of two angles of a triangle are in the ratio $2:3$. The third angle measures $4°$ more than the larger of the other two angles. Determine the degree measure of the exterior angle adjacent to the third angle of this triangle.

19. Square $ABCD$ has E as a midpoint of $\overline{AB}$. $\overline{EB}$ is extended beyond B and point F is positioned on ray $\overrightarrow{EB}$ such that $EF = EC$. $\overline{DC}$ is extended past point C and point G is positioned on ray $\overrightarrow{DC}$ so that $\angle CGF$ is a right angle. The exact ratio $\frac{FG}{DG}$ can be written as $\frac{k + w\sqrt{p}}{q}$ in reduced and simplified radical form with integers k, w, p, and q, and $q > 0$. Determine the sum $(k + w + p + q)$.

20. Three congruent circles with radius 4 are pairwise tangent. Determine the area enclosed by the three circles (the area shaded in the diagram). Express your answer as a decimal correct to four significant digits.



2015 RA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 21

2. $\frac{40}{49}$ (Must be this reduced common fraction.)

3. $15:13:12$ (Must be this extended ratio.)

4. 75 ("square" units optional.)

5. 17 ("miles" optional.)

6. $\frac{169}{12}$ (Must be this reduced improper fraction.)

7. $8\sqrt{1529}$ (Must be this exact answer.)

8. 230

9. 4.991 (Must be this exact decimal.)

10. $(6,8)$ (Must be this ordered pair only.)

11. 60

12. $4\sqrt{130}$ (Must be this exact answer.)

13. $1200-100\pi$ OR OR $100(12-\pi)$ OR $-100\pi+1200$ (Must be this or *exact* equivalent answer.)

14. 32π (Must be this exact answer.)

15. 3π (Must be this exact answer.)

16. $90\sqrt{3}$ (Must be this exact answer.)

17. $\frac{1}{2}$ OR 0.5 OR $.5$

18. 110 ("degrees" optional.)

19. 7

20. 2.580 (Must be this exact decimal, trailing zero necessary.)

1. $\begin{bmatrix} 2 & 1 & 5 \\ 6 & -2 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 9 & -3 \\ k & w & p \end{bmatrix} = \begin{bmatrix} 8 & 10 & 2 \\ 17 & 4 & -7 \end{bmatrix}$. Determine the sum $(k+w+p)$.

2. Determine the maximum value of the function $f(x) = -2x^2 - 16x - 25$.

3. Determine the value of k so that $3x^2 - 6x + k = 12$ has exactly one double root.

4. Determine the number of distinct positive integral factors for the number 635,040.

5. $(f \circ g)(x) = f(g(x))$. $f(x) = 4x + 12$ and $g(x) = 6x - 2$. Determine $(f \circ g)(4)$.

6. Determine all real value(s) for x such that $\sqrt[3]{2x+7} = 3$.

7. The average of two numbers is 36 and their geometric mean is 15. Determine the exact absolute value of the difference of these two numbers.

8. Two of the zeros for $f(x) = x^4 - 7x^3 - 53x^2 + 265x - 350$ are integers and the other two are complex numbers of the form $k \pm wi$ where $i = \sqrt{-1}$ and with $w \geq 0$. Determine the ordered pair (k, w).

9. Determine the exact distance from the center of the circle given by $x^2+8x+y^2-6y+3=2$ to the vertex of the parabola given by $y=x^2+4x+3$ when graphed in the same plane.

10. Given $g(x)=6^{0.6x}$. Determine $g^{-1}(36)$. Express your answer as a common or improper fraction reduced to lowest terms.

11. Determine the ordered triple (x,y,z) that is the solution to the system of equations
$$\begin{cases} x + y - z = -1 \\ 4x - 3y + 2z = 16 \\ 2x - 2y - 3z = 5 \end{cases}$$
Express your answer as an ordered triple (x,y,z).

12. $f(x)$ is a linear function and $g(x)$ is a quadratic function. $f(g(x))=x^2+1$ and $g(f(x))=x^2+4x+3$. Determine the exact value of $f(1)$.

13. $\dfrac{2^{-1}+2^{-3}}{2^{-2}-2^{-4}}=\dfrac{k}{w}$ where k and w are relatively prime positive integers. Determine the sum $(k+w)$.

14. Given $x^2+\dfrac{1}{x^2}=3$. The largest solution for x in this equation can be written in the reduced and simplified radical form $\dfrac{k+w\sqrt{p}}{q}$ where k, w, p, and q are integers. Determine the sum $(k+w+p+q)$.

15. Determine all ordered pair(s) (x, y) that are solutions to the system of equations $\begin{cases} x^2 + y^2 = 10 \\ y = -3x + 10 \end{cases}$. Express your answer as ordered pair(s) (x, y).

16. When $f(x) = x^4 + 3x^2 - 340$ is divided by $(x-4)$ the quotient is $q(x)$ and the remainder is k. Determine the exact value of k.

17. A rectangular pool is to be installed in a rectangular plot with dimensions 50 feet by 70 feet. The sides of the pool are parallel to the sides of the plot of land. By code, the pool can take no more than 65% of the surface area of the plot. A walk with equal widths from the edge of the pool will completely fill out the rectangular plot. The owner wants the maximum size pool. Determine the length in feet of the longer side of the rectangular pool. Express your answer as a decimal rounded to four significant digits.

18. $(3x+2)$ is a factor of $(5kx^4 + 13x^3 + 9x^2 - 4x - 4)$. Determine the exact value of k. Express your answer as a common or improper fraction reduced to lowest terms.

19. An infinite geometric series has a first term of 2 and a sum between 5 and 10, inclusive. The possible values for the common ratio r are represented by the inequality $k \le r \le w$. Determine the sum $(k+w)$. Express your answer as a common or improper fraction reduced to lowest terms.

20. The sum of three integers is 172. The product of these three integers is 13824. The three integers are in a geometric progression. Determine the largest of these three integers.

2015 RA

Algebra II

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 7

2. 7

3. 15

4. 180 ("factors" optional.)

5. 100

6. 10 (Must be this value only.)

7. $6\sqrt{119}$ (Must be this exact answer.)

8. $(2,1)$ (Must be this ordered pair.)

9. $2\sqrt{5}$ (Must be this exact answer.)

10. $\frac{10}{3}$ (Must be this reduced improper fraction.)

11. $(2,-2,1)$ (Must be this ordered triple.)

12. 3

13. 13

14. 9

15. $(3,1)$ (Must have this ordered pair only.)

16. -36

17. 58.73 (Must be this exact decimal, "feet" optional.)

18. $\frac{6}{5}$ (Must be this reduced improper fraction.)

19. $\frac{7}{5}$ (Must be this reduced improper fraction.)

20. 144

1. Determine the *least* value of x *NOT* in the domain of the function $f\left(x\right)=\dfrac{4x+9}{4x^3-9x}$.

2. Given $0\le\theta\le\dfrac{\pi}{2}$. If $\cos\theta=\dfrac{5}{13}$, determine the value of $\sin\left(2\theta\right)$. Express your answer as a common or improper fraction reduced to lowest terms.

3. $f\circ g\left(x\right)=f\left(g\left(x\right)\right)$. If $f\left(x\right)=5x+14$ and $g\left(x\right)=4x-1$, then $f\circ g\left(x\right)=kx+w$. Determine the sum $\left(k+w\right)$.

4. Determine the sum of all real number values of x that are solutions to the equation $6x^3=11x^2+10x$. Express your answer as a common or improper fraction reduced to lowest terms.

5. An airplane travels at a constant rate of 400 mph. The plane departs from Airport A and travels for 1 hour at a bearing $N15°E$ ($15°$ east of due north.) The plane then travels for 3 hours traveling due east and lands at Airport B. The bearing for the direct flight from Airport A to Airport B would have been $Nk°E$. Determine the value of k. Express your answer as a decimal rounded to the nearest tenth of a degree.

6. Determine the sum of all integral multiples of 7 between 7 and 700 inclusive.

7. A parabola has focus $\left(-1,0\right)$ and directrix $y=-4$. An equation for this parabola can be written in standard form $\left(x+k\right)^2=w\left(y+h\right)$. Determine the product $\left(kwh\right)$.

8. Given $3^{(k-2)}+3+3^{(k-2)}=3^{(k+2w)}+3-3^{(k-2)}$, determine the exact value of w. Express your answer as a common or improper fraction reduced to lowest terms.

9. $\sin 2x=\frac{7}{9}$ with $0\le x<\frac{\pi}{2}$. Determine the exact value of $(\sin x+\cos x)$. Express your answer as a common or improper fraction reduced to lowest terms.

10. Determine the numerical coefficient of the sixth term when $(k+2w)^8$ is expanded and written in order of decreasing powers of k.

11. The sides of a triangle have lengths in the ratio $3:5:7$. Determine the exact value of the sine of the smallest angle of the triangle.

12. Atlas has a belt that exactly fits around the circumference of the Earth at the equator. He increases the length of the belt by 3 feet and uses sufficient congruent pegs along the equator so the circular belt again exactly fits around the Earth above the equator. Determine the number of *inches* in the length of one of these pegs. Express your answer as a decimal number of inches, rounded to four significant digits.

13. The greatest negative angle in radian measure that is a solution to $\cos\left(\frac{5\pi}{12}+\theta\right)=-\frac{1}{2}$ is $\theta=k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

14. The first three terms, in order, of an infinite geometric progression are $7\sqrt{2}$, $\frac{21}{4}$, and $\frac{63\sqrt{2}}{32}$. The sum of this progression, written as a fraction reduced to lowest terms, is $S=\frac{k+w\sqrt{2}}{f}$ where k, w, and f are positive integers. Determine the least possible value of the sum $(k+w+f)$.

15. $f(x)$ is represented in parametric form by $\begin{cases} x = \ln(2t) \\ y = 2t^2 \end{cases}$. Determine the value of $f^{-1}(3)$.
Express your answer as a decimal rounded to the nearest ten-thousandth.

16. The letters of the word QUADRILATERAL are to be arranged so that no two consonants are adjacent. Determine the number of distinguishable arrangements possible with this condition satisfied.

17. Determine the exact value of k when $k = \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{\cdots}}}}}$.

18. On the day of the birth of their first grandchild, Ma and Pa Kettle invest \$5000 in a college fund that produces \$10000 when the interest is compounded continuously for 18 years. The interest rate for this college fund is $k\%$. Determine the value of k. Express your answer as a decimal rounded to the nearest hundredth.

19. Determine the value of k such that $\log_8 64 + \log_2 k - 2\log_4 5 = 4$.

20. When $x^2 + kx + 9$ is divided by $(x-2)$, the quotient is $f(x)$ and the remainder is L. When $x^2 + kx + 9$ is divided by $(x-3)$, the quotient is $g(x)$ and the remainder is M. If $M = L + 1$, determine the value of k.

2015 RA

Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-\frac{3}{2}$ OR $\frac{-3}{2}$ OR $-1\frac{1}{2}$ OR -1.5

2. $\frac{120}{169}$ (Must be this reduced common fraction.)

3. 29

4. $\frac{11}{6}$ (Must be this reduced improper fraction.)

5. 73.5 (Degrees or ° symbol optional.)

6. 35350 OR 35,350

7. 16

8. $-\frac{1}{2}$ OR $\frac{-1}{2}$ OR $\frac{1}{-2}$ (Must be this reduced common fraction.)

9. $\frac{4}{3}$ (Must be this reduced improper fraction.)

10. 1792

11. $\frac{3\sqrt{3}}{14}$ OR $\frac{3}{14}\sqrt{3}$ (Must be this exact answer.)

12. 5.730 (Must be this exact decimal, trailing zero necessary, inches optional.)

13. $-\frac{13}{12}$ OR $\frac{-13}{12}$ OR $\frac{13}{-12}$ (Must be this reduced improper fraction)

14. 415

15. 0.8959 OR .8959 (Must be this exact decimal.)

16. 151200 OR 151,200 ("Arrangements" or "ways" optional.)

17. $-1+\sqrt{3}$ OR $\sqrt{3}-1$ (Must be this exact answer only.)

18. 3.85 (Must be this exact decimal, % optional.)

19. 20

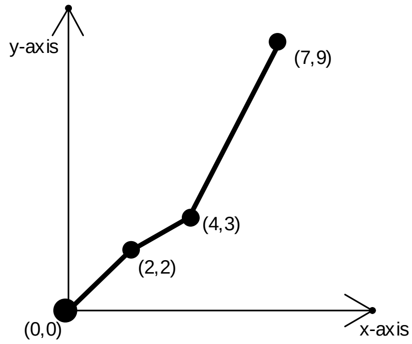
20. -4

NO CALCULATORS

1. Determine the slope of the line passing through the points $(-2,-1)$ and $(4,3)$. Express your answer as a common or improper fraction reduced to lowest terms.

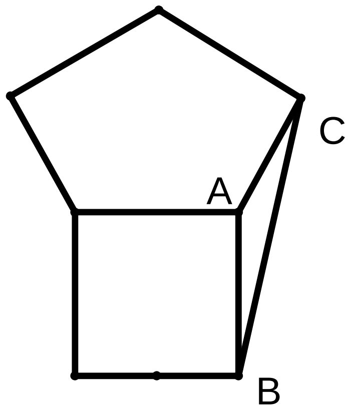
2. $k=\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}$. Determine the value of k when $a+b+c=15$ and $abc=5$.

3. A piecewise function of line segments $f(x)$ is graphed as shown. Determine the value of $f(1)+f(3)+f(6)$. Express your answer as a common or improper fraction reduced to lowest terms.



4. 46 students are enrolled in French, 41 are enrolled in Government, and 33 are enrolled in Statistics. 11 of these students are taking Government and Statistics, 9 are taking Government and French, and 10 are taking French and Statistics. Three of these students take all 3 classes. Determine the probability that one of these students selected at random will be enrolled in French only. Express your answer as a common fraction reduced to lowest terms.

5. On a plane, a regular pentagon and a square share a common side as shown. Determine the degree measure of $\angle ABC$.

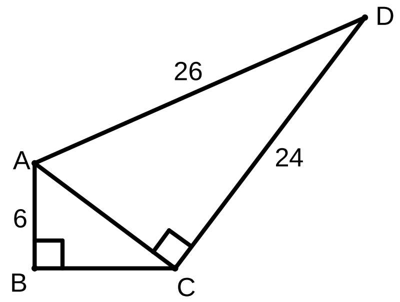


6. A circle is circumscribed about ΔABC with $AB=21$, $BC=72$, and $AC=75$. The circumference of this circle can be expressed as $k\pi$. Determine the value of $(2k-1)$.

NO CALCULATORS

7. The vertical distance between 2 floors in a department store is 35 feet. An escalator, covering a horizontal distance on the floor of 84 feet travels at a speed of $6.5\ ft/\sec$. Determine the number of seconds it takes a person, who stands still on one escalator step, to travel between floors.

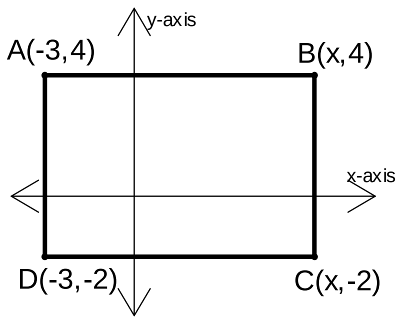
8. Right ΔABC and right ΔACD have a common side $\overline{AC}$ with lengths of three sides marked. Determine the numeric area of quadrilateral $ABCD$.



9. The sum of two numbers is $\frac{11}{2}$ and the product of the same two numbers is 6. Determine the sum of the reciprocals of these two numbers. Express your answer as a common or improper fraction reduced to lowest terms.

10. Let k be an odd integer less than one and let w be an even integer less than zero. Determine the largest possible value of $\left(5^k + 2^w\right)$. Express your answer as a common or improper fraction reduced to lowest terms.

11. Quadrilateral $ABCD$, with coordinates as shown, has perimeter that may be expressed as a function of x, that is as $f(x)$. When $f(x)$ is graphed in the coordinate plane, it has y-intercept k. Determine the value of k.



12. A convex pentagon has vertices with the coordinates $(1,6)$, $(5,3)$, $(3,-4)$, $(-7,-6)$, and $(-3,4)$. Determine the numeric area of this pentagon.

13. A triangle is formed by joining the midpoints of face diagonals of three adjacent faces of a cube with edge length 6. Determine the exact numeric area of this triangle.

NO CALCULATORS

14. A certain line has *x*-intercept $\frac{2}{3}$ and *y*-intercept $\frac{5}{2}$. The equation for this line may be written in standard form $Ax+By=C$ with $A>0$ and A, B, C relatively prime integers. Determine the value of C.

15. $a \oplus b = \frac{a^b}{b}$ and $a \otimes b = |a-b|$ for all valid replacements of real numbers a, b. $3 \otimes (a \oplus 2) = \frac{3}{2}$ has four exact solutions k, w, p, q with $k < w < p < q$. Determine the exact value of p.

16. Four times the reciprocal of the numeric circumference of a circle equals the numeric length of the diameter of the same circle. Determine the exact numeric area of this circle.

17. $k = \dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a^3}+\frac{1}{b^3}}$ for all valid replacements of a, b. Determine the exact value of k when $(a+b)=5$ and $(ab)=5$. Express your answer as an integer or common or improper fraction reduced to lowest terms.

18. The sides of a triangle have numeric lengths in the ratio $2:4:5$ and a perimeter of 88. Determine the sum of the longest and shortest side lengths.

19. Points $A(-4,-13)$, $B(12,17)$, and $C(19,10)$ lie on a circle with center (h,k) and radius r. Determine the sum $(h+k+r)$.

20. Let x and y be integers chosen from the set containing 39 integers: $\{-19,-18,-17,-16,\cdots,15,16,17,18,19\}$. Determine *__the number__* of distinct ordered pairs (x,y) that exist such that $\frac{4}{x}+\frac{1}{y}=\frac{1}{x+y}$.

2015 RA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 3

3. $\frac{21}{2}$ (Must be this reduced improper fraction.)

4. $\frac{10}{31}$ (Must be this reduced common fraction.)

5. 9 ("degrees" optional.)

6. 149

7. 14 ("seconds" optional.)

8. 144 ("square units" optional.)

9. $\frac{11}{12}$ (Must be this reduced common fraction.)

10. $\frac{9}{20}$ (Must be this reduced common fraction.)

11. 18

12. 85 ("square units optional.)

13. $\frac{9\sqrt{3}}{2}$ OR $\frac{9}{2}\sqrt{3}$ OR $4.5\sqrt{3}$

14. 10

15. $\sqrt{3}$ (Must be this exact answer only.)

16. 1 ("square units" optional.)

17. $\frac{5}{2}$ (Must be this reduced improper fraction.)

18. 56

19. 23

20. 18 ("ordered pairs" optional.)

NO CALCULATORS

1. Determine the value of the determinant $\begin{vmatrix} -2 & 3 \\ 4 & 1 \end{vmatrix}$.

2. Determine the exact solution for the system of equations $\begin{cases} x + y = 11 \\ 3x = y + 5 \end{cases}$. Express your answer as an ordered pair (x, y).

3. The sum of two numbers is 7. The sum of the squares of these two numbers is 12. Find the product of these two numbers. Express your answer as a common or improper fraction reduced to lowest terms.

4. When $\sin\theta \neq 0$ and $\tan\theta \neq 0$, $\dfrac{\tan^2\theta - \sin^2\theta}{\tan^2\theta \sin^2\theta}$ can be simplified to an expression involving a single trigonometric function or an exact real number. Determine this trigonometric expression or real number.

5. When $(x-y)^{10}$ is expanded, simplified, and written in order of decreasing powers of x, the numerical coefficient of the term containing y^7 is k. Determine the value of k

6. $i = \sqrt{-1}$. The polynomial $p(x) = x^4 + kx^3 + wx^2 + px + q$ has real-valued coefficients and zeros $(-2+i)$ and $(-1+2i)$. Determine the exact sum $(k + w + p + q)$.

7. A sequence is defined recursively as $A(1)=1$, $A(2)=2$, and $A(n) = \dfrac{A(n-1)}{A(n-2)}$ for all $n > 2$. Determine the exact value of $A(2015)$.

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

NO CALCULATORS

8. Will, Henrik, and Xavier need to make a 100 mile journey, but Henrik only has a 2-person car. They all start at the same point with Henrik driving Xavier at 25 mph for some distance while Will starts out walking at 5 miles per hour until Henrik returns to give him a ride. At some point, Xavier gets out of the car and continues walking to the destination at 5 mph while Henrik retraces his path to meet Will. After picking up Will, Henrik again drives towards the destination and arrives at the same time Xavier arrives. Determine the exact number of hours for this journey. (Assume constant rates and ignore the times to turn around and get into and out of the car.)

9. Determine the number of arrangements for 7 Knights to be seated around an unmarked round table.

10. Determine the coordinates for the focus of the graph of $9y^2 + 36x - 6y - 23 = 0$ in the coordinate plane. Express your answer as an ordered pair (x, y) with entries that are common or improper fractions reduced to lowest terms.

11. Determine the sum of all distinct k such that $x = k$ represents a vertical asymptote of $f(x) = \dfrac{x+3}{x^3 - 4x^2 - 11x + 30}$.

12. The faces of a fair cubical die are labeled with the numbers 3, 5, 7, 11, 13, and 17. If the die is rolled three times, determine the probability the sum of the three top face numbers rolled is less than 20. Express your answer as a common fraction reduced to lowest terms.

13. The distinct real solution(s) for x in the interval $0 \le x < 2\pi$ such that $1 - \cos x = \sqrt{3}\sin x$ may be written as $k\pi$. Determine all possible value(s) of k. Express each answer(s) as an integer or as a common or improper fraction reduced to lowest terms.

NO CALCULATORS

14. Determine the number of positive integers not exceeding 2015 that are multiples of 3 or 4, but not 5.

15. Determine all solutions for x for the equation $-4x^2 + 23x - 15 = 0$. Express your answer(s) as an integer or as a common or improper fraction reduced to lowest terms.

16. $k = \log_2\left(\log_9\left(\log_2\left(\log_2 256\right)\right)\right)$. Determine the exact value of k.

17. $k = \sum_{n=1}^{2015} \frac{1}{n(n+1)}$. Determine the exact value of k. Express your answer as a common or improper fraction reduced to lowest terms.

18. The ellipse $4x^2 + y^2 + 24x - 4y + 36 = 0$ has a major axis of length k and a minor axis of length w. Determine the sum $(k+w)$.

19. Determine the sum of all real-valued k such that k is a zero for $f(x) = x^5 - 4x^3 + x^2 - 4$.

20. On each try, the probability that Ella will win a particular game is $\frac{1}{4}$. On each try, the probability Evie will win this game is $\frac{2}{3}$. Ella and Evie take turns. The game is over when one of them wins. If Ella goes first, determine the probability Ella wins the game. Express your answer as a common fraction reduced to lowest terms.

2015 RA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -14

2. $(4,7)$ (Must be this exact ordered pair only.)

3. $\frac{37}{2}$ (Must be this reduced improper fraction.)

4. 1

5. -120

6. 79

7. $\frac{1}{2}$ OR 0.5 OR $.5$ (Must be this exact answer.)

8. 8 ("Hours" optional.)

9. 720 ("Arrangements optional.)

10. $\left(-\frac{1}{3},\frac{1}{3}\right)$ OR $\left(\frac{-1}{3},\frac{1}{3}\right)$ (Must be this exact ordered pair with reduced common fractions.)

11. 7

12. $\frac{19}{108}$ (Must be this reduced common fraction.)

13. $0, \frac{2}{3}$ (Must have both answers in either order and using a reduced common fraction.)

14. 806 ("Integers" or "positive integers" optional.

15. $\frac{3}{4}, 5$ (Must have both answers in either order and using a reduced common fraction.)

16. -1

17. $\frac{2015}{2016}$ (Must be this reduced common fraction.)

18. 6

19. -1

20. $\frac{1}{3}$ (Must be this reduced common fraction.)

Round answers to four significant digits and write <u>in standard notation</u> unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required. (NOTE: DO NOT USE SCIENTIFIC NOTATION UNLESS SPECIFIED IN THE QUESTION)

1. $k = \log_5\left(\frac{\sqrt[3]{874}}{16}\right)$. Determine the value of k.

2. Determine all possible value(s) for x such that $12^{x^2-2x-4} = 3$.

3. Consider the values $A = \sin 1$, $B = \sin 2$, $C = \sin 3$, $D = \sin 4$, and $E = \sin 5$. Arrange these values in ascending order. Express your answer as an ordered quintuple of ***<u>capital letters</u>*** representing the ascending order.

4. Jeffrey is choosing a 7-character license plate for his car. He wants 3 consonants and 3 vowels in some order followed by a single digit, where no letter is to be repeated. Determine the number of possible license plates arrangements from which Jeffrey may choose.

5. Determine the value of $(J + 3U + 45L)$ when

$$\begin{cases} J + U + L = 1500 \\ J + U + L + I + A = 2500 \\ J - U = 0 \\ A + I - U = 500 \\ I + A = 1000 \\ I + J - A = 500 \end{cases}.$$

6. Determine the 53rd term in the sequence $1, 2, 4, 8, \cdots$. Express your answer in scientific notation.

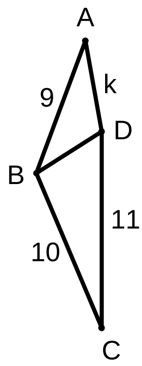
7. The sum of the first k positive integers is greater than 2015. Determine the least possible value for k.

8. The equation for the axis of symmetry for the graph of $y = 0.7437x^2 + \pi x - e$ is either $x = k$ or $y = w$. Determine the appropriate value for either k or w. Express your answer using "$k =$ __" or "$w =$ __", whichever is correct. (NOTE: e is the base of the natural logarithm.)

9. Suzy Softball has a batting average of 0.400. Using this probability for each at-bat, determine the probability she will get a hit in at least two of her next 5 at-bats.

10. The equation of a line perpendicular to $y = \frac{-7.86}{2.4}x + 7.12$ that passes through the point $(3.76, -2.81)$ may be expresses as $y = kx + w$. Determine the ordered pair (k, w) with both k and w rounded to the nearest thousandth.

11. In the diagram marked as shown (but not necessarily drawn to scale), $\angle ABD = 37°$ and $\angle BCD = 23°$. Determine the length k.



12. Define $a \# b = a^b - b^a$ and $a \otimes b = \sqrt{b^2 - a^2}$. Determine the value $(1.2 \otimes 3.2) \# 0.65$.

13. The lengths of the three sides of a right triangle are consecutive terms of an arithmetic sequence. Determine the degree measure of the smaller acute angle of this triangle.

14. A five card hand is dealt from a standard deck of cards (4 suits each with 13 ranks). Determine the probability that the hand contains a pair of aces and no other pairs or other ace.

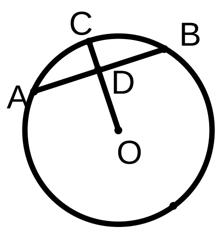
15. An eleven sided regular polygon is inscribed within a circle with radius of 11. Determine the area of the region inside the circle but outside of the eleven sided polygon.

16. Determine the length of the segment joining the points $P(12.12, 8.51)$ and $Q(16.25, 42.01)$.

17. On any given day, the probability your math teacher is in a good mood is $\frac{1}{2}$. The probability she is in a bad mood is $\frac{1}{3}$. The rest of the time she is in a neutral mood. Determine the probability that in a 5 day week, she is in a good mood 3 days and a bad mood one day. Express your answer as a common fraction reduced to lowest terms.

18. A certain sequence is $7\frac{1}{4}, 8\frac{5}{8}, 10, \cdots$, where the terms follow the given pattern and term $n_1 = 7\frac{1}{4}$, $n_2 = 8\frac{5}{8}$ and so forth. Term $n_k = 36\frac{1}{8}$. Determine the value of k.

19. In the diagram (not necessarily drawn to scale), Circle O has radius of length k. Chord $\overline{AB}$ is perpendicular to radius $\overline{OC}$ and intersects at point D. The length of minor arc $\overset{\frown}{AC} = 1.96$ and the length of $\overline{CD} = 0.125$. Determine the value of k, the length of the radius of Circle O.



20. Measured in centimeters, the length L of a tiger shark t years old is given by the formula $L = 337 - 276e^{-0.178t}$. Determine the age in years of a tiger shark measuring 190 centimeters long.

2015 RA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: **All answers must be written legibly. Round answers to four significant digits and write in standard notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required. (NOTE: DO NOT USE SCIENTIFIC NOTATION UNLESS SPECIFIED IN THE QUESTION)**

1. -0.3199 OR $-.3199$

2. $3.333, -1.333$ (Must have both answers in either order.)

3. (E, D, C, A, B) (Must be this quintuple of capital letters.)

4. 95,760,000 ("Combinations" and commas optional.)

5. 24,500 (Comma optional.)

6. 4.504×10^{15} (Must be this answer in scientific notation.)

7. 63

8. $k = -2.112$ (Must be this decimal, and include the k= format.)

9. 0.6630 OR .6630 (Trailing zero necessary.)

10. $(0.305, -3.958)$ OR $(.305, -3.958)$

11. 6.138

12. 1.749

13. 36.87 (Degrees optional.)

14. 0.03251 OR .03251

15. 20.34

16. 33.75

17. $\frac{5}{36}$ (Must be this reduced common fraction.)

18. 22

19. 15.35

20. 3.539 ("years old" or "years" optional.)

1. The expression $\left(-2c^2d\right)\left(2c^3d\right)^2$ can be simplified to the expression $-2^k c^w d^p$. Determine the sum $(k+w+p)$.

2. Determine the center (h,k) and the radius (r) of the circle that has a diameter with endpoints $A(4,-1)$ and $B(2,5)$. Report as your answer the sum $\left(h+k+r^2\right)$.

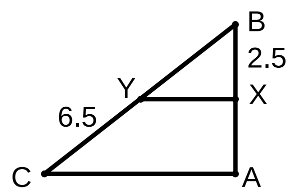
3. Jeffrey participates in a psychology experiment where he participates in Game A and then Game B one time each. The probability of winning Game A has been set to 0.7. The probability of *not* winning Game B is set at 0.4. The probability he will *not* win both of the games is set to 0.6 and there are no ties. Determine the probability Jeffrey will win at least one of the two games. Express your answer as an exact decimal.

4. The circle with center $C(-2,4)$ is tangent to the line $x=3$ and has area $k\pi$. Let w be the probability of rolling a sum of 6 on one roll of two fair, standard cubical dice. Determine the value of $\frac{k}{2w}$.

5. Let R_1 be the $90°$ clockwise rotation about center $A(4,0)$. Let R_2 be the $90°$ counterclockwise rotation about center $B(-8,0)$. Let $P(0,0)$, $R_2R_1(P)=H$ and $R_1R_2(P)=Q$. Determine the exact distance HQ. (NOTE: $R_2R_1(P)$ is the composition of doing R_1 to P and then doing R_2 to the result. $R_1R_2(P)$ reverses the order of the rotations.)

6. Let $k=(3x-3)(3x-6)(3x-9)\cdots(3x-60)$ when $x=15$. Let w be the sum of the zeros of $f(x)=x^3-x^2-6x$. Determine the sum $(k+w)$.

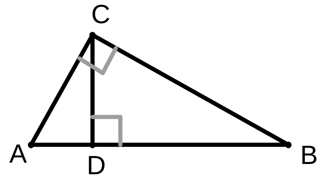
7. In right ΔABC with right angle at A, X and Y are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. Let k be the length of $\overline{XY}$ when $BX=2.5$ and $CY=6.5$. A square is inscribed in a circle. The area of the square is 25. Let w be the numerical length of the diameter of this circle. Report as your answer the sum $(k+w)$ as a decimal rounded to four significant digits.



8. The sides of a triangle have lengths 30 and 60. The length of the third side is also an integer. Determine the sum of all possible lengths of the third side of this triangle.

9. Let k be the number of ounces in one pound, w be the number of seconds in one hour, p be the number of days in one non-leap year, and q be the number of hours in one week. Determine the remainder when $(k + w - p)$ is divided by q.

10. In right ΔABC, $\overline{AC} \perp \overline{CB}$ and $\overline{CD} \perp \overline{AB}$. $CD = \sqrt{96}$ and $AD = 8$. Determine the sum $(AC + DB)$. Express your answer as a decimal rounded to four significant digits.



11. Will is placing a cement border of uniform width around his rectangular garden. The garden is 10 feet long and 6 feet wide. Will has enough material to cover 36 square feet of border. Determine the width of Will's border in feet.

12. Determine all prime numbers less than 100 that are both the sum and the difference of two primes. For your answer, list all possible such primes.

1. The expression $\left(-2c^2d\right)\left(2c^3d\right)^2$ can be simplified to the expression $-2^k c^w d^p$. Determine the sum $\left(k+w+p\right)$.

2. Determine the center (h,k) and the radius (r) of the circle that has a diameter with endpoints $A(4,-1)$ and $B(2,5)$. Report as your answer the sum $\left(h+k+r^2\right)$.

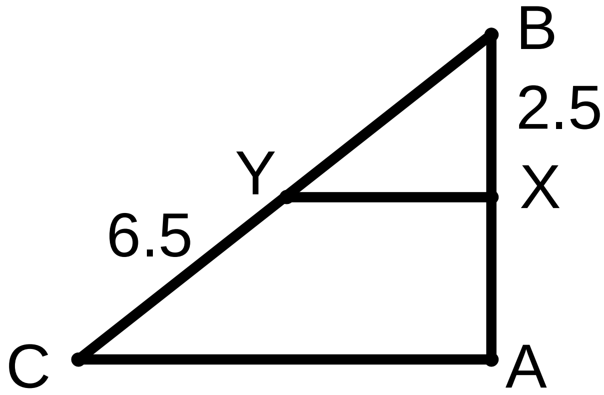
3. Jeffrey participates in a psychology experiment where he participates in Game A and then Game B one time each. The probability of winning Game A has been set to 0.7. The probability of *not* winning Game B is set at 0.4. The probability he will *not* win both of the games is set to 0.6 and there are no ties. Determine the probability Jeffrey will win at least one of the two games. Express your answer as an exact decimal.

4. The circle with center $C(-2,4)$ is tangent to the line $x=3$ and has area $k\pi$. Let w be the probability of rolling a sum of 6 on one roll of two fair, standard cubical dice. Determine the value of $\frac{k}{2w}$.

5. Let R_1 be the $90°$ clockwise rotation about center $A(4,0)$. Let R_2 be the $90°$ counterclockwise rotation about center $B(-8,0)$. Let $P(0,0)$, $R_2R_1(P)=H$ and $R_1R_2(P)=Q$. Determine the exact distance HQ. (NOTE: $R_2R_1(P)$ is the composition of doing R_1 to P and then doing R_2 to the result. $R_1R_2(P)$ reverses the order of the rotations.)

6. Let k be the value of the extended product $(3x-3)(3x-6)(3x-9)\cdots(3x-60)$ when $x=15$.
Let w be the sum of the zeros of $f(x)=x^3-x^2-6x$.
Determine the sum $(k+w)$.

7. In right ΔABC with right angle at A, X and Y are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively.

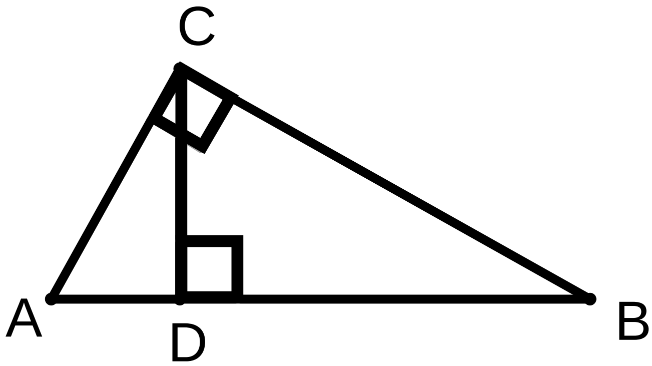


Let k be the length of $\overline{XY}$ when $BX = 2.5$ and $CY = 6.5$.

A square is inscribed in a circle. The area of the square is 25. Let w be the numerical length of the diameter of this circle. Report as your answer the sum $(k + w)$ as a decimal rounded to four significant digits.

8. The sides of a triangle have lengths 30 and 60. The length of the third side is also an integer. Determine the sum of all possible lengths of the third side of this triangle.

9. Let k be the number of ounces in one pound, w be the number of seconds in one hour, p be the number of days in one non-leap year, and q be the number of hours in one week. Determine the remainder when $(k+w-p)$ is divided by q.



10. In right ΔABC, $\overline{AC} \perp \overline{CB}$ and $\overline{CD} \perp \overline{AB}$. $CD = \sqrt{96}$ and $AD = 8$. Determine the sum $(AC + DB)$. Express your answer as a decimal rounded to four significant digits.

11. Will is placing a cement border of uniform width around his rectangular garden. The garden is 10 feet long and 6 feet wide. Will has enough material to cover 36 square feet of border. Determine the width of Will's border in feet.

12. Determine all prime numbers less than 100 that are both the sum and the difference of two primes. For your answer, list all possible such primes.

2015 RA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. 14	______
2. 15	______
3. 0.9 OR .9 (Must be this decimal.)	______
4. 90	______
5. 24	______
6. 1	______
7. 13.07 (Must be this decimal.)	______
8. 3540	______
9. 59	______
10. 24.65 (Must be this decimal.)	______

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. 1 ("foot" optional.)
12. 5 (Must be this prime only.)
13. ______
14. ______
15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Determine the sum of all real numbers x such that $6x^3 = 11x^2 + 10x$.

2. For all valid replacements of x, y, z, and b, $k = \sin(-x)\cos(-x)\sec(x)\csc(x)$ and $w = (\log_{7b} 5y)(\log_z 7b)(\log_{5y} 6)(\log_6 z)$. Determine the sum $(k+w)$.

3. Let k be the number of unique positive integral divisors of 120. k is also the 6th term of an arithmetic sequence whose first term is 1. Find the sum of the first 20 terms of this sequence.

4. Determine the exact sum of the amplitude, period, phase shift (horizontal shift) and vertical shift for the graph of $y = -3\sin(2x+\pi) - 3$.

5. Let p be the perimeter of a triangle with interior angles of $30°$ and $45°$ and shortest side of length $\sqrt{162}$. Let $k = \sqrt{35 - 8\sqrt{6}}$. The sum $(p+k) = A + B\sqrt{2} + C\sqrt{3}$ in simplified and reduced radical form. Determine the exact sum $(A+B+C)$.

6. $f(x) = \sin^2(17x) + \cos^2(17x)$ and $g(x) = \begin{vmatrix} 5 & x \\ -2 & 5 \end{vmatrix}$. $k = f(g(2))$ and $w = g(f(2))$. Determine the sum $(k+w)$.

7. $k = \log_2 \frac{(4)(16)}{(8^2)}$. $w = \sum_{n=8}^{60} \ln n$. Determine the sum $(k+w)$. Express your answer as a decimal correct to four significant digits.

8. k is the volume of the cone formed when a triangle with sides of lengths 3-4-5 is rotated about the side with length 4. w is the volume of the cone formed when the same triangle is rotated about the side with length 3. Determine the exact value of $|k-w|$.

9. The graph of $y = Ax^3 + Bx^2 + Cx + D$ contains the four points $(-4,0)$, $(5,0)$, $(0,7)$, and $(7,0)$. Determine the exact sum $(A+B+C+D)$.

10. Let $k = \frac{1}{\sqrt{11}+\sqrt{9}} + \frac{1}{\sqrt{12}+\sqrt{10}} + \frac{1}{\sqrt{13}+\sqrt{11}} + \cdots + \frac{1}{\sqrt{81}+\sqrt{79}}$. The numeric sum of the areas of the six faces of a cube is 2015. The lengths of each edge of the cube is divided by 3 to form a new, smaller cube. Let w be the numeric length of the diagonal of this solid. Determine the sum $(k+w)$. Express your answer as a decimal rounded to four significant digits.

11. Let $k = \begin{vmatrix} 30 & 50 & 10 \\ 24 & 12 & 12 \\ 16 & 24 & 8 \end{vmatrix}$. Let w be the distance from the point $P(1,3)$ to the line $3x - 4y = 21$. Determine the quotient $\left(\frac{k}{w}\right)$.

12. Let k be the number of distinct ways all 12 students can be arranged into 3 teams of four persons and each person on only one team.
$w = \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \cdots + \frac{1}{8\times9} + \frac{1}{9\times10} + \frac{1}{10\times11}$. Determine the exact product (kw).

1. Determine the sum of all real numbers x such that $6x^3 = 11x^2 + 10x$.

2. For all valid replacements of x, y, z, and b,

$$k = \sin(-x)\cos(-x)\sec(x)\csc(x)$$

and w is the product

$$(\log_{7b} 5y)(\log_z 7b)(\log_{5y} 6)(\log_6 z)$$

Determine the sum $(k+w)$.

3. Let k be the number of unique positive integral divisors of 120. k is also the 6th term of an arithmetic sequence whose first term is 1.

Find the sum of the first 20 terms of this sequence.

4. Determine the exact sum of the amplitude, period, phase shift (horizontal shift) and vertical shift for the graph of $y = -3\sin\left(2x+\pi\right)-3$.

5. Let p be the perimeter of a triangle with interior angles of 30° and 45° and shortest side of length $\sqrt{162}$. Let $k = \sqrt{35 - 8\sqrt{6}}$. The sum $\left(p + k\right) = A + B\sqrt{2} + C\sqrt{3}$ in simplified and reduced radical form. Determine the exact sum $\left(A + B + C\right)$.

6. $f(x) = \sin^2(17x) + \cos^2(17x)$

and $g(x) = \begin{vmatrix} 5 & x \\ -2 & 5 \end{vmatrix}$.

$k = f(g(2))$ and $w = g(f(2))$.

Determine the sum $(k + w)$.

7. $k = \log_2 \frac{(4)(16)}{(8^2)}$.

$$w = \sum_{n=8}^{60} \ln n.$$

Determine the sum $(k + w)$. Express your answer as a decimal correct to four significant digits.

8. k is the volume of the cone formed when a triangle with sides of lengths 3 – 4 – 5 is rotated about the side with length 4. w is the volume of the cone formed when the same triangle is rotated about the side with length 3. Determine the exact value of $|k-w|$.

9. The graph of $y = Ax^3 + Bx^2 + Cx + D$ contains the four points $(-4,0)$, $(5,0)$, $(0,7)$, and $(7,0)$. Determine the exact sum $(A+B+C+D)$.

10. Let

$$k = \frac{1}{\sqrt{11}+\sqrt{9}} + \frac{1}{\sqrt{12}+\sqrt{10}} + \frac{1}{\sqrt{13}+\sqrt{11}} + \cdots + \frac{1}{\sqrt{81}+\sqrt{79}}$$

The numeric sum of the areas of the six faces of a cube is 2015. The lengths of each edge of the cube is divided by 3 to form a new, smaller cube. Let w be the numeric length of the diagonal of this solid. Determine the sum $(k+w)$.

Express your answer as a decimal rounded to four significant digits.

11. Let $k = \begin{vmatrix} 30 & 50 & 10 \\ 24 & 12 & 12 \\ 16 & 24 & 8 \end{vmatrix}$.

Let w be the distance from the point $P(1,3)$ to the line $3x - 4y = 21$. Determine the quotient $\left(\dfrac{k}{w}\right)$.

12. Let k be the number of distinct ways all 12 students can be arranged into 3 teams of four persons and each person is on only one team.

$$w = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots + \frac{1}{8\times 9} + \frac{1}{9\times 10} + \frac{1}{10\times 11}$$

Determine the exact product (kw).

2015 RA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. $\frac{11}{6}$ (Must be this reduced improper fraction.)	
2. 0	
3. 590	
4. $\frac{\pi}{2}$ OR $\frac{1}{2}\pi$ OR 0.5π OR $.5\pi$ (Must be one of these exact answers.)	
5. 48	
6. 28	
7. 180.1 (Must be this decimal.)	
8. 4π (Must be this exact answer.)	
9. 6	
10. 16.47 (Must be this decimal.)	

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. –320
12. 5250 OR 5,250
13.
14.
15.

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

ORAL COMPETITION
ICTM REGIONAL 2015 DIVISION A

1. Comment on the truth of each of the following statements:
   I. $x=3$ is a function in terms of x
   II. $y=3$ is a function in terms of x
   III. $x=|y|$ is a function in terms of x
   Explain how you determined each answer.

2. Suppose the function f is defined as $f(x)=\begin{cases} 3x & \text{for } x\le 1 \\ (x-1)^2 & \text{for } x>1 \end{cases}$
   If a is a constant such that $a<0$, find a simplified expression for $f(1-a)$ in terms of a.
   Explain how you arrived at your answer.

3. Given $f(x)=-2x+1$ and $g(x)=|x-2|$.
   a. Find the domain and range of $f(x)$
   b. If the function $h(x)$ is created by shifting $g(x)$ to the right two units and down three units, express the function $h(x)$ in terms of x.
   c. Find the range of $f(g(x))$
   d. If $k\ge 4$, evaluate $g(f(k))$, expressing your answer without using absolute value

   Explain how you arrived at each answer.

4. 2 and -5 are the zeros of the quadratic function $f(x)$. If $f(3)=16$, write the function $f(x)$.
   Explain how you determined your answer.

ORAL COMPETITION
ICTM REGIONAL 2015 DIVISION A

EXTEMPORANEOUS QUESTIONS

Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. For what value of x is $f(x) = 2$ if $f(x) = 2x - 1$? Explain how you arrived at your answer.

2. If $f(x) = 9$, find the value of $f(f(x-1))$. Explain how you arrived at your answer.

3. If $f(x) = x^2 - 4$, $g(x) = x + 3$ and $h(x) = |1 - x|$. Find the value of $g(h(f(-3)))$. Explain how you arrived at your answer.

4. Find the range of the function $f(x) = x^2 - 6x - 12$. Explain how you arrived at your answer.

JUDGES' SOLUTIONS

1. Comment on the truth of each of the following statements:
   I. $x = 3$ is a function in terms of x
   II. $y = 3$ is a function in terms of x
   III. $x = |y|$ is a function in terms of x

   Explain how you determined each answer.

   SOLUTION:
   I: FALSE; $x = 3$ **represents a vertical line, which IS NOT a function in terms of** ***x***
   II: TRUE; $y = 3$ **represents a horizontal line, which IS a function in terms of** ***x***
   III: FALSE; $x = |y|$ **IS NOT a function in terms of** ***x*** **since it contains points with the same** ***x*****-coordinate and different** ***y*****-coordinates (for example** $(2,2)$ **and** $(2,-2)$**).**

2. Suppose the function f is defined as $f(x) = \begin{cases} 3x & \text{for } x \le 1 \\ (x-1)^2 & \text{for } x > 1 \end{cases}$

   If a is a constant such that $a < 0$, find a simplified expression for $f(1-a)$ in terms of a. Explain how you arrived at your answer.

   SOLUTION:
   Since $a < 0$ **you know that** $1 - a > 1$**. Therefore** $f(1-a) = ((1-a)-1)^2$**. Simplifying gives** $\boxed{f(1-a) = a^2}$

JUDGES' SOLUTIONS

3. Given $f(x) = -2x+1$ and $g(x) = |x-2|$.

a. Find the domain and range of $f(x)$

b. If the function $h(x)$ is created by shifting $g(x)$ to the right two units and down three units, express the function $h(x)$ in terms of *x*.

c. Find the range of $f(g(x))$

d. If $k \geq 4$, evaluate $g(f(k))$, expressing your answer without using absolute value

Explain how you arrived at each answer.

SOLUTION:

a. The domain and range of any non-horizontal linear function is $\boxed{(-\infty, \infty)}$

b. Shifting to the right two units and down three units gives the function $h(x) = g(x-2)-3$**. Then substituting and simplifying,** $g(x-2)-3 = |(x-2)-2|-3$ **so** $\boxed{h(x) = |x-4|-3}$

c. $f(g(x)) = -2|x-2|+1$**. This is an absolute value function which opens downward with a maximum at the vertex of** $(2,1)$**. The range is then** $\boxed{(-\infty, 1] \;\; or \;\; x \leq 1}$**.**

d. $f(k) = -2k+1$**. Then** $g(f(k)) = |(-2k+1)-2|$**, which simplifies to** $|-2k-1|$**. Since** $k \geq 4$**,** $-2k-1$ **is negative and** $|-2k-1| = \boxed{2k+1}$

4. 2 and -5 are the zeros of the quadratic function $f(x)$. If $f(3) = 16$, write the function $f(x)$. Explain how you determined your answer.

SOLUTION: If 2 **and** -5 **are zeros,** $(x-2)$ **and** $(x+5)$ **are factors of** $f(x)$**. Then** $f(x) = a(x-2)(x+5)$**. If** $f(3) = 16$**,** $16 = a(3-2)(3+5)$**. Solving gives** $a = 2$**. The function** $f(x) = \boxed{2(x-2)(x+5) \;\; or \;\; 2x^2+6x-20}$

ORAL COMPETITION
ICTM REGIONAL 2015 DIVISION A

JUDGES' SOLUTIONS

EXTEMPORANEOUS QUESTIONS

Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. For what value of x is $f(x)=2$ if $f(x)=2x-1$? Explain how you arrived at your answer.

   SOLUTION: If $2=2x-1$, solving for x yields $\boxed{x=\frac{3}{2}}$

2. If $f(x)=9$, find the value of $f\left(f(x-1)\right)$. Explain how you arrived at your answer.

   SOLUTION: Since *f* is a constant function whose value is always 9, $\boxed{f\left(f(x-1)\right)=9}$

3. If $f(x)=x^2-4$, $g(x)=x+3$ and $h(x)=|1-x|$. Find the value of $g\left(h\left(f(-3)\right)\right)$. Explain how you arrived at your answer.

   SOLUTION: $f(-3)=(-3)^2-4=5$**; then** $h(5)=|1-5|=4$**, and finally** $g(4)=4+3=\boxed{7}$

4. Find the range of the function $f(x)=x^2-6x-12$. Explain how you arrived at your answer.

   SOLUTION: Rewriting in vertex form: $f(x)=(x-6x+9)-12-9\Rightarrow f(x)=(x-3)^2-21$**.**
   The function represents a parabola that opens up so the range is $\boxed{[-21,\infty)\quad or\quad y\geq -21}$
   (Note: any correct alternate method of determining the vertex should also be given full credit)