1. For all real values of x, the average of 4, $(3x+2)$ and k is $(x+6)$. Determine the value of k.

2. $f(x)=-6x+18$ and $g(x)=x^2-10x+22$. Determine <u>*the number*</u> of distinct real solutions for x for the equation $f(x)=g(x)$.

3. The sum of two numbers is 5 more than twice the smaller of the two numbers. The larger number minus the smaller number is 3 more than the smaller number. Determine the larger of the two numbers.

4. Set A contains the integers from 1 to 6 inclusive. Set B contains all unique possible sums of integers taken two at a time from set A. Set C contains all unique possible products of integers taken two at a time from set A. Determine the sum of the integers that are in set C but not in set B.

5. k is an integer and $1 \le k \le 25$. Determine <u>*the number*</u> of values of k for which the expression $2015+k$ is a multiple of 6.

6. In the three-digit number k with non-zero hundreds digit, the tens digit is the mean (average) of the units digit and the hundreds digit. Determine <u>*the number*</u> of distinct values for k.

7. Determine the value of 5^{3x-4} when $5^x=2$. Express your answer as a common or improper fraction reduced to lowest terms.

8. A vertical line is drawn through the point $(4,-3)$ and a horizontal line is drawn through the point $(6,-1)$. These two lines intersect at the point P. Determine the coordinates of point P. Express your answer as an ordered pair (x, y).

9. The number line distance between x and 9 is less than 8 units. Determine ***the number*** of possible values of x when x is an integer.

10. The points $(4,-2)$, $(k+8,3)$ and $(-2,k-11)$ are collinear. Determine the smallest possible value of k.

11. A rectangle has a numeric perimeter that is 20 units more than its length. If the length of the rectangle is 5 more than the width, find the numeric length of the rectangle.

12. A line ℓ has a $y-\text{intercept}$ of b. If the slope of line ℓ is tripled and the $x-\text{intercept}$ of line ℓ is doubled, the $y-\text{intercept}$ becomes (kb). Determine the value of k.

13. In an election, 75% of the registered voters voted. The winning candidate received 5208 votes, which was 80% of the votes cast. Determine the number of registered voters.

14. Determine ***the number*** of three digit positive integers (with non-zero hundreds digit) in which all of the digits are even and the sum of the digits is 16.

15. Given the points $(4,2)$, $(8,0)$, $(0,-4)$, $(0,4)$, $(4,-2)$, $(1,-3)$, $(10,1)$, $(6,-1)$. One of these points is selected at random. Determine the probability that this point lies on the line $x-2y=8$.

16. When $4^a\left(2^b\right)\left(8^c\right)=16^{x-c}$ is solved for x, the solution can be written as a rational expression reduced to lowest terms in the form $x=\dfrac{ka+mb+nc}{p}$, with integers k, m, n p with $p>0$. Determine the sum $(k+m+n+p)$.

17. A jar contains red, white and blue marbles. The ratio of red to white marbles is 5 to 7 and the ratio of red to blue marbles is 8 to 11. There are two more white marbles than blue marbles. Determine the number of red marbles in the jar.

18. A prime number less than 100 is randomly selected. Determine the probability that this number is a perfect square.

19. Set A contains all points that lie on the line $3x+2y=14$. Set B contains all points that lie on the line $4x+ky=30$. $\mathrm{A}\cap\mathrm{B}=\varnothing$ (the empty set). Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

20. In base b, $(2_b)(2015_b)=4034_b$. Determine the value of b.

2015 RAA

Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 12

2. 1 OR one ("solution" optional.)

3. 7

4. 121

5. 5 ("values" optional.)

6. 45 ("values" optional.)

7. $\frac{8}{625}$ (Must be this reduced common fraction.

8. $(4,-1)$ (Must be this ordered pair.)

9. 15 ("values" optional.)

10. -1

11. 10

12. 6

13. 8680 ("registered voters" and comma optional.)

14. 14 ("integers" optional.)

15. $\frac{5}{8}$ OR 0.625 OR .625

16. 14

17. 80 ("red marbles" optional.)

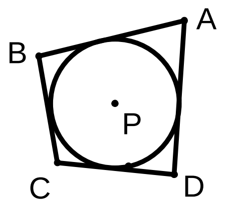
18. 0 OR zero

19. $\frac{8}{3}$ (Must be this reduced improper fraction.)

20. 6

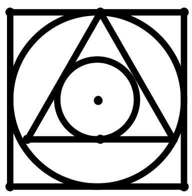
1. A square with side of length 4 is inscribed in a circle. The numeric area of this circle can be written as $k\pi$. Determine the exact value of k .

2. Circle P is tangent to each side of quadrilateral $ABCD$. $AB = 20$, $BC = 11$, and $DC = 14$. Determine the exact length of $\overline{AD}$.

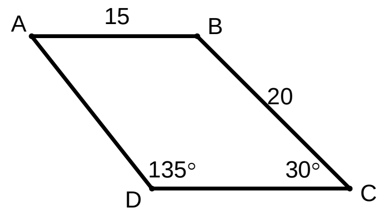


3. The radius of a circle is decreased by 25%. The resulting decrease in area is $k\%$. Determine the value of k . Express your answer as an exact decimal without the % symbol.

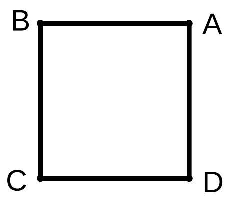
4. The radius of the inscribed circle of an equilateral triangle is 5. The circle that circumscribes that triangle is inscribed in a square. Determine the numeric area of this square.



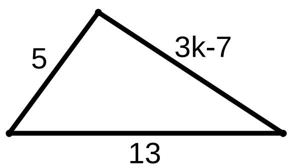
5. $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{CD}$. $AB = 15$, $BC = 20$, $\angle ADC = 135°$, and $\angle BCD = 30°$. The area of trapezoid $ABCD$ can be written in the form $k + w\sqrt{p}$ in simplified and reduced radical form with k, w, and p integers. Determine the sum $(k + w + p)$.



6. In the diagram, $\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$. $AD = 5k - 12$, $AB = 15$, and $BC = 2k + 9$. Determine the value of k .

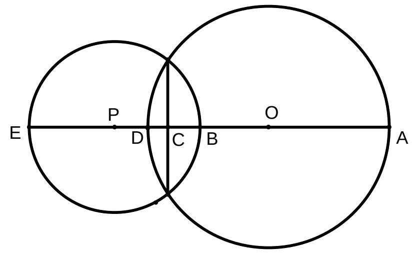


7. Determine *<u>the number</u>* of valid integer replacements for k in the triangle with side lengths as shown.



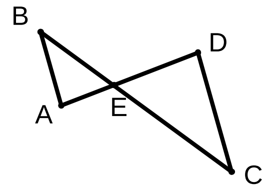
8. A regular polygon is inscribed in a circle with radius 6. The polygon has an exterior angle that measures $60°$. Determine the exact area interior to the circle but exterior to the polygon.

9. Circle O and Circle P intersect as shown. Points A, B, C, D, E, O, and P are collinear. $OB = 3$, $BC = 2$, and $CD = 1$. Determine the length of the radius of Circle P. Express your answer as a common or improper fraction reduced to lowest terms.



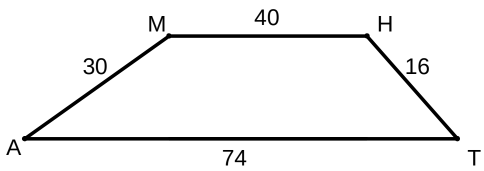
10. A rhombus has diagonals with numeric lengths 24 and 32. The area of the inscribed circle of this rhombus can be expressed in the form $\frac{k}{w}\pi$ where k and w are positive and relatively prime integers. Determine the sum $(k+w)$.

11. In the diagram shown, $\overline{AB} \parallel \overline{CD}$. $AE = 2$, $BE = 3$, $\angle B = 41°$, $CE = 6$, and $DE = k$. Determine the value of k.



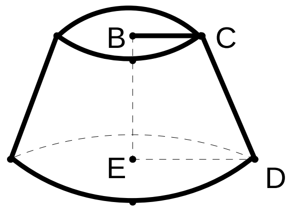
12. Determine the exact numeric area of a triangle with sides of numeric lengths 3, 5, and 6.

13. Trapezoid $MATH$ has $\overline{MH} \parallel \overline{AT}$, $MA = 30$, $AT = 74$, $TH = 16$, and $MH = 40$. Determine the exact numeric area of Trapezoid $MATH$. Express your answer as an improper fraction reduced to lowest terms.



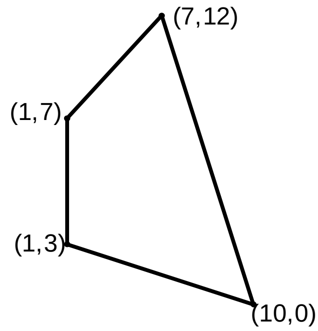
14. The equation of the line that is the perpendicular bisector of the segment joining $P(1,7)$ and $Q(-3,3)$ can be written in the form $y = kx + w$. Determine the sum $(k+w)$.

15. The right frustrum (bottom section of a right cone) has upper radius $BC = 2$, lower radius $ED = 5$ and height $BE = 9$. Determine the exact volume of this frustrum.



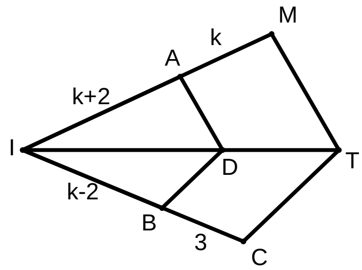
16. An ice cube manufacturer makes cubes of ice with a cylindrical hole through one set of two parallel faces. Each cube is 6 cm on a side and the radius of the base of the right circular cylinder through the cube is 2 cm. Determine the exact volume of ice in one cube in cm^3.

17. Determine the area of the quadrilateral with vertices as shown (but not necessarily drawn to scale.)



18. A circular cylinder is 4 inches in diameter and 6 inches in height. A fly crawls from a point on the top rim to the point on the bottom rim diametrically opposite the starting point. Determine the shortest path the fly could crawl in inches. Express your answer as a decimal correct to four significant digits.

19. Plane figure $ICTM$ has $\overline{AD} \parallel \overline{MT}$, $\overline{BD} \parallel \overline{CT}$, and $IT = 18$. $IA = k + 2$, $AM = k$, $IB = k - 2$, and $BC = 3$. Determine the exact length of $\overline{DT}$. Express your answer as a common or improper fraction reduced to lowest terms.



20. The centers of all six faces of a cube with side length 10 are connected to form a platonic solid. Determine the exact volume of this solid. Express your answer as a common or improper fraction reduced to lowest terms.

2015 RAA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 8

2. 23

3. 43.75 (Must be this exact decimal, no %.)

4. 400 ("square units" optional.)

5. 153

6. 7

7. 3 ("replacements" or "integers" optional.)

8. $36\pi - 54\sqrt{3}$ OR $18\left(2\pi - 3\sqrt{3}\right)$ OR $-54\sqrt{3} + 36\pi$ (Must be this or *exact* equivalent answer.)

9. $\frac{15}{4}$ (Must be this reduced improper fraction.)

10. 2329

11. 4

12. $2\sqrt{14}$ ("square units" optional.)

13. $\frac{13680}{17}$

14. 3

15. 117π (Must be this exact answer, "cubic units" optional.)

16. $216 - 24\pi$ OR $24(9-\pi)$. OR $-24\pi + 216$ (Must be this or *exact* equivalent answer, "cm^3" or "cubic cm" optional.)

17. 61.5 OR $\frac{123}{2}$ OR $61\frac{1}{2}$

18. 8.688

19. $\frac{54}{7}$ (Must be this reduced improper fraction.)

20. $\frac{500}{3}$ (Must be this reduced improper fraction.)

1. The parabolic function $f(x)=ax^2+bx+c$ has vertex $(1,-2)$ and a point $(3,2)$ on the parabola when graphed in the coordinate plane. Determine the sum $(a+b+c)$.

2. Determine the first row, second column entry in the product $\begin{bmatrix}4 & 2\\5 & 3\end{bmatrix}\bullet\begin{bmatrix}1 & 6\\3 & 4\end{bmatrix}$.

3. Determine the largest real value for x such that $\sqrt{5x^4}=30$. Express your answer as a decimal rounded to the nearest hundredth.

4. $\dfrac{\dfrac{1}{2i}+2i}{\dfrac{3}{5i}-\dfrac{i}{3}}=k+wi$ where k and w are real numbers and $i=\sqrt{-1}$. Determine the sum $(k+w)$.
   Express your answer as a common or improper fraction reduced to lowest terms.

5. Determine the ordered triple (x,y,z) that is the solution to the system
   $\begin{cases}4x + y - z = -6\\8x - y - z = 12\\4x + 6y + 4z = -18\end{cases}$. Express your answer as an ordered triple (x,y,z) with exact decimal entries.

6. Determine the exact sum of all distinct values of x such that $\begin{vmatrix}-2x & 3x\\-16 & x^2\end{vmatrix}=64$.

7. $c(\log_{10} x)=\ln x$ for all $x>1$ and $c=\ln k$ for some real value of k. Determine the value of k.

8. The three zeros of the function $f(A) = A^3 - 124A^2 + 3103A - 20460$ can be used to form Aune's birth date as an ordered triple in the form $(MM, DD, YY) = (Month,\ Day,\ Year)$. Determine this ordered triple that represents Aune's birth date. Express your answer as an ordered triple in the form (MM, DD, YY).

9. The values of y such that the graph of the function $f(x) = e^y x^2 + e^{3y} x + e^{4y}$ does not have an x-intercept are $y < \ln k$. Determine the exact value of k.

10. An engineer must design a rectangular computer screen that has a 19-inch diagonal measure and covers 175 square inches of area. Determine the shorter dimension of this rectangular screen in inches. Express your answer as a decimal rounded to four significant digits.

11. Determine all point(s) of intersection when the line $y - x = 3$ and the circle $x^2 - 6x - 27 + y^2 = 0$ are graphed in the same coordinate plane. Express your answer as ordered pair(s) in the form (x, y).

12. Two of the roots for the equation $(x^2 + 1)^2 = 4(x^2 + 4x + 4)$ are integers and the other two are complex numbers of the form $k \pm wi$ where $i = \sqrt{-1}$ and with $w \geq 0$. Determine the ordered pair (k, w).

13. $f(x) = \dfrac{x + a}{bx + k}$ with $a \neq 0$ and $b \neq 0$ is its own inverse function. Determine the value of k.

14. Eight monkeys, all eating at the same rate, can eat 36 bananas in 3 minutes. Determine the number of minutes it will take 5 of these monkeys, all eating at this same rate, to eat 100 bananas. Express your answer as a common or improper fraction reduced to lowest terms.

15. An infinite geometric series has a common ratio of $\frac{1}{2}$ and a sum between 5 and 10, inclusive. Determine the sum of the smallest and largest possible values for the first term of this series. Express your answer as an exact decimal.

16. Determine the equation of a parabola passing through the three points $(1,3)$, $(2,2)$, and $(3,-3)$ in the form $(y-k)=4p(x-h)^2$. Report as your answer as the sum $(h+k+p)$. Express this sum as a common or improper fraction reduced to lowest terms.

17. The sum of 3 numbers is 126 and the product of these 3 numbers is 13824. These 3 numbers are in a geometric progression. Determine the harmonic mean of these 3 numbers. Express your answer as a common or improper fraction reduced to lowest terms.

18. $f(x)=2x^4+21x^3+35x^2-37x+46$. $\frac{f(x)}{2x+7}=Q(x)+\frac{k}{2x+7}$. Determine the value of k.

19. Auntie Frieze's auto has a radiator that contains 20 pints of a 20% solution of antifreeze. Determine the number of pints of this 20% antifreeze solution that must be drained away and replaced with 100% antifreeze so that the final mixture in Auntie's radiator is 50% antifreeze. Express your answer as an exact decimal.

20. A conic represented by $x^2-9y^2+36y-72=0$ has a major axis of length k and a minor axis of length w. Determine the sum $(k+w)$.

2015 RAA
Algebra II

Name **ANSWERS**

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -2

2. 32

3. 3.66 (Must be this exact decimal.)

4. $-\frac{45}{28}$ OR $\frac{-45}{28}$ OR $\frac{45}{-28}$ (Must be this reduced improper fraction.)

5. $(1.2, -6.6, 4.2)$ (Must be this ordered triple with exact decimal entries.)

6. 0

7. 10

8. $(11, 20, 93)$ (Must be this ordered triple.)

9. 4

10. 11.67 (Must be this exact decimal, "inches" optional.)

11. $(3, 6), (-3, 0)$ (Must have both ordered pairs in either order.)

12. $(-1, 2)$ (Must be this ordered pair.)

13. -1

14. $\frac{40}{3}$ (Must be this reduced improper fraction, "minutes" optional.)

15. 7.5 (Must be this exact decimal.)

16. $\frac{31}{8}$ (Must be this reduced improper fraction.)

17. $\frac{96}{7}$ (Must be this reduced improper fraction.)

18. 4

19. 7.5 (Must be this exact decimal, "pints" optional.)

20. 16

1. Points $P=(-3,4)$ and $Q=(-5,2)$. Determine the exact magnitude of vector $\overrightarrow{PQ}$.

2. $f\circ g(x)=f(g(x))$. If $f(x)=5x+6$ and $g(x)=3x-1$, then $f\circ g(x)=kx+w$. Determine the ordered pair (k,w).

3. Determine all complex roots for the equation $x^2+40=-9$.

4. x and y are real numbers such that $x^2+3xy+y^2=60$. Determine the maximum possible value for the product (xy).

5. The largest real value of x such that $\sin 3x+\sin x=0$ when solved over the interval $[0,2\pi)$ is $x=k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

6. Consider the graph of the function $y=\frac{1}{2}\cos(8\pi x-4)+12$. Let k represent the amplitude of this function and w represent its period. Determine the exact value of the product (kw). Express your answer as a common or improper fraction reduced to lowest terms.

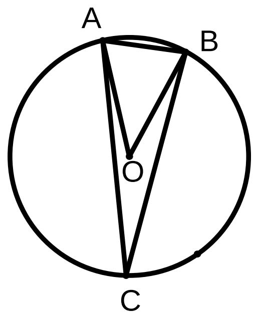
7. A sequence is defined by the rule $F_{n+1}=F_n+F_{n-1}$. $F_4=7$ and $F_7=29$. Determine the value of F_{10}.

8. Solve for all values of x such that $\log_{100}(x+2)=\log x$.

9. In vector notation, $\vec{v} = \langle a,b \rangle = a\vec{i} + b\vec{j}$. Determine the degree measure of the angle between the vectors $\vec{u} = \langle 2,3 \rangle$ and $\vec{v} = \langle -2,5 \rangle$. Express your answer as a decimal rounded to the nearest hundredth.

10. The probability a certain man will be alive 25 years hence is $\frac{3}{7}$ while independently, the probability his wife will be alive 25 years hence is $\frac{4}{5}$. Determine the probability that at least one of this man and woman is alive 25 years hence. Express your answer a common fraction reduced to lowest terms.

11. In the figure shown, the radius of Circle O is 8 and $AB = 6$. Determine the degree measure of $\angle C$. Express your answer as a decimal rounded to the nearest hundredth.



12. If $x > 5$, determine the largest value of k so that $\log(x) - k = \log(x-5) + 3$ has no real solutions.

13. $\dfrac{4x^4 + 2x^3 + 5x^2 + 15}{x+5} = Q(x) + \dfrac{k}{x+5}$. Determine the value of k.

14. $\angle A$ and $\angle B$ are first quadrant angles such that $\sin A = \frac{4}{7}$ and $\cos B = \frac{5}{7}$. In reduced and simplified radical form, $\tan(A+B) = \dfrac{k\sqrt{p} + w\sqrt{q}}{f}$ where k, w, p, q, and f are integers. Determine the value of the sum $(k + w + p + q + f)$.

15. Evie and Xavier are playing a game rolling a fair standard six-sided die. Evie wins if she rolls a 6 before Xavier rolls an odd number. If Evie rolls first, determine the probability Evie wins the game. Express your answer as a common fraction reduced to lowest terms.

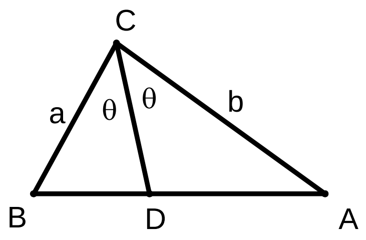
16. Henrik measured the angle of inclination from the ground to the top of a vertical pole along a line on level ground and found the angle to be $50°$. He walked 10 feet further from the pole along the same line and measured the angle of inclination from ground level to the top of the pole to be $40°$. If he walked along the same line to a point 10 feet from the base of the pole, the angle of inclination from ground level to the top of the pole is $k°$. Determine k. Express your answer as a decimal rounded to the nearest hundredth of a degree.

17. Determine the point on the graph of the function $y=\sqrt{2x-5}$ that is closest to the point $(5,0)$. Express your answer as an ordered pair with exact coordinate entries.

18. When written out as an integer, $2015!$ has k trailing zeros (2015 factorial ends in a string of k zeros.) Determine the value of k.

19. Let $\sec\theta=\frac{20}{15}$. Determine the least possible exact value for $\sin\theta$.

20. In the diagram shown, $\overrightarrow{CD}$ bisects $\angle ACB$ in ΔABC. $\cos\theta=\frac{1}{4}$ and $\frac{1}{a}+\frac{1}{b}=\frac{1}{20}$. Determine the exact length CD.



2015 RAA

Pre-Calculus

Name **ANSWERS**

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $2\sqrt{2}$ (Must be this exact answer.)

2. $(15,1)$ (Must be this ordered pair.)

3. $7i, -7i$ OR $0+7i, 0-7i$ (Must have both answers in either form or order, but may use +/-.)

4. 12

5. $\frac{3}{2}$ (Must be this reduced improper fraction.)

6. $\frac{1}{8}$ (Must be this reduced common fraction.)

7. 123

8. 2 (This value only.)

9. 55.49 (Must be this decimal, "degrees" or $^\circ$ optional.)

10. $\frac{31}{35}$ (Must be this reduced common fraction.)

11. 22.02 (Must be this decimal, "degrees" or $^\circ$ optional.)

12. -3

13. 2390

14. 62

15. $\frac{2}{7}$ (Must be this reduced common fraction.)

16. 70.57° (Must be this decimal, "degrees" or $^\circ$ optional.)

17. $\left(4,\sqrt{3}\right)$ (Must be this exact ordered pair.)

18. 502 ("Zeros optional.)

19. $-\frac{\sqrt{7}}{4}$ OR $-\frac{1}{4}\sqrt{7}$ OR $-0.25\sqrt{7}$ (Or exact reduced equivalent.)

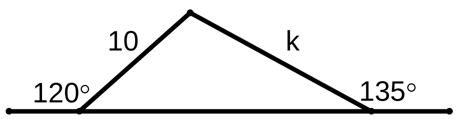
20. 10

NO CALCULATORS

1. For all valid replacements of real numbers a and b, $a \odot b = \frac{(a+b^2)}{(a-b)}$. Determine the value of $4 \odot (3 \odot 2)$. Express your answer as a common or improper fraction.

2. Segment $\overline{AB}$ has endpoints $A(-3,5)$ and $B(-4,3)$. The equation of the line that is the perpendicular bisector of $\overline{AB}$ may be written in the form $y = mx + b$. Determine the sum $(m+b)$. Express your answer as a common or improper fraction reduced to lowest terms.

3. Given the triangle with two exterior angles and two side lengths as marked. Determine the exact value of k.



4. The distance from the center of a regular octagon to one vertex is 10. Determine the exact numeric area of the octagon.

5. $k = \frac{4}{3} + \frac{10}{9} + \frac{28}{27} + \frac{82}{81} + \frac{244}{243} - 6$. Determine the exact value of k. Express your answer as a common or improper fraction reduced to lowest terms.

6. Twenty friends tried out for the baseball and basketball teams this year. Half of the students were chosen for the baseball team and 20% were chosen for the basketball team. $\frac{2}{5}$ of the students were not chosen for either team. Determine the number of students that were chosen for both teams?

7. Determine the numeric area of the triangle whose vertices are $(-1,2)$, $(2,6)$, and $(7,-4)$.

NO CALCULATORS

8. A jar contains 20 red, 15 white, and 15 blue marbles. Three marbles are drawn without replacement. Determine the probability that one of each color marble was drawn. Express your answer as a common fraction reduced to lowest terms.

9. The numeric length of one side of an equilateral triangle is 49. The exact numeric area of this triangle can be expressed as $\frac{k\sqrt{w}}{p}$ in simplified and reduced radical form and k and p relatively prime integers. Determine the sum $(k+w+p)$.

10. For some real value of b, the graphs of $y=x^2$ and $y=-(x-6)^2+b$ intersect at exactly one point. Determine the coordinates of that point. Express your answer as an ordered pair (x,y).

11. The vertex of an angle is located in the exterior of a circle in such a way as the sides of the angle form secant rays through the circle. The angle measures $20°$ and the circle is divided into four arcs, three of which are congruent to each other. Determine all possible degree measure(s) of the larger of the arcs.

12. Determine the number of positive integers less than 2015 that are divisible by 3, 5, or 7.

13. $k=\frac{9\left(\sqrt{2}+\sqrt{10}\right)}{2\left(\sqrt{3+\sqrt{5}}\right)}$. When simplified, $k=a+b\sqrt{c}$ with integers a, b, and c. Determine the sum $(a+b+c)$.

14. The perimeter of a square is 6 more than the perimeter of an equilateral triangle. The side of the square is 50% larger than the side of the triangle. Determine the numeric area of the square.

NO CALCULATORS

15. $k = 2.\overline{54}$ is a repeating decimal where the 54 repeats. k may also be written as a rational number. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

16. The equation $ax^3 + bx^2 + cx + d = 0$ has real coefficients such that $a + b = 13$, $a - b = -1$, $2c - d = -16$, and $3c - 5d = 81$. The largest root of this equation is 2. Determine the smallest root of this equation. Express your answer as an integer or common or improper fraction reduced to lowest terms.

17. A cubic polynomial has three distinct zeros r, s, and w whose sum is 0, whose product is $-\frac{15}{256}$ and the sum of whose squares is $\frac{19}{32}$. Determine the sum of the reciprocals of these zeros $\left(\frac{1}{r} + \frac{1}{s} + \frac{1}{w}\right)$. Express your answer as a common or improper fraction reduced to lowest terms.

18. The radius of a sphere is 50% larger than the side of a cube. The ratio of the volume of the sphere to the volume of the cube is $k\pi$. Determine the value of k. Express your answer as an integer or common or improper fraction reduced to lowest terms.

19. Determine *the number* of integers n, $18 < n < 40$, that have the property that the product of the distinct positive integral divisors of n is n^2.

20. Let G be the centroid of ΔABC where $A(12,2)$, $B(21,2)$ and $C(21,14)$ are the vertices. G is reflected across the x-axis to get point G'. G' is then reflected across the line $y = x$ to get point G''. With O as the origin, let θ be the numeric degree measure of $\angle GOG''$ and let k be the exact numeric distance GG''. Determine the exact numeric sum $(k + \theta)$.

2015 RAA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-\frac{53}{3}$ OR $\frac{-53}{3}$ OR $\frac{53}{-3}$ (Must be this reduced improper fraction.)

2. $\frac{7}{4}$ (Must be this reduced improper fraction.)

3. $5\sqrt{6}$ (Must be this exact answer.)

4. $200\sqrt{2}$ (Must be this exact answer, "square units optional..)

5. $-\frac{122}{243}$ OR $\frac{-122}{243}$ OR $\frac{122}{-243}$ (Must be this reduced common fraction.)

6. 2 ("students" optional.)

7. 25 ("square units" optional.)

8. $\frac{45}{196}$ (Must be this reduced common fraction.)

9. 2408 ("square units" optional.)

10. $(3,9)$ (Must be this ordered pair.)

11. 100, 120 (Must have both answers, either order, "degrees" optional.)

12. 1093 ("integers" optional.)

13. 9

14. 9 ("square units" optional.)

15. $\frac{28}{11}$ (Must be this reduced improper fraction.)

16. $-\frac{5}{3}$ OR $\frac{-5}{3}$ OR $\frac{5}{-3}$ (Must be this reduced improper fraction.)

17. $\frac{76}{15}$ (Must be this reduced improper fraction.)

18. $\frac{9}{2}$ (Must be this reduced improper fraction.)

19. 9 ("integers" optional.)

20. $90+12\sqrt{5}$ OR $12\sqrt{5}+90$ OR $6\left(15+2\sqrt{5}\right)$ OR $6\left(2\sqrt{5}+15\right)$

NO CALCULATORS

1. Determine the dot (or inner) product $\vec{v} \bullet \vec{w}$ when $\vec{v} = \langle 1, 0, -3 \rangle$ and $\vec{w} = \langle -3, 4, 5 \rangle$.

2. A portion of a number triangle that continues indefinitely is shown as
$$\begin{array}{ccccccccc} & & 1 & & 14 & & 1 & & \\ & 1 & & 15 & & 15 & & 1 & \\ 1 & & 16 & & 30 & & 16 & & 1 \end{array}$$
. If this portion represents rows 3, 4, and 5, then the sum of row 32 may be represented as 2^k. Determine the value of k.

3. Determine the sum of all distinct values for x that satisfy the determinant equation
$$\begin{vmatrix} x-1 & 1 & 3 \\ 0 & 2x+1 & 3 \\ 0 & x+1 & 1 \end{vmatrix} = 0.$$

4. $x^3 - 5x^2 - 12x + 36 = 0$. The product of two of the roots for this equation is 12. The three roots of this equation are k, w, and p with $k \le w \le p$. Determine the ordered triple (k, w, p).

5. $f(\theta) = \sin(3\theta)$ where θ is measured in degrees. Determine the exact value for $f(2015)$.

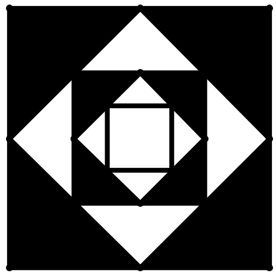
6. Determine the exact real value for x such that $x = \sqrt{20\sqrt{15\sqrt{20\sqrt{15\cdots}}}}$.

7. A sequence is defined recursively by $u_1 = 7$ and $u_{n+1} - u_n = 5n + 3$. u_k is the first term of this sequence that is a three-digit integer. Determine the value of k.

8. Determine the exact solution(s) for x when $5x^2 = 3 - 14x$. Express each answer(s) as an integer or common or improper fractions reduced to lowest terms.

NO CALCULATORS

9. Infinitely many nested squares are formed by connecting the midpoints of the sides of one square in order to make an interior square. The interior of the original square not including the inner square is shaded. The process continues, only the interior of the second square is left unshaded and the inner square shaded. The third square has interior shaded but not including the fourth square, and so on. Several iterations are shown in the diagram. The original square had sides of length 2. Determine the exact total shaded area of these infinitely many squares. Express your answer as a common or improper fraction reduced to lowest terms.



10. Let M be a 3×3 matrix such that each entry is either a 1 or a -1. The only possible distinct non-zero values of the determinant of M are k and w. Determine the value of $|k-w|$.

11. $A=(343)^{\frac{3}{2}}(125)^{\frac{2}{3}}(128)^{\frac{3}{2}}$. $B=(243)^{\frac{4}{3}}(49)^{\frac{1}{2}}(625)^{\frac{4}{3}}(64)^{\frac{1}{3}}$. Determine the greatest common integral factor of A and B.

12. The sum of two numbers is 20 and the sum of the squares of these two numbers is 195. Determine the product of these two numbers. Express your answer as a common or improper fraction reduced to lowest terms.

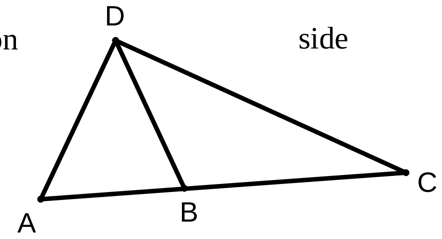
13. $\left(2\sqrt{x}+3\sqrt[3]{y}\right)^4$ is expanded and written in order of decreasing powers of x. Determine the sum of the numerical coefficients of the terms in which the power of x is a positive integer.

14. The sum of all real θ, $0\le\theta<\pi$, such that the graph of the function $f(x)=(\cos\theta)x^2+(\sin\theta)x+\left(\frac{1}{4}\cos\theta\right)$ has exactly one x-intercept may be written as $k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

NO CALCULATORS

15. Determine the exact value of $5^{2\log_5 3 - 4\log_5 2 + \log_6 36}$. Express your answer as a common or improper fraction reduced to lowest terms.

16. ΔACD (not necessarily drawn to scale) is shown with point B on side $\overline{AC}$. $\cos\angle DBC = -\frac{3}{4}$, $BD = 6$, and $AD = 10$. Determine the exact length AB.



17. Determine the number of distinct palindromic arrangements of the letters in the word MISSISSIPPI. (Palindromic arrangements read the same backwards as forward.)

18. In this problem, $x = k$ represents a vertical asymptote and $y = w$ represents a horizontal asymptote. $f(x) = \frac{(x+2)}{(3x+5)}$, $g(x) = \frac{(3x^3+24)}{(3x^2+7x+2)}$, and $p(x) = \frac{(5x^3+13x^2-6x)}{(2x^3+2x^2-12x)}$. One of the vertical and/or horizontal asymptotes for these three functions is selected at random. Determine the probability the corresponding value of k or w is an integer. Express your answer as a common fraction reduced to lowest terms.

19. $\sin(kx) = 4\sin x\cos^2 x - \sin x$ for all x. Determine the value of k.

20. $k = \sum_{n=1}^{\infty} \frac{3n}{7^n}$. Determine the exact value of k. Express your answer as a common or improper fraction reduced to lowest terms.

2015 RAA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = ☐

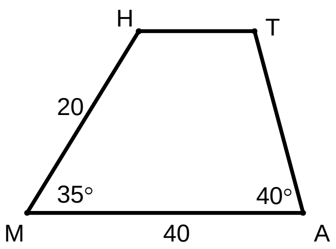
Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -18
2. 33
3. -1
4. $(-3, 2, 6)$ (Must be this exact ordered triple.)
5. $-\frac{\sqrt{2}+\sqrt{6}}{4}$ OR $\frac{-\sqrt{2}-\sqrt{6}}{4}$ (Or exact equivalent.)
6. $10\sqrt[3]{6}$ (Must be this exact answer.)
7. 7
8. $-3, \frac{1}{5}$ (Must have both answers in either order and with the exact reduced common fraction.)
9. $\frac{8}{3}$ (Must be this exact improper fraction, "square units" optional.)
10. 8
11. 700
12. $\frac{205}{2}$ (Must be this reduced improper fraction.)
13. 232
14. $\frac{3}{2}$ (Must be this reduced improper fraction.)
15. $\frac{225}{16}$ (Must be this reduced improper fraction.)
16. (Must be this single, exact answer shown below or exact, simplified equivalent.) $\frac{9+\sqrt{337}}{2}$ OR $\frac{1}{2}\left(9+\sqrt{337}\right)$
17. 30
18. $\frac{1}{5}$ (Must be this reduced common fraction.)
19. 3
20. $\frac{7}{12}$ (Must be this reduced common fraction.)

Round answers to four significant digits and write <u>in standard notation</u> unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required. (NOTE: DO NOT USE SCIENTIFIC NOTATION UNLESS SPECIFIED IN THE QUESTION)

1. Determine the least positive real-valued solution for the equation
   $4\left(\sin\left(2x-1\right)\right)^{\frac{1}{5}}=\ln\left(8x+3\right)$.

2. *MATH* is a trapezoid with measures as shown (but not necessarily drawn to scale). Determine the numeric area of trapezoid *MATH*.



3. $k=\begin{vmatrix}-3 & 2 & 5\\ 7 & 4 & 8\\ -3 & -9 & 6\end{vmatrix}-2\begin{vmatrix}-3 & 3 & 2\\ 6 & 7 & 4\\ -4 & -3 & 6\end{vmatrix}-3\begin{vmatrix}8 & 13\\ 2 & 44\end{vmatrix}-4\begin{vmatrix}33 & 9\\ 2 & 6\end{vmatrix}+5\begin{vmatrix}4 & 3 & 6\\ -22 & 3 & -2\\ 4 & 3 & 5\end{vmatrix}-\begin{vmatrix}9 & -8\\ -8 & 16\end{vmatrix}$.
   Determine the value of k.

4. Determine the number of digits in the expansion of $\left(2015\right)^{2015}$.

5. Determine the largest value for x in the domain of the inverse sine function $y=\sin^{-1}\left(0.0024x+2\sqrt{3}\right)$. Express your answer as a decimal rounded to the nearest hundredth.

6. Determine the decimal representation for the degree measure of the angle whose measure is $42°24'36"$.

7. A certain disease can be detected by a laboratory test with 92% accuracy. Unfortunately, the test also gives a false positive result in 2% of healthy patients. Suppose 1000 people have been tested of whom 6 are known to have the disease. One of these 1000 people is chosen at random. Determine the probability that the patient is healthy given the patient's test was positive.

8. Liz invests $2015 in an account earning 3.75% annual interest, compounded monthly. Determine the number of years it will take for Liz's investment to double.

9. $i=\sqrt{-1}$. Determine the absolute value of $(6.71+2.05i)$.

10. A hollow metal sphere is enclosed in a spherical shell that is 2 cm thick. The total volume of the metal used to form the shell is $483\pi\ cm^3$. Determine the radius of the sphere measured in cm.

11. $f(x)=25.325\sqrt{(\pi x)}$. Determine the value of $\underbrace{f\left(f\left(f\left(f\left(\cdots 42\cdots\right)\right)\right)\right)}_{42}$. (That is, 42 iterations where the input for the first iteration is 42 and each subsequent input is the value of the previous iteration.)

12. The numeric total surface area of a right circular cone with height 5 is 13π. Determine the numeric lateral surface area for this cone.

13. In ΔABC, $\angle CAB = 60°$ and $\angle ABC = 75°$. Determine the decimal value for the ratio $AB : AC$.

14. Determine *the number* of ordered pairs with integral entries that are solutions to the system $\begin{cases} f(x) \geq x^2 - 2 \\ g(x) \leq -x^2 + 3 \end{cases}$.

15. ΔABC is isosceles with base $\overline{AC}$ that has length 14. The numeric area of ΔABC is 77. Determine the degree measure of $\angle ABC$.

16. $k = \prod_{j=2}^{j=10} (\ln j)$. Determine the value of k.

17. A parabolic function $f(x)$ contains the points $(3,5)$, $(7,21)$, and $(-1,53)$. The vertex of this parabola is also the center of a circle with radius 3. The point(s) of intersection of the graphs of this circle and this parabola may be denoted (k,w). Determine the sum of all the coordinates of these possible point(s) of intersection..

18. $i = \sqrt{-1}$. Determine the product of the roots for x in the equation $e^{i\pi} = x^3 + 3x^2 - 6x - 9$.

19. A sequence is given recursively by $A(n) = \dfrac{A(n-1)}{1 + A(n-2)}$, for all $n \geq 3$ and with $A(1) = 1$ and $A(2) = 2$. Determine $A(20)$.

20. Determine the number of trailing zeros when $2015!$ is expanded and written as a integer. (Trailing zeros are the rightmost zeros after the last non-zero digit.)

2015 RAA

Calculator Team

School **ANSWERS**

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in standard notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required. (NOTE: DO NOT USE SCIENTIFIC NOTATION UNLESS SPECIFIED IN THE QUESTION)

1. 0.5142 OR .5142

2. 286.5

3. –2247

4. 6659 ("digits" optional.)

5. –1026.71 (Must be this decimal.)

6. 42.41 ("degrees" optional.)

7. 0.7827 OR .7827

8. 18.51 ("years" optional.)

9. 7.016

10. 6.749 ("cm" optional.)

11. 2015

12. 30.43 ("square units" or "units squared" optional.)

13. 0.7321 OR .7321

14. 14

15. 64.94 ("degrees" optional.)

16. 62.32

17. 19.52

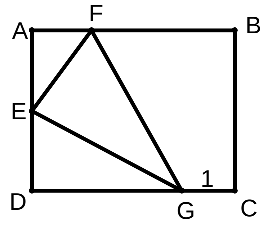
18. 8

19. 0.04272 OR .04272

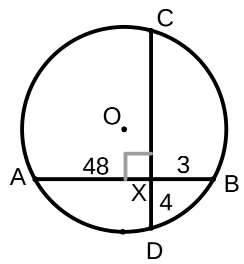
20. 502 ("zeros" optional.)

1. Let $k = 0.399999\ldots$. Let the line through points $(20,15)$ and $(2,w)$ have slope $\frac{2}{3}$. Determine the sum $(k+w)$. Express your answer as a common or improper fraction reduced to lowest terms.

2. Determine the product of ***all*** solutions to the equations $3x^2+7x+2=0$ and $12y^2-4y-56=0$. Express your answer as a common or improper fraction reduced to lowest terms.



3. $ABCD$ (shown, but not drawn to scale) is a 4×6 rectangle. E is the midpoint of $\overline{AD}$, $AF:FB=1:2$, and $GC=1$. Determine the sum of possible areas for ΔEFG.



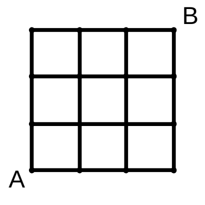
4. In Circle O with radius length 52, chords $\overline{AB}$ and $\overline{CD}$ are perpendicular at point X. $AX=48$, $XB=3$, and $XD=4$. Determine the exact distance from the center of the circle to the chord $\overline{CD}$.

5. Let k and n be integers such that $2^n+2^n+2^n+2^n=k\left(2^{(n+1)}\right)$. An edge of a cube has length $8\sqrt{3}$. Let w be the length of the radius of the sphere circumscribing the cube. Determine the sum $(k+w)$.

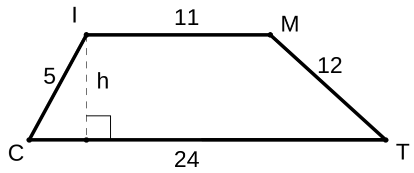
6. A sphere of radius 3.5 cm has numeric volume $k\ cm^3$ and numeric surface area $w\ cm^2$. Determine the value of $(k-w)$. Express your answer as a decimal correct to 4 significant digits.

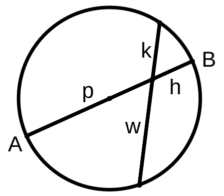
7. The arithmetic mean (average) of the 5 members of the set $\{106, 112, 92, 86, k\}$ is 104. The arithmetic mean of the 7 members of the set $\{98, 56, 27, 289, 652, k, w\}$ is 204. Determine the value of w.

8. On a 3×3 chessboard, there are 20 ways to travel from point A to point B moving only upwards or to the right along the edges of the squares. On a $k\times k$ chessboard, there are 40,116,600 ways to move in this fashion from the lower left corner to the upper right corner. Determine k.



9. In trapezoid $ICTM$, h is the distance between parallel sides $\overline{IM}$ and $\overline{CT}$. $IC = 5$, $CT = 24$, $TM = 12$, and $IM = 11$. In the circle with area 36π, $\overline{AB}$ is a diameter divided by a second chord. The same value of h is the length of one part of the diameter and the second chord is divided into two pieces k and w. Determine the product (kw). Express your answer as a common or improper fraction reduced to lowest terms.





10. A bowl contains 6 orange, 8 blue, 5 red, and 6 white marbles. Evie reaches in and draws two marbles and then returns them to the bowl. Xavier then reaches in and draws two marbles and returns them to the bowl. Let k be the probability Evie did not draw a white marble. Let w be the probability Xavier did not draw a blue or a red marble. Determine $(k+w)$. Express your answer as an exact decimal.

11. Twelve is divisible by the sum of its digits (3) and by the product of its digits (2). Find the sum of the least two integers larger than 12 with this property.

12. $f\left(g\left(x\right)\right)=20x$ and $g\left(f\left(x\right)\right)=15x$. Let $f\left(g\left(g\left(f\left(x\right)\right)\right)\right)=kx$ and $g\left(f\left(f\left(g\left(x\right)\right)\right)\right)=wx$. Determine the sum $\left(k+w\right)$.

ICTM Math Contest

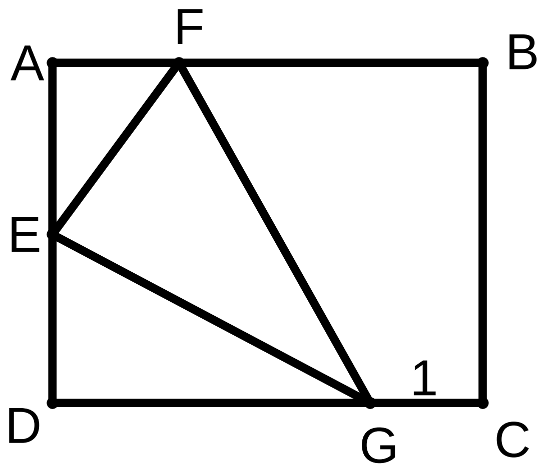
Freshman – Sophomore

2 Person Team

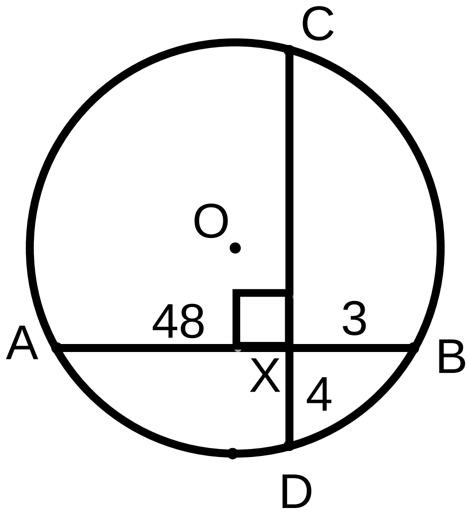
Division AA

1. Let $k = 0.399999\ldots$. Let the line through points $(20,15)$ and $(2,w)$ have slope $\frac{2}{3}$. Determine the sum $(k+w)$. Express your answer as a common or improper fraction reduced to lowest terms.

2. Determine the product of ***<u>all</u>*** solutions to the equations $3x^2 + 7x + 2 = 0$ and $12y^2 - 4y - 56 = 0$. Express your answer as a common or improper fraction reduced to lowest terms.



3. ABCD (shown, but not drawn to scale) is a 4×6 rectangle. E is the midpoint of $\overline{AD}$, $AF : FB = 1 : 2$, and $GC = 1$. Determine the sum of possible areas for ΔEFG.



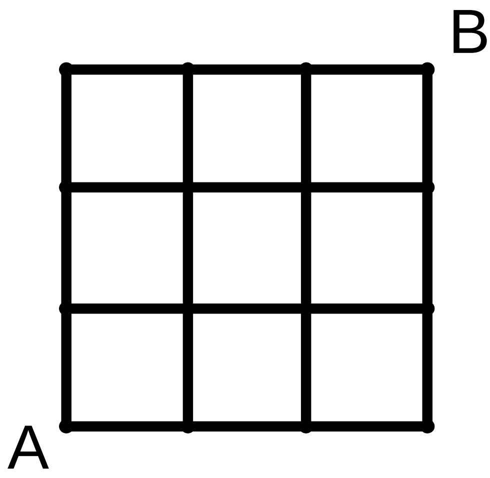
4. In Circle O with radius length 52, chords $\overline{AB}$ and $\overline{CD}$ are perpendicular at point X. $AX = 48$, $XB = 3$, and $XD = 4$. Determine the exact distance from the center of the circle to the chord $\overline{CD}$.

5. Let k and n be integers such that $2^n + 2^n + 2^n + 2^n = k\left(2^{(n+1)}\right)$ An edge of a cube has length $8\sqrt{3}$. Let w be the length of the radius of the sphere circumscribing the cube. Determine the sum $(k+w)$.

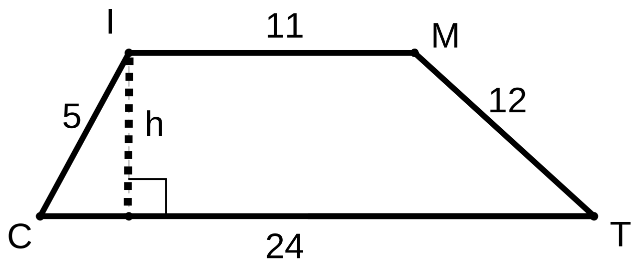
6. A sphere of radius 3.5 cm has numeric volume k cm^3 and numeric surface area w cm^2. Determine the value of $(k-w)$. Express your answer as a decimal correct to 4 significant digits.

7. The arithmetic mean (average) of the 5 members of the set $\{106, 112, 92, 86, k\}$ is 104. The arithmetic mean of the 7 members of the set $\{98, 56, 27, 289, 652, k, w\}$ is 204. Determine the value of w.

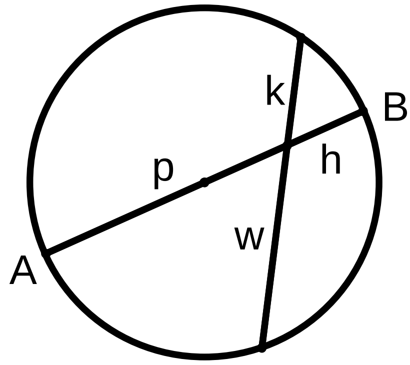
8. On a 3×3 chessboard, there are 20 ways to travel from point A to point B moving only upwards or to the right along the edges of the squares. On a $k\times k$ chessboard, there are 40,116,600 ways to move in this fashion from the lower left corner to the upper right corner. Determine k.



9. In trapezoid ICTM, h is the distance between parallel sides $\overline{IM}$ and $\overline{CT}$. $IC = 5$, $CT = 24$, $TM = 12$, and $IM = 11$.



In the circle shown with area 36π, $\overline{AB}$ is a diameter divided by a second chord. The same value of h is the length of one part of the diameter and the second chord is divided into two pieces k and w. Determine the product (kw). Express your answer as a common or improper fraction reduced to lowest terms.



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12. $f\left(g\left(x\right)\right)=20x$ and $g\left(f\left(x\right)\right)=15x$. Let $f\left(g\left(g\left(f\left(x\right)\right)\right)\right)=kx$ and $g\left(f\left(f\left(g\left(x\right)\right)\right)\right)=wx$. Determine the sum $\left(k+w\right)$.

2015 RAA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. $\frac{17}{5}$ (Must be this reduced improper fraction.)	
2. $-\frac{28}{9}$ OR $\frac{-28}{9}$ Or $\frac{28}{-9}$ (Must be this reduced improper fraction.)	
3. $\frac{27}{2}$ OR 13.5 OR $13\frac{1}{2}$	
4. 48	
5. 14	
6. 25.66 (Must be this decimal.)	
7. 182	
8. 14	
9. $\frac{5760}{169}$ (Must be this reduced improper fraction.)	
10. 0.79 OR .79 (Must be this decimal.)	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 60
12. 600
13. ______
14. ______
15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Determine the value of x and y such that $\begin{bmatrix} x & 0 \\ 2y & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix} = 2\begin{bmatrix} -10 & 0 \\ 23 & 4.5 \end{bmatrix}$. Express your answer as the ordered pair (x, y).

2. The roots for the equation $x^3 + ax^2 + bx + c = 0$ are 2, –3 and 5. Determine the exact sum $(a+b+c)$.

3. In ΔABC, $\tan A + \tan B = \tan A \tan B - 1$. The exact radian measure of $\angle C$ is $k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

4. $f(x) = x^2 + x + 1$ and $g(x) = \dfrac{1}{x-1}$. $k = f\left(g\left(\dfrac{5}{4}\right)\right)$ and $w = g\left(f\left(\dfrac{5}{4}\right)\right)$. Determine the product (kw). Express your answer as a common or improper fraction reduced to lowest terms.

5. Each of the angles μ, α, and θ is an angle whose radian measure is between 0 and $\dfrac{\pi}{2}$, inclusive. $2\sin\mu\cos\mu = 1$, $\tan(2\alpha) = 1$, and $\csc(2\theta) = \sqrt{2}$. The largest value of the sum $(\mu + \alpha + \theta)$ is $k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

6. A bee on a three dimensional grid flies directly from point $A(0,-6,0)$ to point $B(3,2,1)$ then directly to point $C(0,0,3)$, then directly to point $D(0,-2,3)$, and finally directly back to point A. Determine the total distance the bee flew. Express your answer as a decimal rounded to four significant digits.

7. A triangle with sides of lengths 3-4-5 is rotated about the side of length 5 to form a solid of revolution. The volume of this solid is $k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

8. A slot machine has 5 wheels and each wheel has 5 symbols: an apple, banana, cherry, grape, and pear. A lever is pulled, the five wheels spin, and each wheel stops randomly on one of the symbols. You win if the wheels stop on three or more cherries. Determine the probability you will win at least once if you pull the lever five times. Express your answer as a decimal rounded to four significant digits.

9. Let $k = A \times 10^B$ in scientific notation be the number of distinct sets of 3-person teams that can be formed from a class of 40 students using as many students as possible but with each person being on no more than one team. Determine the value of A as a decimal rounded to four significant digits.

10. The expressions $(x^2 - 9x)$, $(-x-6)$, and $(x^2 - 7x)$ taken in that order form an arithmetic progression. Determine the sum of all the terms in the possible arithmetic progression(s).

11. Determine the exact area enclosed by the conic given by $9x^2 + 4y^2 - 54x + 16y + 61 = 0$.

12. Determine the ten's digit in the number $k = 20^{15} + 15^{20} + 21^{50}$.

ICTM Math Contest

Junior – Senior

2 Person Team

Division AA

1. Determine the value of x and y such that

$$\begin{bmatrix} x & 0 \\ 2y & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix} = 2\begin{bmatrix} -10 & 0 \\ 23 & 4.5 \end{bmatrix}.$$

Express your answer as the ordered pair (x, y).

2. The roots for the equation $x^3 + ax^2 + bx + c = 0$ are 2, –3 and 5.

Determine the exact sum $\left(a+b+c\right)$.

3. In ΔABC, $\tan A + \tan B = \tan A \tan B - 1$. The exact radian measure of $\angle C$ is $k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

4. $f(x) = x^2 + x + 1$ and $g(x) = \dfrac{1}{x-1}$. $k = f\left(g\left(\dfrac{5}{4}\right)\right)$ and $w = g\left(f\left(\dfrac{5}{4}\right)\right)$. Determine the product (kw). Express your answer as a common or improper fraction reduced to lowest terms.

5. Each of the angles μ, α, and θ is an angle whose radian measure is between 0 and $\frac{\pi}{2}$, inclusive. $2\sin\mu\cos\mu = 1$, $\tan\left(2\alpha\right) = 1$, and $\csc\left(2\theta\right) = \sqrt{2}$. The largest value of the sum $\left(\mu + \alpha + \theta\right)$ is $k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

6. A bee on a three dimensional grid flies directly from point $A(0,-6,0)$ to point $B(3,2,1)$ then directly to point $C(0,0,3)$, then directly to point $D(0,-2,3)$, and finally directly back to point A. Determine the total distance the bee flew. Express your answer as a decimal rounded to four significant digits.

7. A triangle with sides of lengths 3 – 4 – 5 is rotated about the side of length 5 to form a solid of revolution. The volume of this solid is $k\pi$. Determine the value of k. Express your answer as a common or improper fraction reduced to lowest terms.

8. A slot machine has 5 wheels and each wheel has 5 symbols: an apple, banana, cherry, grape, and pear. A lever is pulled, the five wheels spin, and each wheel stops randomly on one of the symbols. You win if the wheels stop on three or more cherries. Determine the probability you will win at least once if you pull the lever five times. Express your answer as a decimal rounded to four significant digits.

9. Let $k = A \times 10^{B}$ in scientific notation be the number of distinct sets of 3-person teams that can be formed from a class of 40 students using as many students as possible but with each person being on no more than one team. Determine the value of A as a decimal rounded to four significant digits.

10. The expressions $\left(x^2 - 9x\right)$, $\left(-x - 6\right)$, and $\left(x^2 - 7x\right)$, taken in that order, form an arithmetic progression. Determine the sum of all the terms in the possible arithmetic progression(s).

11. Determine the exact area enclosed by the conic given by $9x^2 + 4y^2 - 54x + 16y + 61 = 0$.

12. Determine the ten's digit in the number

$$k = 20^{15} + 15^{20} + 21^{50}.$$

2015 RAA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are _NO CALCULATOR_

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer		Score (to be filled in by proctor)
1.	$(-5,5)$	(Must be this ordered pair.)	
2.	15		
3.	$\frac{1}{4}$	(Must be this reduced common fraction.)	
4.	$\frac{112}{15}$	(Must be this reduced improper fraction.)	
5.	$\frac{3}{4}$	(Must be this reduced common fraction.)	
6.	19.73	(Must be this decimal.)	
7.	$\frac{48}{5}$	(Must be this reduced improper fraction.)	
8.	0.2579 OR .2579	(Must be this decimal.)	
9.	1.003	(Must be this decimal.)	
10.	-57		

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 6π (Must be this exact answer.)

12. 2

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

ORAL COMPETITION
ICTM REGIONAL 2015 DIVISION AA

1. Tom's refrigerator contains 6 apples, 2 of which are rotten. Tom randomly selects 3 apples.

   a) What is the probability that none of the selected apples are rotten?

   b) What is the probability that Tom selects at least one rotten apple?

2. Suppose a 5 character license plate must contain exactly 3 distinct digits and 2 distinct letters.

   a) How many such license plates are possible?

   b) What is the probability that the two letters appear before any of the digits in the license plate?

3. a) How many DISTINCT ways are there to arrange the letters of the word "COMBINATION" ?

   b) What is the probability that the letter "B" appears first ?

   c) What is the probability that all "I"s appear before all "O"s AND all "O"s appear before all "N"s ?

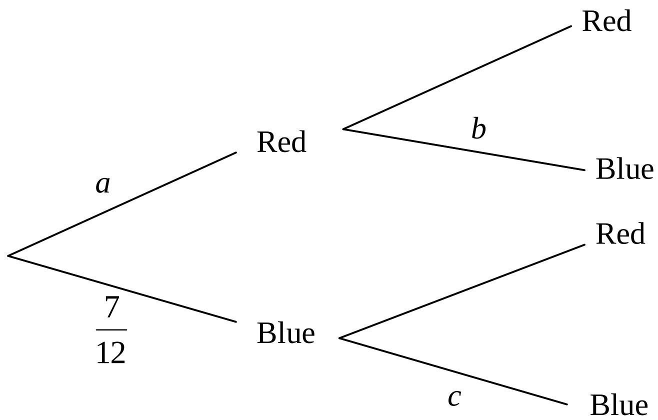
ORAL COMPETITION
ICTM REGIONAL 2015 DIVISION AA

EXTEMPORANEOUS QUESTIONS

<u>Give this sheet to the students at the beginning of the extemporaneous question period.</u>

<u>STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.</u>

1. The following tree diagram shows the probabilities related to the following situation: A bag contains 5 red and 7 blue marbles. A marble is selected at random and its color noted. It is not returned to the bag. A second marble is then selected and its color noted. Explain what the probabilities marked as *a, b* and *c* should be. You may, but do not have to, draw the tree in your explanation.



2. A bucket contains 100 balls labeled 1 through 100. Two distinct balls are selected at random. Explain how to determine the probability that the two selected balls are both even. You do not need to actually complete the calculations.

3. For some value *n*, $C(n,20)=C(n,15)$. Find the value of *n* and justify your answer.

4. Refer to problem #3b from the initial contest (the probability of the letter B appearing first when arranging the letters in the word COMBINATION). How would your answer change if the letter N must appear first?

ORAL COMPETITION
ICTM REGIONAL 2015 DIVISION AA

JUDGES' SOLUTIONS

(NOTE: There may be alternate solution methods to some or all of these problems. Any mathematically correct solution should be given full credit)

1. Tom's refrigerator contains 6 apples, 2 of which are rotten. Tom randomly selects 3 apples.

   a) What is the probability that none of the selected apples are rotten?

# of ways to select 3 apples = C(6,3) = 20
# of ways to select 3 apples from the 4 which are not rotten = C(4,3) = 4
Probability is $\frac{4}{20} = \boxed{\frac{1}{5}}$

   b) What is the probability that Tom selects at least one rotten apple?

Selecting at least one rotten apple is the complement of selecting no rotten apples.
(Students may refer to the "backdoor" method in the reference)
Using the result of part a: (Probability of selecting no rotten apples), the probability of selecting at least one rotten apple is then $1 - \frac{1}{5} = \boxed{\frac{4}{5}}$**.**
Alternately, students may find the number of ways to select at least one rotten apple:
C(6,3) – C(4,3) = 20 – 4 = 16 and then find the probability as $\frac{16}{20} = \frac{4}{5}$**.**

Another potential option would be for students to find the number of ways to select one, two or three rotten apples: C(2,1)*C(4,2) + C(2,2)*C(4,1) + 0 = 16 and then find the probability as above.

2. Suppose a 5 character license plate must contain exactly 3 distinct digits and 2 distinct letters.

   a) How many such license plates are possible?

We select the two letters (without repetition):
# of ways to select two letters: C(26,2) = 325

We then select the three digits (without repetition):
# of ways to select three digits: C(10,3) = 120

There are 5! = 120 ways to arrange the 5 characters.

The total number of ways is $(5!)(325)(120) = \boxed{4,680,000}$

An alternate method would be to choose the positions of the three digits (C (5,3) = 10) or for the 2 letters (C(5,2)=10) and then use the multiplication rule with the number of ways to arrange the letters (26*25) and the numbers (10*9*8): 26*25*10*9*8 = 468,000 and multiply by 10 to get 4,680,000.

b) What is the probability that the two letters appear before any of the digits in the license plate?

We select and arrange the two letters (without repetition):
# of ways to select two letters: P(26,2) = 650 (or 26*25=650)

We then select and arrange the three digits (without repetition):
# of ways to select three digits: P(10,3) = 720 (or 10*9*8=720)

The total number of ways is then $(650)(720)=468,000$ **and the probability is** $\frac{468,000}{4,680,000}=\boxed{\frac{1}{10}}$

3.
a) How many DISTINCT ways are there to arrange the letters of the word "COMBINATION" ?

Using the Permutation Theorem involving repetitions with 11 letters, including 2 O's, 2 I's, and 2 N's, the total number of ways is $\frac{11!}{2!2!2!}=\boxed{4,989,600}$

b) What is the probability that the letter "B" appears first ?

The "B" is fixed in the first place, and there is only 1 way for that to happen. It then becomes the number of ways to arrange the remaining 10 letters, including the same repetitions as in part a:
$1*\frac{10!}{2!2!2!}=453,600$ **out of the total 4,989,600 for a probability of** $\frac{453,600}{4,989,600}=\boxed{\frac{1}{11}}$

c) What is the probability that all "I"s appear before all "O"s AND all "O"s appear before all "N"s ?

We will start with the following "template":
***I*I*O*O*N*N* and fill in with the remaining letters.**
This effectively forces the condition on I's, O's, and N's.
First, there are 5! ways to sort the remaining distinct characters.
Once these are sorted, we only need to know what 5 positions to put them in.
# of ways to distribute the 5 letters among the 11 positions = C(11,5).
The probability is then $\frac{5!\left(C(11,5)\right)}{4,989,600}= =\boxed{\frac{1}{90}}$

ORAL COMPETITION
ICTM REGIONAL 2015 DIVISION AA

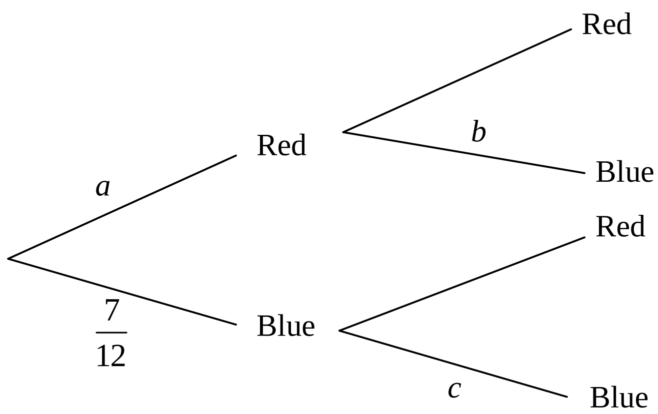
EXTEMPORANEOUS QUESTIONS

JUDGES' SOLUTIONS

Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. The following tree diagram shows the probabilities related to the following situation: A bag contains 5 red and 7 blue marbles. A marble is selected at random and its color noted. It is not returned to the bag. A second marble is then selected and its color noted. Explain what the probabilities marked as *a, b* and *c* should be. You may, but do not have to, draw the tree in your explanation.



The probability $\boxed{a=\frac{5}{12}}$ **either because there are 5 of 12 red marbles or by using** $1-\frac{7}{12}$**.**

If a red marble is selected first, there are still 7 blue marbles, but only 11 total, so $\boxed{b=\frac{7}{11}}$

If a blue marble is selected first, there are 6 blue marbles left and 11 total, so $\boxed{c=\frac{6}{11}}$

2. A bucket contains 100 balls labeled 1 through 100. Two distinct balls are selected at random. Explain how to determine the probability that the two selected balls are both even. You do not need to actually complete the calculations.

# of ways to pick 2 evens = C(50,2)
# of ways to pick any 2 = C(100,2)

Probability is $\boxed{\frac{C(50,2)}{C(100,2)}}$

3. For some value *n*, $C(n,20) = C(n,15)$. Find the value of *n* and justify your answer.

Since $C(n,r) = C(n,n-r)$ and $r + n - r = n$, the value of *n* is $20 + 15 = \boxed{35}$. Students should explain that $C(35,20) = \frac{35!}{(20!)(15!)}$ and $C(35,15) = \frac{35!}{(15!)(20!)}$. These expressions are equivalent, since the denominators are equal by the commutative property of multiplication.

4. Refer to problem #3b from the initial contest (the probability of the letter B appearing first when arranging the letters in the word COMBINATION). How would your answer change if the letter N must appear first?

Since there are 2 N's, the probability would be twice the probability if the B appeared first.