1. If $\frac{3}{5-x}=1$, find the value of x.

2. If $\left(x^2-y^2\right)=0$ and $x+y=17.534$, find the value of $x-y$.

3. If x is 75% of y, then $y=kx$. Find the value of k. Express your answer as an improper fraction reduced to lowest terms.

4. Find the **ordered pair** that represents the point at which the line whose equation is $7x+3y=84$ intersects the *x*-axis.

5. Let x, y, and z represent integers greater than 1. If $\left(x^7\right)^y = x^7x^z$, find the sum of the two smallest distinct values of $|z-y|$.

6. Let x and y be positive integers such that the arithmetic mean (average) of $5x$ and $7y$ is 60. Find the sum of all distinct possible values of x.

7. Gusher invested part of his $100,000 in oil bonds, which earned interest at 6% annual percentage rate and invested the rest of his $100,000 in natural gas bonds, which earned interest at 5% annual percentage rate. At the end of one year his interest income from these two investments came to a total of $5320. Find the number of dollars Gusher invested in natural gas bonds.

8. If $kx+11.2=106.4$ and $wx+15.4=1738.52$, find the value of $\frac{w}{k}$. Express your answer as a **decimal**.

9. If twice the sum of the squares of the roots for the equation $x^2 - 70x + k = 0$ is added to the product of the roots for the equation, the result is 6632. Find the smaller of the two roots for x.

10. The sum, the product, and the average (arithmetic mean) of three different numbers are equal. If two of the numbers are 7.486 and –7.486, find the third number.

11. Bob sells x articles at 4 cents each, with a profit of $\frac{2}{5}$ cents on each article. On the same day, Bob sells y articles at 2 for 5 cents (must be sold in multiples of 2), with a profit of $\frac{1}{7}$ cents on each article. Judy tells him it would be simpler just to mix the articles and sell them at 3 cents each. So the next day, Bob does sell $(x+y)$ articles at 3 cents each. Obviously, x and y are positive integers. If the overall profit was the same each day, find the smallest possible value of $(x+y)$.

12. There are two numbers formed by the same two digits in reverse order. The sum of the two numbers is 22 times the difference between the two digits, and the difference between the squares of the two numbers is k where $k > 1000$. If none of the digits of k is the same as any of the digits of the two original numbers, find the larger of the two original numbers.

13. An auto was originally priced at $25,000. The auto's price was reduced by $x\%$ to y dollars. Then the price of y dollars was reduced by $k\%$ to $16170. If both x and k are positive integers such that $x < k$, find the smallest possible value of $2k$.

14. A woman bought a 36-ounce Pepsi at the Big Pep. On the first day, she drank 1 ounce from the container and refilled the remainder of the container with 7-up. On the second day, she drank 2 ounces from the container and refilled the remainder of the container with 7-up. On the third day, she drank 3 ounces from the container and refilled the remainder of the container with 7-up. This procedure was continued for succeeding days until the container was empty. Find the total number of ounces of 7-up that the woman drank during this process.

15. Find the value of a such that the solution set for x of $|x-a|<16$ is $\{x: -34<x<-2\}$.

16. How many integers in the set of all integers from 1 to 100 (including 1 and 100) are **not** the cube of an integer?

17. To a 30 gallon mixture that is 30% silver nitrate are added an x gallon mixture that is 25% silver nitrate and a y gallon mixture that is 42% silver nitrate. If the resulting $(30+x+y)$ mixture is 35% silver nitrate and if x and y are positive integers, find the ordered pair (x, y) such that $(x+y)$ is a minimum. Be sure to express your answer as an ordered pair of the form (x, y).

18. A village had a large tank that contained k gallons of water. If each person in the village consumed an equal amount each day, there would be enough water in the tank to last for 11 days. If the village had 400 more persons, each person would need to consume 2 fewer **ounces** per day than the original allotment for the water in the tank to last for 11 days. If the village had 600 fewer persons, each person could consume 2 more **ounces** per day than the original allotment, and there would be enough water in the tank to last for 12 days. Find the value of k. **Note:** Remember that k is the number of gallons of water. Express your answer as a **decimal**.

19. The slope of the line whose equation is $10x+ky-70=0$ is 5. Find the value of k.

20. A number is five more than the product of two consecutive positive integers and is also an integral multiple of both 55 and 121. Let the smaller of the two consecutive positive integers be represented by k. Find the sum of all distinct values of k if $k<157$.

2009 SA
Algebra I

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

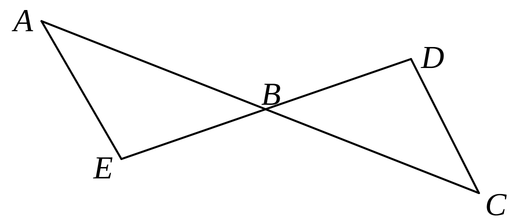
______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

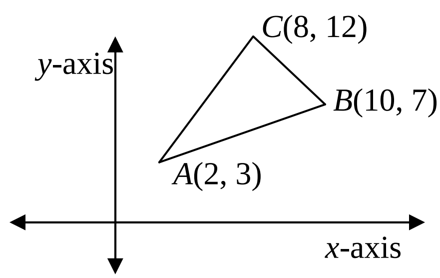
1. 2
2. 0
3. $\frac{4}{3}$ **(Must be this reduced improper fraction.)**
4. $(12,0)$ **(Must be this ordered pair.)**
5. 16
6. 30
7. $68,000$ **($ optional.)**
8. 18.1 **(Must be decimal answer.)**
9. 22
10. 0
11. 3
12. 93
13. 46
14. 630 **(“ounces” optional.)**
15. -18
16. 96
17. $(6,30)$ **(Must be this ordered pair.)**
18. 6187.5 **(Must be decimal answer, “gallons” optional.)**
19. -2
20. 169

1. Find the sum of the degree measures of the interior angles of a convex polygon that has 19 sides.

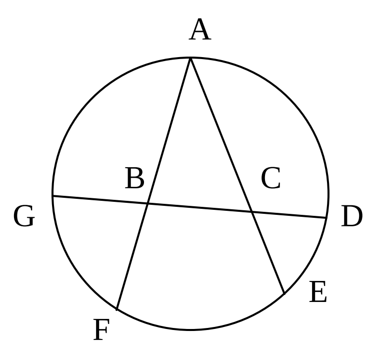
2. In the diagram, $\overline{AC}$ and $\overline{ED}$ intersect at B. $\overline{AE} \perp \overline{ED}$, $\overline{CD} \perp \overline{ED}$. $AE = 5$, $DC = 7$, and $AC = 30$. Find AB. Express your answer as a **decimal**.



3. Using the diagram with coordinates as shown, find the **ordered pair** that represents the midpoint of $\overline{AB}$.



4. In the diagram, points A, G, F, E, and D lie on the circle, points A, B, and F are collinear; points A, C, and E are collinear; and points G, B, C, and D are collinear. $GB = 5$, $BF = 6$, $CD = 4$, and $AB = AC = 10$. Find AE. Express your answer as a **decimal**.



5. One circle has a radius whose length is 8 and another circle has a diameter whose length is 64. The area of the larger circle is how many times the area of the smaller circle?

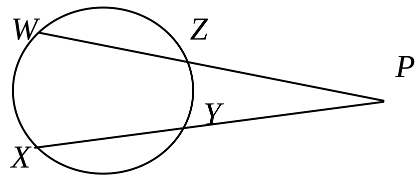
6. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   The locus of points, in a plane, that are equidistant from two given points is:

   A) Two parallel lines
   B) A line
   C) A point
   D) Two points
   E) Three points.
   F) Four points.
   G) A Circle.

Note: Be certain to write the correct capital letter as your answer.

7. In the diagram, W, Z, X, and Y lie on the circle. W, Z, and P are collinear; and X, Y, and P are collinear. $\overset{\frown}{WXY} = 236°$, and $\overline{WZ} \cong \overline{XY}$. Find the degree measure of $\angle WPX$.

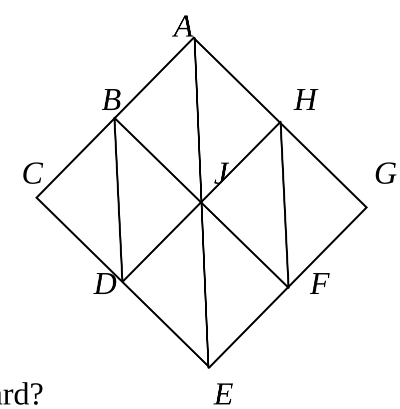


8. The number of inches in the length of the minor arc intercepted by a chord whose length is $\sqrt{1200}$ inches is $k\pi$ inches. If the radius of this circle is 20 inches, find the value of k. Express your answer as an improper fraction reduced to lowest terms.

9. Each of the three sides of a right triangle has a length that is an integer. The difference between the length of the hypotenuse and the length of the longer leg is the same as the difference between the length of the longer leg and the length of the shorter leg. If the area of the triangle is 600, find the length of a radius of the circumscribed circle of the right triangle.

10. The perimeter of an equilateral triangle is $72\sqrt{3}$. Find the distance from a vertex of the triangle to the centroid of the triangle.

11. *A*, *B*, and *C* are collinear, *C*, *D*, and *E* are collinear, *E*, *F*, and *G* are collinear, *G*, *H*, and *A* are collinear, *B*, *J*, and *F* are collinear, *A*, *J*¸ and *E* are collinear, and *H*, *J*, and *D* are collinear. How many different paths are possible to travel from point *A* to point *E* moving only downward?



12. From a square, a triangle, a pentagon, and a hexagon, two polygons are selected at random without replacement. Find the probability neither polygon selected was a triangle. Express your answer as a common fraction reduced to lowest terms.

13. By how much does the length of a diagonal of a square whose perimeter is $8\sqrt{2}$ exceed the length of a diagonal of a square whose perimeter is $\sqrt{32}$?

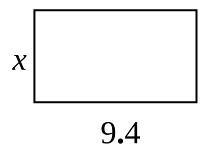
14. Three points $(9,-12)$, $(10,5)$, and $(0,9)$ lie on Circle P. An element is selected at random from the set $\{-11,-7,2\}$ and is called k; an element is selected at random from the set $\{-3,6,7,10\}$ and is called w. Find the probability that the point (k,w) lies in the interior of Circle P. Express your answer as a common fraction reduced to lowest terms.

15. Find the length of a radius of a circle inscribed in a triangle whose sides have lengths of 34, 34, and 32. Express your answer as a **decimal**.

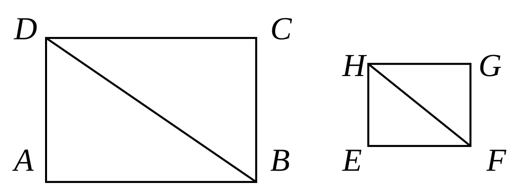
16. Let k represent a **positive integer**. If the length of a chord of a circle whose equation is $x^2+2.2x+y^2+6.8y=35.69$ is k, find the largest possible value of k.

17. A sphere with center at J is inscribed in a right circular cone with a peak (top vertex) at P. The sphere is tangent to the lateral surface of the right circular cone at E and F, and the sphere is also tangent to the base of the cone at O, the center of the base of the cone. P, J, E and F are coplanar. If $\angle EPF = 56°$ and the length of a radius of the base of the right circular cone is 16.78, find the volume of the sphere. Express your answer rounded to the nearest **whole number**.

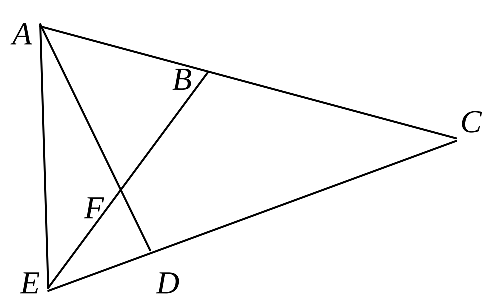
18. If the perimeter of the rectangle shown is 25.2, find the area of the rectangle. Express your answer as a **decimal**.



19. In the diagram, $ABCD$ and $EFGH$ are rectangles such that the lengths of all sides and diagonals are integers. The area of rectangle $ABCD$ is $\frac{7}{6}$ of the area of Rectangle $EFGH$. If $HF=41$, find the sum of all possible distinct lengths of $\overline{DB}$.



20. In the diagram, ΔAEC is acute, B lies on $\overline{AC}$, D lies on $\overline{EC}$, and $\overline{AD}$ and $\overline{BE}$ intersect at F. If $DC=5(DE)$ and $AB:BC=4:5$, then the ratio of the area of ΔABF to the area of quadrilateral $BCDF$ is $k:w$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.



2009 SA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 3060 (Degrees optional.)
2. 12.5 (Must be decimal answer.)
3. $(6,5)$ (Must be this ordered pair.)
4. 15.2 (Must be decimal answer.)
5. 16
6. B (Must be this capital letter.)
7. 56 (Degrees optional.)
8. $\frac{40}{3}$ (Must be this reduced improper fraction.)
9. 25
10. 24
11. 13
12. $\frac{1}{2}$ (Must be this reduced common fraction.)
13. 2
14. $\frac{5}{12}$ (Must be this reduced common fraction.)
15. 9.6 (Must be decimal answer.)
16. 13
17. 4293
18. 30.08 (Must be decimal answer.)
19. 66
20. 87

1. A set of pool balls bearing the whole numbers 1 through 15 inclusive is placed in a bag. Kay draws one of the pool balls at random. If the number is less than 10, she replaces it and draws again. If the number is greater than or equal to 10, she does **not** draw another ball. Find the probability that Kay did **not** need to draw a second ball. Express your answer as a common fraction reduced to lowest terms.

2. **(Always, Sometimes, or Never)** For your answer, write the _whole word_ **Always, Sometimes,** or **Never**—whichever is correct.

   If k represents a real number and m and n represent positive integers, then $(k^m)^n = k^{mn}$.

3. Find the exact length of the major axis of an ellipse whose equation is $16x^2 + 64y^2 = 2048$.

4. Let $C(n,k) = \dfrac{n!}{k!(n-k)!}$. Find the value of $C(10,5)$.

5. On each of Karen's ten tests, it is possible to score any of the 101 integral grades from 0 to 100 inclusive. On the first six of Karen's tests that Tom took, his average was 85. If x was Tom's average for the ten total tests, the set of possible averages for Tom for the ten tests was $\{x : k \le x \le w\}$. Find the value of $(k+w)$.

6. The fifth term of an arithmetic sequence is -18, and the ninth term is 4. Find the sum of the first thirty-two terms of this sequence.

7. Find the positive number that is **not** a member of the solution set of the inequality $9x^3 + 21x^2 - 17x + 3 > 0$.

8. If $x^2 - 6 + x^4$ is divided by $x + 2$, the quotient is $x^3 + kx^2 + wx + p$. Find the value of $(k + w + p)$.

9. A person writes down 5 different integers at random from the 25 integers from 1 to 25 inclusive. Each of the 25 integers is then called off one at a time in a random order. As soon as all 5 of the person's numbers have been called off, the person yells: "Bingo." Find the probability that the person will yell "Bingo" when the 16^{th} number is called. Express your answer as a **decimal** rounded to 4 significant digits.

10. Find the **sum** of all negative integers that are members of the solution set for x when $|-3x| < 18$.

11. Let y be a positive integer greater than 1, and let $0 < x < 1$. Find the value of x such that $(\log_4 y)(\log_y x) = \log_x 4$.

12. There is always a winner when there is a two-player match among Lee, Cindy, and Jeffrey. Lee beats Cindy $\frac{2}{3}$ of the time; Cindy beats Jeffrey $\frac{3}{4}$ of the time; Jeffrey beats Lee $\frac{4}{5}$ of the time. Lee plays Cindy first, and then the winner plays Jeffrey for the championship. Find the probability that Jeffrey will win the championship. Express your answer as a common fraction reduced to lowest terms.

13. Given the following system: $\begin{cases} x+y+z+w=14 \\ x^2+y^2+z^2+w^2=54 \\ x^3+y^3+z^3+w^3=224 \\ xyzw=120 \end{cases}$. If x, y, z, and w are positive integers and (x,y,z,w) is a member of the solution set for the given system, find the second largest possible value of $(6x+5y+4z+3w)$.

14. Bob's daughters—Marnie, Christie, and Katie—all own large ranches in the form of a rectangle with each ranch containing the same number of acres. The number of miles in the length of Marnie's ranch is 2 more than the number of miles in the width. The number of miles in the length of Christie's ranch is 28 more than the number of miles in the width. The number of miles in the length of Katie's ranch is 16 more than the number of miles in the width. All lengths and widths of each ranch are an integral number of miles. Find the number of square miles in each ranch.

15. If $g(x)=\dfrac{x}{x^3+4x^2-7x-10}$, find the **product** of all distinct real numbers that are **not** in the domain of $g(x)$.

16. If $f(x)=7x+9$ and $g(x)=x^2-3$, find $f(g(-11))$.

17. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

A four digit number of the form $abcd$ where a is the thousands digit, b is the hundreds digit, etc., is k times the four digit number of the form $bcda$ where b is the thousands digit, etc. If k is a member of the set $\{1.5, 2.5, 4.5\}$, for which value(s) of k is the problem as stated **not** possible?

A) 1.5 only.
B) 2.5 only.
C) 4.5 only.
D) 1.5 and 2.5 only.
E) 1.5 and 4.5 only.
F) 2.5 and 4.5 only.
G) All 3 values of k.

Be certain to write the correct capital letter as your answer.

18. The sum of three numbers in an increasing geometric progression is 258. If 75 were added to the second term of this geometric progression, then the three terms in the same order would form an arithmetic progression. Find the third term of this geometric progression.

19. If $f(x)=3(x-5)^3+2x^2+5(18-x)+3$, find the value of $f(7)$.

20. Let k be a positive integer such that $23<k<64$. Find the sum of all possible distinct values of k such that the product of all distinct positive integral divisors of k is k^2.

2009 SA
Algebra II

Name ANSWERS

School ______

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{5}$ (Must be this reduced common fraction.)
2. Always (Must be this whole word.)
3. $16\sqrt{2}$ (Must be this exact answer.)
4. 252
5. 142
6. 1448
7. $\frac{1}{3}$ or $0.\bar{3}$ or $.\bar{3}$
8. -7
9. 0.02569 or .02569 or 2.569×10^{-2} (Must be this decimal.)
10. -15
11. $\frac{1}{4}$ or 0.25 or .25
12. $\frac{37}{60}$ (Must be this reduced common fraction.)
13. 67
14. 960 (Square miles optional.0
15. 10
16. 835
17. D (Must be this capital letter.)
18. 216
19. 180
20. 561

1. If $\cos(\theta) = \frac{15}{17}$, find the smallest possible value of $\sin(\theta)$. Express your answer as a common fraction reduced to lowest terms.

2. Find the **sum** of the first five terms of a geometric sequence whose first term is 2 and whose second term is 6.

3. Find the value of p such that the three-dimensional vectors $\langle 1, -2, 2 \rangle$ and $\langle 2, 3, p \rangle$ are perpendicular.

4. The graph of $y = \frac{-5x + x^2}{2x^2 - 8}$ has a horizontal asymptote when $y = k$. Find the value of k.

5. Find the sum of the first 50 terms of the arithmetic progression: 2, 6, 10, $\cdots$.

6. Find the value of the indicated sum: $\sum_{1}^{4} 2^x$

7. A prone surveyor on a mountain peak observes below him on a horizontal water surface two vessels lying at anchor 1 mile apart and in the same vertical plane with his position. He finds the angles of depression of the ships to be 16° and 10° respectively. Find the number of **feet** in the vertical height of the mountain peak above the water. Express your answer as a whole number rounded to the nearest foot.

8. Let $G=\{a,b,c,d,e\}$. If $2a=5b=6c=8d=15e$, then the median of G is ka. Find the value of k. Express your answer as a common fraction reduced to lowest terms.

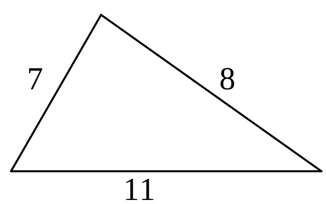
9. Jerry has 3 sticks, one of length 6, a second with a length less than 5, and a third with a length less than 4. Find the probability the 3 sticks can form a triangle. Express your answer as a common fraction reduced to lowest terms.

10. **(Always, Sometimes, or Never)** For your answer, write the <u>*whole word*</u> **Always, Sometimes, or Never**—whichever is correct.

If a function is continuous at $x=c$, then the function has an absolute maximum at $x=c$.

11. Find the sum of the infinite sequence: $\frac{1}{3},\frac{1}{3},\frac{5}{27},\frac{7}{81},\cdots,\frac{2n-1}{3^n},\cdots$.

12. The triangle (not necessarily drawn to scale) has sides of lengths as shown. Find the **exact** value of the tangent of the largest angle of the triangle.



13. Given the following seven points: $(8,2)$, $(10,-3)$, $(7,5)$, $(13,-4)$, $(-8,-5)$, $(-3,-2)$, $(-1,1)$. If three of these seven points are selected at random without replacement, find the probability that fewer than two of the points selected lie below the x-axis. Express your answer as a common fraction reduced to lowest terms.

14. Two parallel chords of a circle with lengths of 8 and 10 serve as bases of a trapezoid inscribed in the circle. If the length of a radius of the circle is 12, find the largest possible area of such a described inscribed trapezoid. Round your answer to the nearest whole number, and express your answer as that whole number.

15. In the vector sum shown, find the value of $(k+w)$: $\langle 2,3\rangle + \langle -7,8\rangle + \langle w,k\rangle + \langle 4,-31\rangle = \langle 17,6\rangle$.

16. Let k be a positive integer and let n be an integer such that every value for x of the form $(44+180n)°$ satisfies the equation $\sin(kx+2)° = \cos(3x)°$. Find the value of k.

17. Doc and Sandy are together at a point on an infinitely paved plane. At noon, Sandy heads directly east at a constant rate of 50 mph. Doc will also leave at noon and head in a direction of N5°E (5 degrees East of North) at a constant rate of 40 mph. If a random moment is picked between 9:00 and 10:00 in the evening of the same day that Sandy left, find the probability the two will be less than 600 miles apart. Express your answer as a **decimal** rounded to 4 significant digits.

18. Find the value of $\lim_{x\to 5}\left(\frac{x^2+6x-55}{x-5}\right)$.

19. If $(2x-y)^6$ is expanded and completely simplified, one of the terms is kx^3y^3. Find the value of k.

20. How many distinct strings of 5 letters (repetition permitted) of the English alphabet have exactly 3 distinct letters?

2009 SA

Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-\frac{8}{17}$ (Must be this negative reduced common fraction.)

2. 242

3. 2

4. $\frac{1}{2}$ or 0.5 or $.5$

5. 5000

6. 30

7. 2418 (Feet optional.)

8. $\frac{1}{3}$ (Must be this reduced common fraction.)

9. $\frac{9}{40}$ (Must be this reduced common fraction.)

10. Sometimes (Must be the whole word.)

11. 1

12. $-\sqrt{195}$ (Must be this exact radical.)

13. $\frac{13}{35}$ (Must be this reduced common fraction.)

14. 200

15. 44

16. 5

17. 0.7962 or $.7962$ or 7.962×10^{-1} (Must be this decimal.)

18. 16

19. -160

20. $390{,}000$

NO CALCULATORS

1. The equation of a line that is always 4 units from the line whose equation is $y=1$ can be written in the form $y=k$. Find the smallest possible value of k.

2. In the addition problem shown, let x and y stand for single digits. Find the value of y. .

$$\begin{array}{rrr} & 8 & x \\ + & x & 2 \\ \hline 1 & y & 7 \end{array}$$

3. A chord of a circle, the radius of the circle, and the distance from the center of the circle to the chord are all whole numbers in length. Find the smallest possible length of the chord.

4. Two bikers, A and B, are at opposite ends of a perfectly straight 70 km road. At the same time, A and B start riding toward each other. A rides at the constant speed of 5 kph. while B rides at the constant speed of 9 kph. A bug was on the handlebars of A when the two bikers started riding. The bug immediately starts flying at a constant speed of 17 kph. toward B. When it reaches B, the bug immediately turns around and heads toward A at the same constant speed of 17 kph. The bug continues this process until A and B meet. Find the number of kilometers that the bug travels.

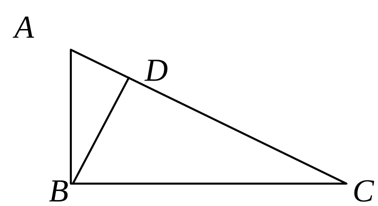
5. In which quadrant does the circle whose equation is $(x+5)^2+(y-7)^2=9$ lie? Express your answer as a **Roman numeral**.

6. The equation of a circle is $(x-5)^2+(y+3)^2=25$. The equation of the tangent to the circle at $(1,0)$ can be expressed in the form $y=mx+b$. Find the value of b. Express your answer as an improper fraction reduced to lowest terms.

7. **(Always, Sometimes, or Never)** For your answer, write *the whole* word **Always, Sometimes,** or **Never—**whichever is correct.

   If a square and a circle have equal perimeters, then the area of the square is more than the area of the circle.

8. In the diagram, $\overline{AB} \perp \overline{BC}$, $AB=6$, $BC=12$, and $\angle DBA=30°$. If D lies on $\overline{AC}$, then, expressed in simplest radical form, $BD=\dfrac{k\sqrt{w}-f}{p}$ where k, w, f, and p are positive integers. Find the smallest possible value of $(k+w+f+p)$.



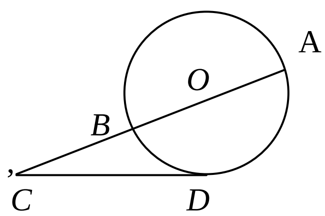
NO CALCULATORS

NO CALCULATORS

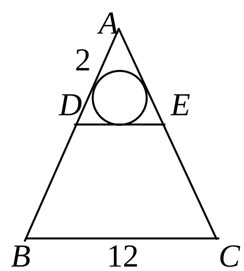
9. By how much does the volume of a 3 by 4 by 6 rectangular solid exceed the volume of a 5 by 2 by 3 rectangular solid?

10. Let $x > 0$, $y > 0$, and $z > 0$. If $xy = 8$, $yz = 25$, and $xz = 32$, find the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$.
Express your answer as a common fraction reduced to lowest terms.

11. In the diagram, A, B, and D lie on circle O, $\overline{CD}$ is tangent to the circle at D, and C, B, O, and A are collinear. If $\overline{AB}$ is a diameter, $CD = 40$, and $CB = 32$, find the perimeter of ΔCOD.



12. In the diagram, the circle is inscribed in ΔADE, $AD = 2$, $AB = 8$, $BC = 12$, $\overline{DE} \parallel \overline{BC}$, and $\overline{AB} \cong \overline{AC}$. The area of the circle shown can be expressed in the form $\frac{k\pi}{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$. (Note: A, D, B are collinear, and A, E, C are collinear.)



13. A, B, C, and D are four people who live in a land where each person always makes true statements or always makes false statements. They state:
A: C is a truth-teller.
B: C and D are opposite types of people.
C: A and B are opposite types of people.
D: At least one of A and B is a liar.
Using T for truth teller and L for liar, write the ordered quadruple that describes (A, B, C, D).
For example, if A, B, and C are truth tellers and D is a liar, your ordered quadruple would be: (T, T, T, L).

14. May 13, 2009 is a Wednesday. Assuming that the way we determine our calendar does not change, on what day of the week will May 13, 2137 occur? Hint: Are you certain you know when leap years occur? For your answer, write the <u>*whole word*</u> for the day of the week.

NO CALCULATORS

15. Zeke is standing on a flat, horizontal plane and is 5 feet away from a concession stand. He walks away at a steady pace. After 7.5 seconds, Zeke is 35 feet away from the concession stand. The linear equation that indicates Zeke's distance in feet, y, from the stand as a function of time elapsed in seconds, x, can be expressed in the form $y = mx + b$. Find the value of $(m+b)$.

16. When written in simplest radical form, $\frac{1}{\sqrt{3}+\sqrt{6}-\sqrt{2}} = \frac{a+b\sqrt{c}-\sqrt{d}-7\sqrt{2}}{e}$ where a, b, c, d, and e are positive integers. Find the value of $(a+b+c+d+e)$.

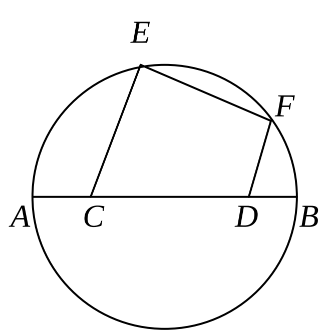
17. **(Always, Sometimes, or Never)** For your answer, write the <u>*whole word*</u> **Always, Sometimes,** or **Never**—whichever is correct.

    The circumcenter of an acute triangle lies on the longest side of the acute triangle.

18. From 68 scalene triangles, 140 triangles that are isosceles but not equilateral, and x equilateral triangles, one triangle is selected at random. If the probability that an equilateral triangle was selected is $\frac{2}{15}$, find the value of x.

19. $(1,4)$, $(1,3,1)$, and $(3,1,1)$ are 3 examples of **distinct ordered groups** of **positive** integers for which the sum of the members of each ordered group is 5. Find the number of **distinct ordered groups** of **positive** integers for which the sum of the members of each ordered group is 6. Assume that each ordered group must contain at least two members.

20. In the diagram, points A, E, F, and B are distinct points that lie on the circle. $\overline{AB}$ is a diameter, $\angle CEF$ and $\angle DFE$ are right angles, and points A, C, D, and B are collinear in that order. $AB = 200$, and $CE = 113$. Find the sum of all distinct integer values for the length of $\overline{EF}$ such that the length of $\overline{DF}$ is an **odd integer**.



NO CALCULATORS

2009 SA

School ________ANSWERS__________

Fr/So 8 Person

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -3
2. 3
3. 6
4. 85 **(km. optional.)**
5. II **(Must be this Roman Numeral, quadrant optional.)**
6. $-\frac{4}{3}$ **(Must be this reduced improper fraction.)**
7. Never **(Must be the whole word.)**
8. 86
9. 42
10. $\frac{13}{16}$ **(Must be this reduce d common fraction.)**
11. 90
12. 37
13. (T, L, T, T) **(Must be this ordered quadruple.)**
14. Monday **(Must be the whole word.)**
15. 9
16. 49
17. Never **(Must be the whole word.)**
18. 32
19. 31
20. 336

NO CALCULATORS

1. $\left[x\right]$ represents the greatest integer function. Find the value of the expression $\left[\log_3 712\right]$.

2. Find the value of the determinant: $\begin{vmatrix} 6 & 5 & -1 \\ 2 & 8 & 3 \\ -7 & 0 & 4 \end{vmatrix}$.

3. When $x^{12}-1$ is factored completely with respect to the integers, how many of the factors are trinomials?

4. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   For all values of x where $\sin(x)$, $\cos(x)$, and $\tan(x)$ are all defined, $\left(\sin(x)\right)\left(\cos(x)\right)\left(\tan(x)\right)=$

   A) $\sin^2(x)$
   B) $\cos^2(x)$
   C) $\left(\sin(x)\right)\left(\cos(x)\right)$
   D) $\tan^2(x)$
   E) 1
   F) -1
   G) 0

   Note: Be certain to write the correct capital letter as your answer.

5. An ellipse has its minor axis lying on the *y*-axis, has foci at $(3,0)$ and $(-3,0)$, and has an *x*-intercept of 7. The equation of this ellipse can be expressed in the form $\frac{x^2}{k}+\frac{y^2}{w}=1$. Find the value of $(2k+3w)$.

6. Let $i=\sqrt{-1}$. Expressed in trigonometric form, one of the cube roots of $4-4\sqrt{3}\,i$ is $2cis\left(k°\right)$ where $170° < k < 290^\circ$. Find the value of k.

7. In a plane, a segment 90 units in length is located in such a way that one of its endpoints lies on the *y*-axis and the other endpoint lies on the line $y=3$. Let $(x,2y+1)$ represent the trisection point on this line segment that lies closer to the endpoint on the *y*-axis. Then $x^2+kx+y^2+wy=p$. Find the value of $(k+w+p)$.

8. If $\lim_{x\to 2} f(x)=4$ and $\lim_{x\to 2} g(x)=9$, find $\lim_{x\to 2}\left(2f(x)+\pi g(x)\right)$.

NO CALCULATORS

NO CALCULATORS

9. An ellipse has the equation $9x^2+4y^2+72x+8y=77$. The least distance between a vertex and a focus, written as a single reduced fraction, is of the form $\frac{a+b\sqrt{c}}{d}$. Find the sum $\left(a+b+c+d\right)$.

10. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

Which graph(s) has (have) an amplitude of 3?

I) $y=-3\cos(3\theta)$
II) $y=3\sin(\theta)$
III) $y=\sin(3\theta)$

A) I
B) II
C) III
D) I and II
E) I and III
F) II and III
G) I, II and III

Note: Be certain to write the correct capital letter as your answer.

11. The explicit value for the sum: $3^2+3^5+3^8+\cdots+3^{(3n-1)}+\cdots+3^{2003}$ can be written in the form $\frac{3^a(3^b-1)}{c}$ where a, b, and c are positive integers. Find the smallest possible value of $\left(a+b+c\right)$.

12. If $\cos^2 75°$ is a root for x of $64x^2+kx+w=0$ where k and w are integers, find the value of $\left(k+w\right)$.

13. It is known that y varies directly as the square of x and inversely as the cube of r. If $y=\frac{40}{9}$ when $x=4$ and when $r=3$, find the constant of proportionality. Express your answer as an improper fraction reduced to lowest terms.

NO CALCULATORS

NO CALCULATORS

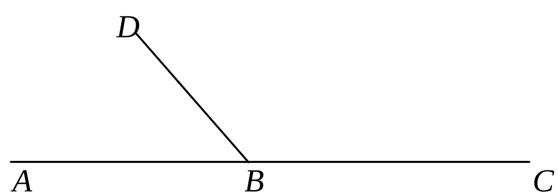
14. Karen prepares for her math exam by studying, praying, or bowling. (She does only **one** of these per exam.) Six out of ten times she studies in preparation, three out of ten times she prays, and she bowls one out of ten times. She has a $\frac{4}{5}$ probability of passing the exam when she studies, a $\frac{1}{2}$ probability of passing if she prays, and a $\frac{1}{5}$ probability of passing when she bowls. If Karen fails this exam, find the probability that she studied in preparation. Express your answer as a common fraction reduced to lowest terms.

15. The vector $(5,10)$ is perpendicular to the vector (k,w), and the vector (k,w) is parallel to the vector $(p,-36)$. Find the value of $\left(\frac{k}{w}+p\right)$.

16. Line ℓ passes through the point represented by $(6,-19)$ and is parallel to the line passing through points represented by $(12,18)$ and $(15,39)$. Line ℓ can be represented as $\{(6+t,-19+kt)\}$ Find the value of k.

17. Let a, b, and c each represent a single non-zero digit. Find the number of distinct **ordered triples** of the form (a,b,c) for which $1<\frac{abc}{a+b+c}<2$.

18. In the diagram, A, B, and C are collinear, and $\angle ABD$ is acute.



$\frac{\sin(\angle CBD)}{\sin(\angle ABD)}=k$. Find the value of k.

19. Let $A=\{1,3,5,7\}$. Let $B=\{1,3,5,y\}$. There are 2 different values for y such that the total population standard deviation (σ_x) of A is equal to the sample population standard deviation (S_x) of B. The larger of these two values for y, expressed in simplest radical form, is $\frac{k+w\sqrt{p}}{f}$ where k, w, p, and f are positive integers. Find the value of $(k+w+p+f)$.

20. Let k be a positive integer such that $k<25$. Find the sum of all possible distinct values of k for which $(k+1)!+(k+2)!+(k+3)!$ is an integral multiple of 1183.

NO CALCULATORS

2009 RA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 5
2. –9
3. 3
4. A (Must be this capital letter.)
5. 218
6. 220 (Degrees optional.)
7. 897
8. $8+9\pi$ or $9\pi+8$
9. 17
10. D (Must be this capital letter.)
11. 2032
12. –60
13. $\frac{15}{2}$ (Must be this reduced improper fraction.)
14. $\frac{12}{35}$ (Must be this reduced common fraction.)
15. 70
16. 7
17. 70
18. 1
19. 35
20. 33

1. The lengths of two of the sides of a right triangle are 39 and 80. The length of the third side is also an integer. Find that integer. Express your answer as an **exact integer**. Do **not** use scientific notation.

2. Instruments on a lunar probe rocket measure the amount of power available to maintain operation. The function $P(t) = 50e^{\left(\frac{-t}{300}\right)}$ gives the amount of power available in **days** (24-hour periods). The probe has 100% power at the instant of launch. How long, calculated to the nearest second after launch, will the critical measure of 50% power be reached? Give your answer as an ordered quadruple of integers, correctly rounded, in the form $(\text{days,hours,minutes,seconds})$.

3. How many fluid ounces are equivalent to 27.83 quarts?

4. Find the length of a radius of a circle whose center is at the origin if the point $(87.41, 96.13)$ lies on the circle.

5. A sidewalk of uniform width x feet surrounds a rectangular garden with dimensions of 200 feet by 80 feet. If the area of the sidewalk is 1600 square feet, find the value of x.

6. The area of a big circle is five times the area of a small circle. The **diameter** of the big circle is k times the **radius** of the small circle. Find the value of k.

7. In this problem, assume that the standard deviation is calculated according to the standard method of calculating the standard deviation for a set of sample proportions. Also, assume the following table of *z*-scores with the accompanying standard normal probabilities is accurate. On the table – 2(.0228) means that there is a normal probability of 0.0228 of obtaining a *z*-score of -2 or less.

– 2(.0228)	– 1.9(.0287)	– 1.8(.0359)	– 1.7(.0446)	– 1.6(.0548)
– 1.5(.0668)	– 1.45(.0735)	– 1.4(.0808)	–1.35(.0885)	– 1.3(.0968)
– 1.25(.1056)	– 1.2(.1151)	– 1.1(.1357)	–1(.1587)	– 0.9(.1841)
– 0.8(.2119)	– 0.75(.2266)	– 0.7(.2420)	– 0.6(.2743)	– 0.5(.3085)
– 0.4(.3446)	– 0.3(.3821)	– 0.25(.4013)	– 0.1(.4602)	0(.5000)
0.1(.5398)	0.2(.5793)	0.25(.5987)	0.3(.6179)	0.4(.6554)
0.5(.6915)	0.6(.7257)	0.7(.7580)	0.75(.7734)	0.8(.7881)
0.9(.8159)	1.0(.8413)	1.1(.8643)	1.2(.8849)	1.25(.8944)
1.3(.9032)	1.4(.9192)	1.5(.9332)	1.6(.9452)	1.7(.9554)
1.75(.9599)	1.8(.9641)	1.9(.9713)	2.0(.9772)	2.1(.9821)
2.2(.9861)	2.25(.9878)	2.3(.9893)	2.4(.9918)	2.5(.9938)

Professor Mortgalis announces to his class that anyone who receives a *z*-score whose absolute value is less than or equal to x will receive a grade of B on the exam. Assume the scores of the students on the exam will be normally distributed. To the nearest tenth, find the smallest possible value of x for which more than 50% of the students will receive a grade of B on the exam. Express your answer as a **decimal**.

8. On a plane surface, two circles with radii of lengths 12 and 9 are x units apart where x is a positive integer and $x < 29$. If the length of a common external tangent segment of the two circles is $\sqrt{w}$ where w is an integral multiple of 11, find the sum of all possible distinct values of x. Express your answer as an exact **integer.**

9. Find the sum of the solution(s) *in radians* for x in the equation $2\cos(2x) = 0.7x$.

10. If $\begin{vmatrix} 1 & x & -1 \\ 2 & y & 3 \\ 3 & z & 1 \end{vmatrix} = 27.85$, find the value of $40.852x + 23.344y - 29.18z$.

11. Ten fair coins are tossed. Find the probability that at least 7 coins land heads-up. Express your answer as a **decimal**.

12. On May 1, Tom bought a used automobile for $8000 for which he took out a loan at 7% annual percentage rate with monthly compounding. At the end of May and at the end of each succeeding month, Tom will make a payment of $247.02. After he has made the twentieth payment, how much will he still owe on the principal? Round your answer to the nearest dollar and express your answer as an integer. Do **not** use scientific notation.

13. According to Wikipedia, rifle target shooters use the term "click" to represent "one minute of an arc" in their rifle sighting system. Thus, one click moves the projectile impact point *approximately* 1 inch left or right when aimed at a perpendicular target at the distance of 100 yards. What is the actual distance, in inches, that one "click" moves the impact point on the target horizontally left or right?

14. In ΔABC, a, b, and c are the sides opposite the respective angles $\angle A$, $\angle B$, and $\angle C$. $b = 4$, $c = 6$ and $m\angle B = 20°$. Find the sum of all possible length(s) for side a.

15. An observer on a tower that rises vertically notes that two objects on a horizontal road below have respective angles of depression of 17° and 8° respectively. If the eye of the observer is 100 feet above the horizontal road and if the road runs directly away from the observer, find the number of feet in the distance between the objects.

16. Find the value of $\sin(1°)+\sin(2°)+\cdots+\sin(n°)+\cdots+\sin(46°)$.

17. The three points $(14.26,16.84)$, $(0.000,12.92)$, and $(-5.868,10.24)$ lie on a circle. Find the length of a radius of this circle.

18. If $x^7+x^6-2.112x^5+1.021x^4-x^3+x^2+4.113x-7.118$ is divided by $(x-1.114)$, find the numerical remainder.

19. A soda pop can in the shape of a right circular cylinder is to hold exactly 230 cubic units. Find the length of the height of this right circular cylindrical can that will yield a can of minimal total surface area.

20. The three points $(7,1)$, $(17,3)$, and $(112,8)$ lie on a parabola whose axis of symmetry is parallel to the x-axis. Find the absolute value of the shortest possible total distance from $(3,17)$ to a point on the parabola and then from that point on the parabola to the focus of the parabola.

2009 SA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 89 (Must be this integer, no scientific notation.)

2. $(207, 22, 39, 35)$ (Must be this ordered quadruple of integers.)

3. 890.6 or 8.906×10^{2} (Ounces optional.)

4. 129.9 or 1.299×10^{2}

5. 2.801 or 2.801×10^{0} (Feet optional.)

6. 4.472 or 4.472×10^{0}

7. 0.7 or $.7$ or 7.000×10^{-1} (Must be decimal answer, trailing zeroes necessary in sci. not.)

8. 74 (Must be this integer.)

9. -2.263 or -2.263×10^{0}

10. 162.5 or 1.625×10^{2}

11. 0.1719 or $.1719$ or 1.719×10^{-1}

12. 3763 (Must be this integer, no scientific notation, \$ optional.)

13. 1.047 or 1.047×10^{0} (Inches optional.)

14. 11.28 or 1.128×10 or 1.128×10^{1}

15. 348.5 or 3.845×10^{2} (Feet optional.)

16. 17.85 or 1.785×10 or 1.785×10^{1}

17. 66.41 or 6.641×10 or 6.641×10^{1}

18. -0.6883 or $-.6883$ or -6.883×10^{-1}

19. 6.641 or 6.641×10^{0}

20. 16.74 or 1.674×10 or 1.674×10^{1}

1. How many of the first 250 members of the Fibonacci sequence: $1,1,2,3,5,8,13,21,34,55,\cdots$ are even integers?

2. The prime factorization of 80000 is $a^b c^d$ where a, b, c, and d are positive integers. Let k be the absolute value of the difference between the two roots of the quadratic equation $4x^2+28x-95=0$. Find the value of $(b+d+k)$.

3. Let k be the greatest common factor of 860 and 2107. Let w be the length of the altitude to the hypotenuse of a right triangle whose legs have length of 45 and 60. Find the value of $(k+w)$.

4. Let k be the absolute value of the difference between the solutions of $\frac{x}{\sqrt{2x-12}}=4$. Let w be the length of a radius of a circle inscribed in a triangle whose sides have lengths of 6, 8, and 10. Find the value of $(k+w)$.

5. Let $ABCD$ be a rectangle with $AB=28$ and $BC=45$. Let p be the perimeter of the quadrilateral that joins the midpoints of the sides, taken in order, of the rectangle. Let E be the midpoint of $\overline{AB}$. Find the exact value of $(p+EC)$.

6. One of the edges of a rectangular solid has a length of 2. The total surface area of the rectangular solid is 356. If the length of each edge of the solid is a whole number, find the sum of all possible distinct volumes of the rectangular solid.

7. Bob rides his motorcycle at a constant rate of w mph. for 24 miles and then at a constant rate of $4w$ mph. for 48 miles and his average rate for the 72 miles is 12 mph. Let k be the area of a triangle whose vertices are at $(-8,0)$, $(2,0)$, and $(0,4)$. Find the value of $(k+w)$.

8. Define a prime decade as a sequence of ten consecutive integers starting with a positive multiple of 10 that contains exactly four numbers that are primes. The smallest prime decade starts with the integer 10 and contains the primes 11, 13, 17, and 19. Find the **sum of the primes** in the next smallest prime decade.

9. The measures of the exterior angles of a triangle are in the ratio of $5:6:7$. The measures of the corresponding interior angles of this triangle are in the ratio of $a:b:c$ where a, b, and c are positive integers. Find the smallest possible value of $(2a+3b+4c)$.

10. For how many distinct groups of consecutive odd positive integers will the sum of its distinct members be 288?

2009 SA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer	Score (to be filled in by proctor)
1.	83	
2.	23	
3.	79	
4.	18	
5.	$106+\sqrt{2221}$	
6.	504	
7.	26	
8.	420	
9.	25	
10.	6	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 512
12. 51.414 or 5.1414×10^{1}
13. 24
14. 804
15. 42

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Given the following four quadratic equations: $x^2 = 4$, $x^2 - 7x + 10 = 0$, $(x-5)^2 = 0$, and $2x^2 + 7x - 4 = 0$. If one of these equations is selected at random, find the probability that the solution set for that equation would consist of two **distinct** integers. Express your answer as a common fraction reduced to lowest terms.

2. Given the sequence: $12^{35}, 12^{34}, 12^{33}, 12^{32}, 12^{31}, \cdots$. If the product of the terms of the sequence must exceed 144^{149}, find the least number of terms in the sequence.

3. Let $[x]$ represent the greatest integer function (also know as integer floor function.) If $S = \left[r + \frac{1}{50}\right] + \left[r + \frac{2}{50}\right] + \cdots + \left[r + \frac{50}{50}\right] = 38$, then the solution interval for r is $r \in [k, w)$. Find $(k + w)$ as an exact reduced proper or improper fraction.

4. Let N be the greatest common divisor of 14,040 and 202,800. Let M be the greatest common divisor of 140,448 and 428,868. Find the least common integral multiple of N and M. Express your answer as an **exact integer**. Do **not** use scientific notation.

5. Let $i = \sqrt{-1}$. If x and y are real numbers such that $x + y + 1 + xi - yi + 3i = 1 + 7i$, find the **ordered pair** (x, y).

6. When polynomial $P(x)$ is divided by $(x-2)$, the quotient is $Q(x)$ and the remainder is 5. When $Q(x)$ is divided by $(x-4)$, the remainder is 7. Let k be the remainder when $P(x)$ is divided by $(x-4)$. Isosceles ΔABC with $\overline{AB} \cong \overline{AC}$ has vertex A corresponding to the complex number $25(\cos 40° + i \sin 40°)$, B at the origin, and C on the positive x-axis. The complex number corresponding to C is $a + bi$ in standard form. Find $(a + b - k)$ correct to four significant digits.

7. Let p be the perimeter of a **right** triangle whose area is 1620 and which has an angle whose cosecant is 1.025. Let k be the smallest positive integer that has exactly 36 distinct positive integral divisors. Find the value of $(p + k)$.

8. Let k be the remainder when $3x^{23} - 14x^{14} - 12x^3 - 7$ is divided by $(x+1)$. Let w be the greatest integral value of x such that $\sum_{n=0}^{\infty} \frac{(4x+14)^n}{(10)^{(n+1)}}$ is a convergent sequence (has a finite sum). Find the sum $(k + w)$.

9. The three vertices of a triangle are $(4,7)$, $(8,13)$, and $(0,7)$. Find the least possible sum of the squares of the distances of a point in the plane to the three vertices of the triangle.

10. Let $A=\{3,7,15,31,\cdots,2^{(n+1)}-1,\cdots,524287\}$. Let $B=\{3,5,9,17,\cdots,2^{n}+1,\cdots,262145\}$. Each member of A is divided into each member of B. Find the sum of all distinct integral values obtained by these divisions.

2009 SA

Jr/Sr 2 Person Team

School ________ANSWERS______

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. $\frac{1}{2}$ **(Must be this reduced common fraction.)**	________
2. 10	________
3. $\frac{3}{2}$ **(Must be this reduced improper fraction.)**	________
4. 326,040 **(Must be this integer.)**	________
5. $(2,-2)$ **(Must be this ordered pair.)**	________
6. 19.30 or 1.930×10 or 1.930×10^1 **(Trailing zero necessary.)**	________
7. 1530	________
8. –14	________
9. 56	________
10. 58,257	________

TOTAL SCORE: ________

(*enter in box above)

Extra Questions:

11. 9.5 **(Must be decimal answer.)**

12. –5

13. $(0.445,-1.95)$ or $(.445,-1.95)$ **(Must be this ordered pr.)**

14. –28

15. 10

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

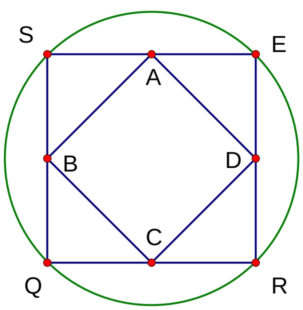
FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 1

ICTM 2009 DIVISION A STATE FINALS

1. $\frac{12x^6}{9x} \div \frac{36x^3}{81x^2} = kx^w$. Find the sum $(k+w)$.

2. Christie bikes 10 miles from home before a broken chain forces her to stop and walk home on the same route she biked. She travels at the two constant rates; her total time is 4 hours. If she bikes ANS times as fast as she walks, how fast does she bike (in mph)?

3. Square $SQRE$ is inscribed in the circle shown. A, B, C, and D are midpoints of consecutive sides of $SQRE$. If $AB = ANS$, the area of the circle shown is $k\pi$. Find the value of k.



4. Consider the earth a sphere with diameter 8000 miles. Skylab is ANS miles above the earth and an Astronaut on Skylab views Houston just as it comes over the horizon. How far from Houston is the Astronaut? Answer as a whole number rounded to the nearest mile.

ANSWERS:

1. 7

2. 20 (mph optional.)

3. 400

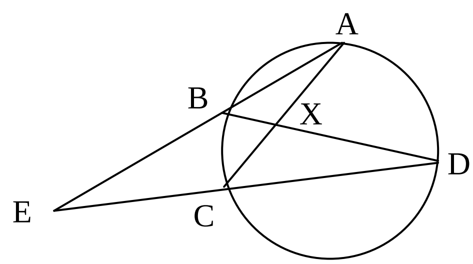
4. 1833 (miles optional)

1. Shana has an average of 88% on the first four exams in her class. She must score $k\%$ on the fifth exam to raise her average to for the five exams to 90%? Find the value of k. **Do NOT attach the % sign**.

2. The product of 2 consecutive odd integers added to five times the smaller odd integer is ANS. Find the larger of the 2 consecutive odd integers.

3. If 4 times the measure of the supplement of an angle is $30°$ more than ANS times the measure of the complement of an angle, find the measure of the supplement of the angle.

4. In Circle O (not drawn to scale) $m\overset{\frown}{AD} = ANS°$ and $m\angle E = 20°$. Find $m\angle AXB$.



ANSWERS:

1. 98

2. 9

3. 156 (Degrees optional)

4. 44 (Degrees optional)

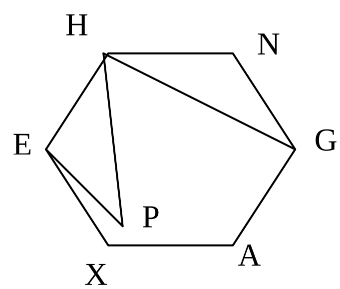
FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 3

ICTM 2009 DIVISION A STATE FINALS

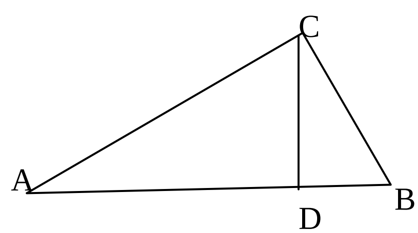
1. Let $f(x) = 3.7x - 4.2$ and $g(x) = 5.7x + 1.9$. Find $g(4.4) - f(5.5)$. Report your answer as an exact decimal.

2. The median of a set of numbers is the number that is numerically in the middle. For example, the median of the set $\{0, 5, 2, 6.4, -8, 9, 1.3\}$ is 2 because it is numerically in the middle (three numbers are smaller and three numbers are larger). Let $R = \{7.50, 9.75, 11.00, 12.27, ANS\}$ be a set of hourly wages with wages shown in dollars. By how much does the median hourly wage exceed the average hourly wage in Set R? Report your answer as a whole number of **cents**.

3. $HEXAGN$ is a regular hexagon with $\overline{HG}$ a diagonal and P a point in the interior. If $m\angle XEP = 20°$ and $m\angle EPH = ANS°$, find $m\angle GHP$.



4. ΔABC is a right triangle with right angle at C. $\overline{CD}$ is the altitude to $\overline{AB}$. $CB : CA = 11 : 60$. If the area of ΔCDB is ANS, find the area of ΔABC. Write your answer as a reduced improper fraction reduced to lowest terms.



ANSWERS:

1. 10.83 (Must be this exact decimal.)

2. 56 (Cents optional.)

3. 66 (Degrees optional.)

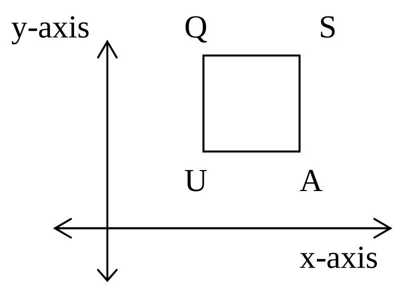
4. $\frac{22326}{11}$ (Must be this reduced improper fraction.

1. Ryan has only dimes and quarters in his piggy bank. If the ratio of dimes to quarters is $3:2$ and the total value in the piggy bank is $3.20, how many quarters does Ryan have?

2. $25^x \bullet 5^{(4-x)} = 125^{ANS}$. Solve for x.

3. Let $k = ANS$. Two triangles are similar. The larger triangle has an area of 484; the smaller triangle has an area of k^2. Find the ratio of the length of a side of the smaller triangle to the length of a corresponding side of the larger triangle. Express your ratio as a common fraction reduced to lowest terms.

4. ANS should be a common fraction $\frac{a}{b}$ reduced to lowest terms. Let $k = \frac{1}{2}a$. Square $SQUA$ lies entirely in the first quadrant with coordinates $U(5,k)$, $A(b,5)$, and $Q(k,b)$. Q' is the reflection of Q over the y-axis and A' is the reflection of A over the x-axis. Find the area of the quadrilateral $Q'QAA'$.



ANSWERS:

1. 8 (Quarters optional.)
2. 20
3. $\frac{10}{11}$ (Must be this reduced common fraction.)
4. 110

1. Tickets to a water park cost \$7.50 for residents and \$11.50 for non-residents. If 650 people attend one day and the total amount collected is \$5715.00, then how many non-residents attended that day?

2. A computer printer can print ANS lines per second. How many **minutes** will it take the printer to print 1,000,000 lines? Give your answer as an improper fraction reduced to lowest terms.

3. A circle contains a sector subtended by a $140°$ central angle and the sector has area of $ANS\pi$. Find the exact radius of the circle. Write you answer as an improper fraction reduced to lowest terms.

4. ANS should be in the form $\frac{a}{b}$. Find the area of an obtuse triangle where 2 sides have length a and b and the included angle is $150°$.

ANSWERS:

1. 210 (Non-residents optional.)

2. $\frac{5000}{63}$ (Must be this reduced improper fraction, minutes optional.)

3. $\frac{100}{7}$ (Must be this reduced improper fraction.)

4. 175

JUNIOR-SENIOR RELAY COMPETITION ROUND 1

ICTM 2009 DIVISION A STATE FINALS

1. If $5^{(5x-3)} = 25^{(2x+3)}$, find the value of x.

2. Find the value of the expression $-2(7-3x)-[8x-(3-x)]$ when $x = ANS$.

3. Solve for k if $\begin{vmatrix} 1 & 0 & -10 \\ k & 0 & 0 \\ 0 & 12 & 0 \end{vmatrix} = ANS$. Write your answer as a common fraction reduced to lowest terms.

4. ANS should be in the form $\frac{k}{w}$. $f(x) = ax^2 + bx + c$ is a quadratic function with a zero at $x = 2$, and the graph of $y = f(x)$ contains the points $(4, k)$ and $(5, w)$. Find the sum $(a+b+c)$.

ANSWERS:

1. 9

2. -38

3. $\frac{19}{60}$ (Must be this reduced proper fraction.)

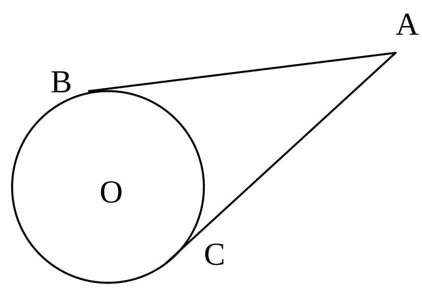
4. 22

JUNIOR-SENIOR RELAY COMPETITION ROUND 2

ICTM 2009 DIVISION A STATE FINALS

1. Set $C = \{x\}$. Set C has 2 subsets: $\varnothing$, $\{x\}$.
   Set $D = \{2,3\}$. Set D has 4 subsets: $\varnothing, \{2\}, \{3\}, \{2,3\}$.
   Set $E = \{a,b,c\}$. Set E has 8 subsets: $\varnothing, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$.
   Set A has 96 more subsets than Set B. How many **more** elements are in Set A than in Set B?

2. Given Circle O with $\overline{AB}$ and $\overline{AC}$ tangent segments to the circle. $AB = 2x - 3y$ and $AC = 8 - xy$. Find the value of x when $y = ANS$. Write your answer as an improper fraction reduced to lowest terms.



3. Let $k = ANS$. $f(x)$ is a function such that $f(kx) = 2x + 1$ and $f(t) = 25$. Find the value of t.

4. Let e be the base for the natural logarithms $(\ln)$. The exponential equation $e^{2x} = ANS - e^{x}$ has an exact solution of the form $x = \ln k$. Find the value of k.

ANSWERS:

1. 2

2. $\frac{7}{2}$ (Must be this reduced improper fraction.)

3. 42

4. 6

1. If x represents a real number, find the least possible value for the sum $|x-3|+|x|+|x+3|$.

2. Let $k = 12(ANS)$. Find the sum of the roots for x of the fifth degree equation $(x-2400)(x-k)(x^3-28x^2+77x-50)=0$

3. An investor has $10,000 to invest. He invests half in a guaranteed deposit that will earn 5% APR. The other half he invests in a speculative but potentially high-yield land deal. The investor's goal is to receive ANS dollars at the end of the first year in total interest from the two halves of his $10,000 invested. What rate of return must the investor receive on the land deal in order to achieve this goal? The answer is $k\%$ APR where k is a whole number. Write the value of k on your answer sheet. **Do not include "%" in your answer.**

4. $\log_2 3 = M$, $\log_2 5 = N$, $\log_2 7 = P$ and R is a real number constant. Let $k = ANS$. $\log_2\left(\frac{70k}{8}\right)$ can be written in the form $aM + bN + cP + R$. Find the sum $(a+b+c+R)$.

ANSWERS:

1. 6

2. 2500

3. 45 (Must not have "%" sign.)

4. 3

JUNIOR-SENIOR RELAY COMPETITION ROUND 4

ICTM 2009 DIVISION A STATE FINALS

1. $P(x)$ is a polynomial in x. If $\left(3x^4 - 7x^3 + 5x^2 - 4x + 2\right) - P(x) = x^4 + 6x^3 + x^2 - 5$, what is the coefficient of the degree 3 term of $P(x)$?

2. Let r and t represent the solutions to $x^2 - 5x + ANS = 0$. Find the value of $\frac{2}{rt^2} + \frac{2}{r^2 t}$. Write your answer as a common fraction reduced to lowest terms.

3. Find the value of the eccentricity of the conic $(ANS)x^2 - \frac{10}{27}y^2 = 1$. Write your answer as a common fraction reduced to lowest terms.

4. ANS should be an improper fraction reduced to lowest terms in the form $\frac{k}{w}$. Let $p = k + w - 1$. Let $f(x) = 3 + 7\tan(13x + p)$. Find the negative value for x closest to zero for which $f(x) = 3$. Write your answer as a decimal correct to **4 decimal places.**

ANSWERS:

1. -13

2. $\frac{10}{169}$ (Must be this reduced common fraction.)

3. $\frac{14}{13}$ (Must be this reduced improper fraction.)

4. -0.0667 or $-.0667$ (Must be this exact decimal.)

1. How many positive integers k leave a remainder of 5 when 65 is divided by that integer k?

2. Let $k = ANS + 24$. Find the sum of the first k terms of the increasing sequence $1, 6, 11, 16, 21, 26, \cdots$ where each term is one more than a multiple of 5.

3. a and b are integers in polynomial $P(x)$. Applying the Rational Root Theorem, $P(x) = 19x^7 + 8x^6 + ax^5 - bx^3 + 3x^2 + 2x + ANS$ has *potential* rational roots (which may or may not be actual roots.) Find the sum of the positive *potential* rational roots that are integers.

4. Let $p = \frac{ANS}{28}$. When written in completely simplified standard form and in decreasing powers of x, the fourth term of $\left(\frac{4}{3}x + ky\right)^{10}$ has numeric coefficient p. Find the exact value of k. Write your answer as a common fraction reduced to lowest terms.

ANSWERS:

1. 7

2. 2356

3. 4480

4. $\frac{9}{16}$ (Must be this simplified common fraction.)

ICTM State Contest 2009
Voting Theory Questions

1. The AP Top 25 College Men's Basketball poll is a ranking system that assigns total points based on 25 points for a first-place vote, 24 points for a second-place vote, and so on down through one point for a 25th place vote. The voters' points are summed to determine a team's ranking for the week.

In the AP ranking based on team records through February 15, Oklahoma, with four first place votes and 1709 total points, was ranked second among the top 25 teams.

a. If the total number of points to be distributed was 23,400, how many voters were there?

b. If no voters ranked Oklahoma lower than 3rd place, how many second place votes and how many third place votes did Oklahoma receive for the week ending February 15?

2. Voters A, B, C, D, and E use the weighted voting system [8**:** 5, 3, 1, 1, 1].

a. What are the minimal winning coalitions in this system?

b. Does any voter in this system have veto power? If so, who?

c. Calculate the Banzhaf index for this voting system.

d. Under the Banzhaf model, what percent of the voting power does voter B have?

3. Consider again the weighted voting system [8**:** 5, 3, 1, 1, 1].

a. How many permutations of the voters are there?

b. Under the Shapley-Shubik model, in how many permutations will B be the pivotal voter?

c. What is the Shapley-Shubik index for voter B?

2009 ICTM State Contest
Voting Theory Solutions

1. a. Each voter has $25+24+\cdots+2+1=\frac{25\cdot 26}{2}=325$ points to assign. The number of voters is thus $\frac{23,400}{325}=72$.

b. Since there are 72 voters, and 4 of them ranked Oklahoma in first place, the second and third place votes must total 68. Let x be the number of second place votes. Then the number of third place votes is $68-x$. An expression for the number of points Oklahoma received is thus $25\cdot 4+24\cdot x+23\cdot(68-x)$. We solve the equation

$$25\cdot 4+24\cdot x+23\cdot(68-x)=1709$$
$$100+24\cdot x+1564-23\cdot x=1709$$
$$x=45$$

$$68-x=68-45=23$$

Oklahoma received 45 second place votes and 23 third place votes.

2. a. The minimal winning coalitions are {A, B} and {A, C, D, E}.

b. Yes, A has veto power. No issue can pass without A's vote.

c.

winning coalition	**weight**	**extra votes**	**critical voters** A	B	C	D	E
{A, B}	8	0	1	1	0	0	0
{A, B, C}	9	1	1	1	0	0	0
{A, B, D}	9	1	1	1	0	0	0
{A, B, E}	9	1	1	1	0	0	0
{A, B, C, D}	10	2	1	1	0	0	0
{A, B, C, E}	10	2	1	1	0	0	0
{A, B, D, E}	10	2	1	1	0	0	0
{A, C, D, E}	8	0	1	0	1	1	1
{A, B, C, D, E}	11	3	1	0	0	0	0
			9	7	1	1	1

Each voter is a critical voter in the same number of blocking coalitions as winning coalitions. Thus, the Banzhaf index for this system is (18, 14, 2, 2, 2).

d. B's share of the voting power is $\frac{14}{18+14+2+2+2}=\frac{14}{38}=\frac{7}{19}\approx 37\%$.

3. a. There are 5! = 120 permutations of the five voters.

b. B has only 3 votes. Thus, B will never be a pivotal voter if he is the first voter in a permutation. Also, the other four voters have a combined total of 8 votes. Thus, B will never be a pivotal voter if he is the last member of a permutation. We consider the cases where B is the 2nd, 3rd, and 4th member of the permutation.

case 1: B is the second member of the permutation ___ _B_ ___ ___ ___

B is pivotal only when A is the first member of the permutation; since the remaining one-vote members can be arranged in 3! = 6 ways, there are 6 permutations in which B is pivotal

case 2: B is the third member of the permutation ___ ___ _B_ ___ ___

B will be pivotal only when A is the first or second member of the permutation; in each case there will be 6 ways to arrange the other one-vote members; thus, B will be pivotal in twelve of these permutations

case 3: B is the fourth member of the permutation ___ ___ ___ _B_ ___

Again, B will be pivotal only when A is the first, second, or third member of the permutation; in each of these cases there will be 6 permutations of the other members; thus, B will be pivotal in 18 of these permutations

The total number of permutations in which B is pivotal is thus 6 + 12 + 18 = 36.

c. The Shapley-Shubik index for voter B is $\frac{36}{120} = \frac{3}{10}$. In the Shapley-Shubik model, B has 30% of the voting power.

2009 ICTM State Competition
Voting Theory Extemporaneous Questions

Voters A, B, and C want to use the weighted voting system [q: 8, 4, 1].

1. What are the possible values they can use for q?

2. Which values of q will result in exactly one voter with veto power? Who? Why?

3. Which values of q will result in one or more dummies? Who? Why?

2009 ICTM State Competition
Voting Theory Extemporaneous Solutions

1. Since we require that a quota be greater than half the sum of the weights, in this system we require $q > \frac{8+4+1}{2} = \frac{13}{2}$. Possible values for q are thus 7, 8, 9, 10, 11, 12, 13.

2. A voter has veto power if all coalitions consisting of only other voters are losing coalitions. Using values of 7, 8, or 9 for q would ensure that voter A and only voter A has veto power. (Using a quota of 10, 11, or 12 would give both voter A and voter B veto power. Using a quota of 13 would give all three voters veto power.)

3. A dummy is a voter that is not a critical voter in any winning coalition. Using values of 7 or 8 for q would make both voter B and voter C dummies. Using values of 10, 11, or 12 for q would make voter C a dummy. (Letting q = 9 or q = 13 yields a voting system with no dummies.)