1. If $x \neq 0$, and y is a real number such that $x^3 = y$ and $x^4 = 7y$, find the value of x.

2. Let n be a positive integer, and let x be a positive integral multiple of 8. If $n = \frac{x}{4} + \frac{x}{5} + \frac{x}{6}$, find the smallest possible value of x.

3. If $\frac{2}{3}x$ is 10% of 120, find the value of 10% of $\frac{1}{3}x$. Express your answer as a **decimal.**

4. If $xy = 13$, find the value of $7(\frac{x}{y})(\frac{x^2}{y})(\frac{y^3}{x^2})$.

5. If you add the sum of two positive integers to their product, you obtain one less than a three-digit positive factor of 2001. Find the positive difference of the two integers.

6. The value of $x = \sqrt{2010 - \sqrt{2010 - \sqrt{2010 - \sqrt{2010 - \cdots}}}}$ can be expressed $\frac{-1+\sqrt{k}}{w}$ where k and w are positive integers. Find the smallest possible value of $(k + w)$.

7. Let $x = 13.47$ and let $3.2x - 1.342y$ represent a positive integer. Find the smallest possible value of y if y also represents a positive integer.

8. Working at a constant rate, Paul can mop the kitchen floor in 10 minutes. Working at a constant rate, Wanda can mop the same kitchen floor in 8 minutes. Paul has been mopping the kitchen floor for 5 minutes when Wanda joins him. Assuming no loss of efficiency, how many more **seconds** should it take the two to finish mopping the kitchen once Wanda joins Paul in mopping? Express your answer as an improper fraction reduced to lowest terms.

9. The arithmetic mean (or average) of a set of 78 numbers is 32. The arithmetic mean (or average) of a set of x numbers is 29. The arithmetic mean (or average) of a set of y numbers is 33. If the arithmetic mean (or average) of the $(78+x+y)$ numbers is 31, find the ordered pair of positive integers (x, y) such that $(x+y)$ is a minimum. Be sure to express your answer as an ordered pair of the form (x, y).

10. If $x \in \{95, 96, 97, 98, 99, 100\}$, find the sum of all distinct values of x such that when x is increased by 25% and then the result is decreased by 25%, the final answer will be 93.75% of x.

11. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

    If x represents a negative number, which of the following expressions represents the **least** value?

    A) x
    B) $-x$
    C) $x-x$
    D) $x+2x$
    E) x^2
    F) x^4

    Note: Be certain to write the correct capital letter as your answer.

12. In a certain high school, $37\frac{1}{2}\%$ of the girls and 50% of the boys attended a football game. If 48% of all the students at this high school are girls, what fraction of all the students went to the football game? Express your answer as a common fraction reduced to lowest terms.

13. The combined weight of the baggage of two airplane passengers was 130 pounds. Each person is permitted k pounds of baggage without charge. Any excess baggage weight per person is charged w cents per excess pound. One of the passengers had to pay \$1.92 for excess baggage weight, and the other passenger had to pay \$9.12 for excess baggage weight. If all the baggage had belonged to one person, that person would have had to pay \$21.12 for excess baggage weight. Find the value of k.

14. A clock reads the correct time at 7:00 A. M. Starting at 7:00 A. M. that day, the clock is stopped for 1 minute at the end of the first hour, for 2 minutes at the end of the next full hour after the stoppage, for 3 minutes at the end of the next full hour after the second stoppage, for 4 minutes at the end of the next full hour after the third stoppage, and so forth. What is the correct time when the clock reads 7:30 P. M.? Express your answer in the form hours:minutes and be sure to attach either **A. M.** or **P. M.**, whichever is correct, to your answer.

15. The slope of a line that is perpendicular to a line whose equation is $y = 3kx + 18$ is $-\frac{1}{6}$. Find the value of k.

16. The difference between the squares of two positive numbers is 576, and one number is exactly 60% of the other number. Find the larger of the two positive numbers.

17. Let $AB = BC = CD = DE$. Karen goes directly from A to B at a constant rate of x mph.; then immediately goes directly from B to C at a constant rate of $0.92x$ mph.; then immediately goes directly from C to D at a constant rate of $0.85x$ mph.; then immediately goes directly from D to E at a constant rate of $0.82x$ mph. Relative to the total distance and total time involved from A to E, Karen's average rate was 38.96 mph. Find the value of x. Express your answer as a decimal rounded to the nearest hundredth.

18. One root of the equation $x^2 + kx + 6 = 0$ is five times the other root. If $k > 0$, then k can be expressed in the form $\frac{p\sqrt{w}}{f}$ where *p*, *w*, and *f* are positive integers. Find the smallest possible value of $(p + w + f)$.

19. If $16^{8888} = 4^x$, find the value of x.

20. x, y, and z are integers in the system $\begin{cases} 3x + 5y + z = 10 \\ x + 2y + 3z = 36 \\ kx + y - 2z = 40 \end{cases}$, and k is a positive integer. Find the sum of all possible distinct values of z.

2009 SAA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 7
2. 120
3. 0.6 (Must be decimal answer.)
4. 91
5. 6
6. 8043
7. 12
8. $\frac{400}{3}$ (Must be this reduced improper fraction.)
9. $(40,1)$ (Must be this ordered pair.)
10. 585
11. D (Must be this capital letter.)
12. $\frac{11}{25}$ (Must be this reduced common fraction.)
13. 42 (Pounds optional.)
14. 8 : 48 PM (Must be in "hours:minutes" with PM attached.)
15. 2
16. 30
17. 43.66 (Must be this decimal, mph optional.)
18. 41
19. 17,776
20. 47

1. Find the PERIMETER of a right triangle if the length of the hypotenuse is 61 and the length of one of the legs is 60.

2. A circle with a radius whose length is 5.6 is superimposed upon a rectangle whose diagonal has a length of 15.4 and whose base has a length of 13.2 so that the center of the circle coincides with the intersection of the diagonals of the rectangle. In how many distinct points will the circle and the rectangle intersect?

3. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   How many non-congruent right triangles in which one of the acute angles is 30° have 3 sides all of whose lengths are whole numbers?

   A) 0
   B) 1
   C) more than 1 but less than 10
   D) more than 10 but not an infinite number
   E) an infinite number

   Note: Be certain to write the correct capital letter as your answer.

4. The ratio of the measure of an interior angle of a regular polygon to the measure of an exterior angle of this regular polygon is 23**:**1. Find the number of sides of the regular polygon.

5. The length of exactly one side of an isosceles triangle is 10. If the lengths of all sides of this triangle are whole numbers, find the smallest possible perimeter of any such triangle.

6. The midpoint of a line segment is $(-3,5)$; one endpoint of the segment is $(-5,10)$. Find the **ordered pair** for the other endpoint.

7. Find the length of a radius of the circle that is inscribed in a triangle whose sides have lengths of 17, 55, and 60.

8. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   The locus of points in a plane that satisfy the equation $x^2 + y^2 + 2y + 1 = 0$ is:

   A) two parallel lines
   B) a line
   C) a point
   D) two points
   E) three points
   F) four points
   G) a circle

   Note: Be certain to write the correct capital letter as your answer.

9. Each of the three sides of a right triangle has a length that is an integer. The difference between the length of the hypotenuse and the length of the longer leg is the same as the difference between the length of the longer leg and the length of the shorter leg. If the area of the triangle is 600, find the length of a radius of the inscribed circle of the right triangle.

10. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

    Which of the following lines is perpendicular to the line $2x + y = 5$?

    A) $x + y = 4$
    B) $2x + y = 10$
    C) $2x - y = 10$
    D) $x - 2y = 8$
    E) $x + 2y = 10$
    F) $4x + 2y = 10$

    Note: Be certain to write the correct capital letter as your answer.

11. The points $(5, k)$ and $(9, w)$ lie on a circle whose equation is $x^2 + y^2 = p$. If k and w are both positive integers, and if $k + w < 117$, find the sum of all possible distinct values of p.

12. A circle is inscribed in a rhombus whose diagonals have lengths of 40 and 42. If a point is selected at random in the interior of the rhombus, find the probability that the point selected is in the exterior of the inscribed circle of the rhombus. Express your answer as a decimal rounded to the nearest thousandth.

13. In ΔABC, the length of the median from point A to $\overline{BC}$ is 56.34. Find the distance from point A to the centroid of ΔABC. Express your answer as a **decimal**.

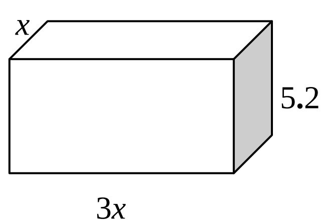
14. Three points, $(9,-12)$, $(10,5)$, and $(0,9)$, lie on Circle P. An element is selected at random from the set $\{-11,-7,2\}$ and is called k; an element is selected at random from the set $\{-3,6,7,10\}$ and is called w. Find the probability that the point (k,w) does not lie in the interior of Circle P. Express your answer as a common fraction reduced to lowest terms.

15. In a circle with a radius whose length is $\sqrt{76}$, two parallel chords of unequal length could either be distances of 3 or 7 apart. The exact sum of the lengths of the two chords is given, in simplest radical form, by $k\sqrt{p}+w\sqrt{f}$. Compute the sum $(k+p+w+f)$.

16. By how much does the length of a side of a square with a diagonal of length $14\sqrt{2}$ exceed the length of a diagonal of a square with a perimeter of $12\sqrt{2}$?

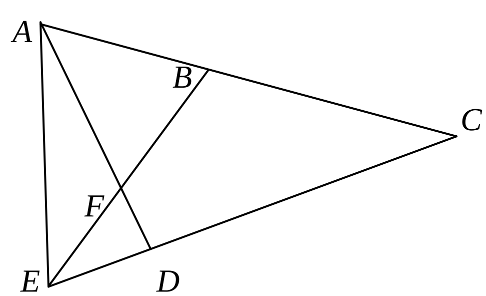
17. A sphere with center at J is inscribed in a right circular cone with a peak (top vertex) at P. The sphere is tangent to the lateral surface of the right circular cone at E and F, and the sphere is also tangent to the base of the cone at O, the center of the base of the cone. P, J, E and F are coplanar. If $\angle EPF = 56°$ and the length of a radius of the base of the right circular cone is 16.78, find the volume of the sphere. Express your answer rounded to the nearest **whole number**.

18. If the rectangular solid shown has a volume of 1100.736, find the value of x. Express your answer as a **decimal**.



19. Let x and y be integers representing the lengths of the edges of two cubes with $x < y$. The number of cubic units in the sum of the volumes of the two cubes is $\frac{223}{24}$ times the number of units in the perimeters of all faces of the cubes. Find the sum of all the distinct numbers of possible cubic units in the volume of the smaller cube.

20. In the diagram, ΔAEC is acute, B lies on $\overline{AC}$, D lies on $\overline{EC}$, and $\overline{AD}$ and $\overline{BE}$ intersect at F. If $DC = 5(DE)$ and $AB:BC = 4:5$, then the ratio of the area of ΔABF to the area of quadrilateral $BCDF$ is $k:w$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.



2009 SAA

Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 132
2. 4
3. A (Must be this capital letter.)
4. 48
5. 22
6. $(-1,0)$ (Must be this ordered pair.)
7. 7
8. C (Must be this capital letter.)
9. 10
10. D (Must be this capital letter.)
11. 356
12. 0.216 or .216 (Must be this decimal.)
13. 37.56 (Must be decimal answer.)
14. $\frac{7}{12}$ (Must be this reduced common fraction.)
15. 67
16. 8
17. 4293
18. 8.4 (Must be decimal answer.)
19. 1547
20. 87

1. Let $C(n,k)=\dfrac{n!}{k!(n-k)!}$. Find the value of $C(12,4)$.

2. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never—**whichever is correct.

   If h, k, and j are real numbers such that $h>k$ and $k>j$, then $k+j>h+k$.

3. Given are the following four algebraic fractions: $\dfrac{3}{x-2}$, $\dfrac{16-4x}{2}$, $\dfrac{16}{x^2-5x+6}$, and $\dfrac{15}{x}$. From the set $\{1,2,3,4\}$, one number is selected at random and substituted for x in every one of the four fractions. Find the probability that the value of at least one of the four fractions will be undefined. Express your answer as a common fraction reduced to lowest terms.

4. The equation of a hyperbola whose foci are $(8,0)$ and $(-8,0)$ and whose vertices are $(3,0)$ and $(-3,0)$ can be expressed in the form $\dfrac{x^2}{k}-\dfrac{y^2}{w}=1$. Find the value of $(2k+3w)$.

5. The arithmetic mean of 23, 30, 16, 33, and two other numbers that differ by 2 is 21. If the smallest of these six numbers is discarded, find the arithmetic mean of the remaining five numbers.

6. If $f(x)=5(2x-3)^4+(x+2)^3-(x-5)^2+17$, find the value of $f(5)$.

7. The fifth term of an arithmetic sequence is -18, and the sum of the first thirty-two terms is 1448. Find the ninth term.

8. Find the **sum** of all negative integers that are members of the solution set for x given $\left|1-2x\right|<15$.

9. Each person in a room writes down an integer at random from one of the 100 integers from 1 to 100 inclusive. Find the minimum number of persons in the room such that the probability that at least two persons wrote down the same number exceeds 90%.

10. If $f(x)=2^{(2x+1)}$ and $\left(f\left(2-x\right)\right)\left(f\left(x\right)\right)\left(f\left(2+x\right)\right)=kf\left(x\right)$, find the value of k.

11. Let 2 be a root for x of the quartic equation: $ax^4+bx^3+cx^2+bx+a=0$. All the roots are real, the sum of the roots is $\frac{26}{3}$, and a, b, and c are integers. Find the smallest possible value of $\left(a+b+c\right)$.

12. Find the shortest distance from the origin to the line whose equation is $3x-4y=20$.

13. Find the value of $\prod_{x=5}^{15624}\log_x(x+1)$.

14. By substituting 1, 2, 3, 4, and 5 for x in order in a polynomial function in x, the first five terms are respectively: -1, 6, 17, 32, and 51. If $P(x)$ is the polynomial function of lowest degree satisfying the given, find $P(243)$.

15. Given the following seven points: $(8,2)$, $(10,-3)$, $(7,5)$, $(13,-4)$, $(-8,-5)$, $(-3,-2)$, $(-1,1)$. If three of these seven points are selected at random without replacement, find the probability that at least two of the points selected lie below the *x*-axis. Express your answer as a common fraction reduced to lowest terms.

16. If $f(x)=7x+9$ and $g(x)=x^2-3$, find $g(f(-11))$.

17. If $x^2+y^2=7$ and $x^3+y^3=10$, find the largest possible value of $(x+y)$.

18. Given the following system: $\begin{cases} x+y+z+w=14 \\ x^2+y^2+z^2+w^2=54 \\ x^3+y^3+z^3+w^3=224 \\ xyzw=120 \end{cases}$. If x, y, z, and w are positive integers and (x,y,z,w) is a member of the solution set for the given system, find the second least possible value of $(6x+5y+4z+3w)$.

19. Let $i=\sqrt{-1}$. If $(k+3i)(-7+2i)=11i-118$, find the value of k.

20. Let k be a positive integer such that $48<k<90$. Find the sum of all possible distinct values of k such that the product of all distinct positive integral divisors of k is k^2.

2009 SAA

Algebra II

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 495
2. Never (Must be the whole word.)
3. $\frac{1}{2}$ (Must be this reduced common fraction.)
4. 183
5. 23
6. 12,365
7. 4
8. –21
9. 22
10. 1024
11. 117
12. 4
13. 6
14. 118,337
15. $\frac{22}{35}$ (Must be this reduced common fraction.)
16. 4621
17. 4
18. 59
19. 16
20. 908

1. Let $C(n,k)=\dfrac{n!}{k!(n-k)!}$. Find the value of $C(12,4)$.

2. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never—**whichever is correct.

   If h, k, and j are real numbers such that $h>k$ and $k>j$, then $k+j>h+k$.

3. Given are the following four algebraic fractions: $\dfrac{3}{x-2}$, $\dfrac{16-4x}{2}$, $\dfrac{16}{x^2-5x+6}$, and $\dfrac{15}{x}$. From the set $\{1,2,3,4\}$, one number is selected at random and substituted for x in every one of the four fractions. Find the probability that the value of at least one of the four fractions will be undefined. Express your answer as a common fraction reduced to lowest terms.

4. The equation of a hyperbola whose foci are $(8,0)$ and $(-8,0)$ and whose vertices are $(3,0)$ and $(-3,0)$ can be expressed in the form $\dfrac{x^2}{k}-\dfrac{y^2}{w}=1$. Find the value of $(2k+3w)$.

5. The arithmetic mean of 23, 30, 16, 33, and two other numbers that differ by 2 is 21. If the smallest of these six numbers is discarded, find the arithmetic mean of the remaining five numbers.

6. If $f(x)=5(2x-3)^4+(x+2)^3-(x-5)^2+17$, find the value of $f(5)$.

7. The fifth term of an arithmetic sequence is -18, and the sum of the first thirty-two terms is 1448. Find the ninth term.

8. Find the **sum** of all negative integers that are members of the solution set for x given $|1-2x|<15$.

9. Each person in a room writes down an integer at random from one of the 100 integers from 1 to 100 inclusive. Find the minimum number of persons in the room such that the probability that at least two persons wrote down the same number exceeds 90%.

10. If $f(x)=2^{(2x+1)}$ and $\left(f(2-x)\right)\left(f(x)\right)\left(f(2+x)\right)=kf(x)$, find the value of k.

11. Let 2 be a root for x of the quartic equation: $ax^4+bx^3+cx^2+bx+a=0$. All the roots are real, the sum of the roots is $\frac{26}{3}$, and a, b, and c are integers. Find the smallest possible value of $(a+b+c)$.

12. Find the shortest distance from the origin to the line whose equation is $3x-4y=20$.

13. Find the value of $\prod_{x=5}^{15624}\log_x(x+1)$.

14. By substituting 1, 2, 3, 4, and 5 for x in order in a polynomial function in x, the first five terms are respectively: -1, 6, 17, 32, and 51. If $P(x)$ is the polynomial function of lowest degree satisfying the given, find $P(243)$.

15. Given the following seven points: $(8,2)$, $(10,-3)$, $(7,5)$, $(13,-4)$, $(-8,-5)$, $(-3,-2)$, $(-1,1)$. If three of these seven points are selected at random without replacement, find the probability that at least two of the points selected lie below the *x*-axis. Express your answer as a common fraction reduced to lowest terms.

16. If $f(x)=7x+9$ and $g(x)=x^2-3$, find $g(f(-11))$.

17. If $x^2+y^2=7$ and $x^3+y^3=10$, find the largest possible value of $(x+y)$.

18. Given the following system: $\begin{cases} x+y+z+w=14 \\ x^2+y^2+z^2+w^2=54 \\ x^3+y^3+z^3+w^3=224 \\ xyzw=120 \end{cases}$. If x, y, z, and w are positive integers and (x,y,z,w) is a member of the solution set for the given system, find the second least possible value of $(6x+5y+4z+3w)$.

19. Let $i=\sqrt{-1}$. If $(k+3i)(-7+2i)=11i-118$, find the value of k.

20. Let k be a positive integer such that $48<k<90$. Find the sum of all possible distinct values of k such that the product of all distinct positive integral divisors of k is k^2.

2009 SAA

Algebra II

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 495
2. Never **(Must be the whole word.)**
3. $\frac{1}{2}$ **(Must be this reduced common fraction.)**
4. 183
5. 23
6. 12,365
7. 4
8. –21
9. 22
10. 1024
11. 117
12. 4
13. 6
14. 118,337
15. $\frac{22}{35}$ **(Must be this reduced common fraction.)**
16. 4621
17. 4
18. 59
19. 16
20. 908

1. Find the amplitude of the graph of the equation $y = -16\sin\left(80(4x^\circ - 64^\circ)\right)$.

2. Find the **ordered pair** (k, w) in the following vector sum: $(k, w) + (-7, 11) = (-15, -29)$.

3. The three-dimensional vectors $(k, -2, 2)$ and $(2, 3, w)$ are perpendicular. Find the value of $(k + w)$.

4. The graph of $y = \dfrac{-5x + x^2}{2x^2 - 8}$ has a horizontal asymptote when $y = k$ and a vertical asymptote at $x = w$ with $w > 0$. Find the value of $(k + w)$. Write your answer as a simplified proper or improper fraction, whichever is appropriate.

5. Find the sum of the first 20 terms of the geometric progression: 2, 6, 18, $\cdots$.

6. Find the value of the indicated sum: $\sum_{2}^{4} 2^{(x+1)}$.

7. For all real values of x for which the fractions in this problem are defined, $\dfrac{2x-1}{x^2+4x+3} = \dfrac{A}{x+3} + \dfrac{B}{x+1}$. Find the value of $(2A + 16B)$.

8. Let $F = \{a, b, c\}$. If $2a = 3b = 8c$, then the arithmetic mean of F is ka. Find the value of k. Express your answer as a common fraction reduced to lowest terms.

9. In this problem, assume a normal, flat football field, 100 yards from goal line to goal line. The fifty-yard line is midway between the goal lines, with the numbering of the yardlines decreasing from the fifty-yard line in both directions until the goal line, which is yard-line zero. Ted kicks off from his 30-yard line across the fifty-yard line and in the direction of his opponents' goal line at an angle of 42.46° with the horizontal and with an initial velocity of 74.89 feet per second. Ted continues running directly down the middle of the field at a constant speed of 29.49 feet per second. Assume the effect-on-gravity vector is $(0,-16t^2)$ where t is measured in seconds. Devon catches the ball 4.816 feet above the ground in the middle of the field. At what yard line of his opponent will Ted be when the ball is caught? Express your answer rounded to the nearest whole number.

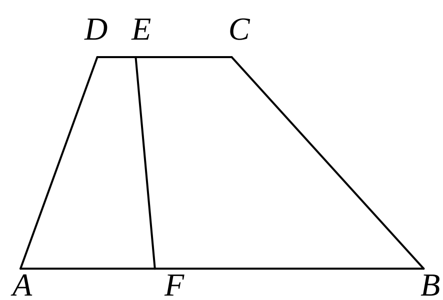
10. **(Always, Sometimes, or Never)** For your answer, write the <u>*whole word*</u> **Always, Sometimes, or Never**—whichever is correct.

If a function is continuous at $x = c$, then the value of the function at c equals the limit of the function as $x \to c$.

11. Let $0° \le x \le 360°$. Then the solution set for x, in degrees, of the equation $\sin(kx-7)° - \cos(3x+9)° = 0$ is $\{11, 53, 56, 101, 146, 191, 233, 236, 281, 326\}$. Find the value of k.

12. Assume that Jack has a 40% probability of making any free throw he attempts. Assume that Karen has a 50% probability of making any free throw she attempts. Jack and Karen agree to the following procedure. Jack will get the first free throw. Thereafter, they will alternate, but Karen will get two successive shots while Jack will only get one. For example, Jack shoots first, Karen second and third, Jack fourth, Karen fifth and sixth, Jack seventh, etc. Find the probability that Jack will be the first to make a free throw that either Jack or Karen attempts. Express your answer as a common fraction reduced to lowest terms.

13. In the diagram,
$ABCD$ is a trapezoid with $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$.
$AB = 32.00$, $DC = 10.00$, $\angle DAB = 62°$,
$\angle DCB = 152°$. $DE = 2.500$, $AF = 8.000$
E lies on $\overline{DC}$, F lies on $\overline{AB}$. Find EF.
Express your answer as a decimal rounded to 4 significant digits.



14. In Triangle ABC, $AB = 16$, $BC = 21$, and $AC = 19$. $\overrightarrow{AD}$ bisects $\angle CAB$, and $\overrightarrow{CD}$ bisects $\angle ACB$. Expressed in simplest radical form, $AD = k\sqrt{w}$ where k and w are positive integers. Find the value of $(k+w)$.

15. The **sum** of the terms of a geometric sequence whose first term is $\frac{1}{3}$ and whose second term is 1 is $\frac{40}{3}$. Find the number of terms in this geometric sequence.

16. The two lines represented by $3x-5y=k$ and $wx-37y=-4694$ meet at a point whose *y*-coordinate is 2. If the tangent of the positive acute angle formed by the lines is $\frac{21}{20}$ and if $w>0$, find the value of $(k+w)$.

17. The equations of L_1 and L_2 are $y=kx$ and $y=wx$, respectively. L_1 makes an angle θ with the horizontal and L_2 makes an angle ∂ with the horizontal (measured counterclockwise from the positive *x*-axis). The slope of L_1 is 7 times the slope of L_2. If $0°<\theta<90°$ and $\theta=2(\partial)$, find the value of θ (in degrees). Express your answer as a decimal rounded to the nearest hundredth of a degree.

18. Find the value of $\lim_{x\to 3}(\frac{x^3-27}{x-3})$.

19. If $(3x-y)^6$ is expanded and completely simplified, one of the terms is kx^3y^3 with k a real number. Find the value of k.

20. Doc and Sandy are together at a point on an infinitely paved plane. At noon, Sandy heads directly east at a constant rate of 50 mph. There is a 50% chance that Doc will also leave at noon and head in a direction of N5°E at a constant rate of 40 mph. Otherwise, Doc will stay at the point at which he was at noon. If a random moment is picked between 9:00 and 10:00 in the evening of the same day that Sandy left, find the probability the two will be less than 600 miles apart. Express your answer as a decimal rounded to 4 significant digits.

2009 SAA

Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: **All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.**

1. 16

2. $(-8,-40)$ **(Must be this ordered pair.)**

3. 3

4. $\frac{5}{2}$ **(Must be this reduced improper fraction.)**

5. $3{,}486{,}784{,}400$

6. 56

7. -17

8. $\frac{23}{36}$ **(Must be this reduced common fraction.)**

9. 40 **(Opponents, yard and/or yard-line optional.)**

10. Always **(Must be the whole word.)**

11. 5

12. $\frac{8}{17}$ **(Must be this reduced common fraction.)**

13. 9.143 or 9.143×10^{0} **(Must be this Decimal.)**

14. 21

15. 4

16. 71

17. 80.41 **(Must be this decimal, degrees optional.)**

18. 27

19. -540

20. 0.8981 or $.8981$ or 8.981×10^{-1} **(Must be this decimal.)**

NO CALCULATORS

1. Rosa sliced a pizza into equal sixths. She then sliced each of these equal sixths into two equal tiny pieces. How many of these tiny pieces would Placido need to eat so that he would have eaten one-fourth of the original pizza?

2. The area of a triangle is 630. The lengths of the shortest side and the longest side of the triangle are respectively 25 and 63. Find the sum of the lengths of the shortest and the longest altitudes of this triangle. Express your answer as a **decimal**.

3. A line that is tangent to a circle at the point $(1,-2)$ has the equation of $3x+4y+5=0$. The center of the circle is on the line whose equation is $x+y=6$. Find the center of the circle. Express your answer as an **ordered pair** of the form (x, y).

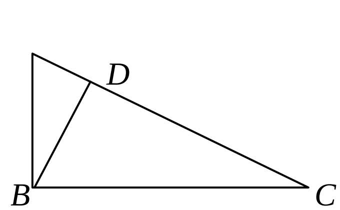
4. Cindy is 90 feet from the bumper cars. She walks toward the bumper cars at a steady rate. After 8.5 seconds, Cindy has not yet reached the bumper cars and is 56 feet away from the bumper cars. The linear equation that indicates Cindy's distance in feet, *y*, from the bumper cars as a function of time elapsed in seconds, *x*, can be expressed in the form $y = mx + b$. Find the value of $(m+b)$.

5. Given the system: $\begin{cases} 3x+2y=7 \\ x+y=3 \end{cases}$. Find the value of $(4x-5y)$.

6. A bag contains only red, white, blue, and green marbles. The bag contains 11 red marbles, 27 white marbles, *x* blue marbles, and more than 13 green marbles. If the probability that a marble drawn at random from the bag is blue is exactly 0.25, find the minimum possible value of x.

7. If $x+3 \neq 0$, by how much does $\dfrac{x^2-8x-33}{x+3}$ exceed $x-15$?

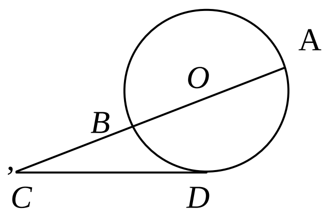
8. In the diagram, $\overline{AB} \perp \overline{BC}$, $AB = 6$, $BC = 12$, and $\angle DBA = 30°$. If D lies on $\overline{AC}$, then, expressed in simplest radical form, $AD = \dfrac{k}{w}\sqrt{f - p\sqrt{3}}$ where k, w, f, and p are positive integers. Find the smallest possible value of $(k+w+f+p)$.



NO CALCULATORS

NO CALCULATORS

9. In the diagram, A, B, and D lie on circle O, $\overline{CD}$ is tangent to the circle at D, and C, B, O, and A are collinear. If $\overline{AB}$ is a diameter, $CD = 40$, and $CB = 32$, find the area of ΔCOD.

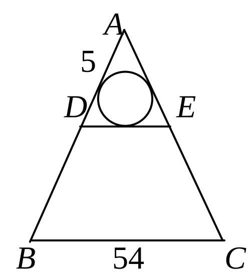


10. $\dfrac{717}{1150} = \dfrac{1}{k} + \dfrac{1}{w} + \dfrac{1}{f} + \dfrac{1}{6900}$ where k, w, and f are positive integers. Find the smallest possible value of $(k + w + f)$.

11. Let $x > 0$, $y > 0$, and $z > 0$. If $xy = 8$, $yz = 10$, and $xz = 20$, find the value of $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{1}{z^2}$. Express your answer as a common fraction reduced to lowest terms.

12. By how much does the total surface area of a 3 by 4 by 6 rectangular solid exceed the total surface area of a 5 by 2 by 3 rectangular solid?

13. In the diagram, the circle is inscribed in ΔADE, $AD = 5$, $DB = 25$, $BC = 54$, $\overline{DE} \parallel \overline{BC}$, and $\overline{AB} \cong \overline{AC}$. The area of the circle shown can be expressed in the form $\dfrac{k\pi}{w}$ where k and w are positive integers. Find the smallest possible value of $(k + w)$. (Note: A, D, B are collinear, and A, E, C are collinear.)



14. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

A quadrilateral is inscribed in a circle. If the degree measure of one of the angles of this quadrilateral is 121, which one of the following **must** be the degree measure of one of the other angles of this quadrilateral?

A) 239
B) 121
C) 79
D) 59
E) 31

Note: Be certain to write the correct capital letter as your answer.

NO CALCULATORS

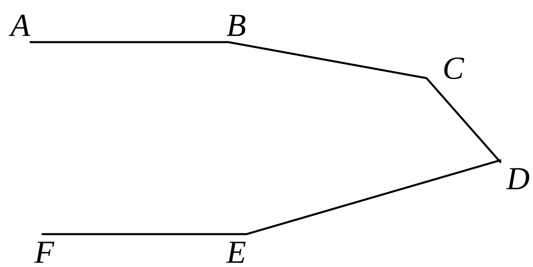
NO CALCULATORS

15. Together, Al and Bob weigh 324 pounds. Together, Bob and Chuck weigh 338. Together, Chuck and Dick weigh 322 pounds. Together, Al, Chuck, and Ed weigh 530 pounds. Together, Dick and Ed weigh 356 pounds. Find the number of pounds in the combined weight of the four people Al, Bob, Chuck, and Ed.

16. When written in simplest radical form, $\frac{1}{\sqrt{3}+\sqrt{6}-\sqrt{2}} = \frac{a+b\sqrt{3}+c\sqrt{6}+d\sqrt{2}}{e}$ where a, b, c, d, and e are integers. Find the absolute value of $(a+b+c+d+e)$.

17. If x and y are two different positive integers such that $x^3y^2 = 1323$, find the value of xy.

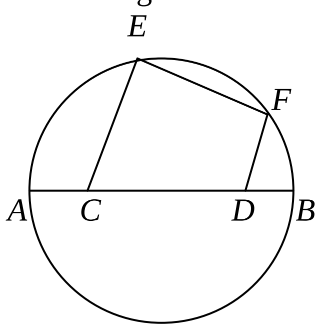
18. In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{FE}$.



Find: $m\angle ABC + m\angle BCD + m\angle CDE + m\angle DEF$. Express your answer in degrees.

19. $(1,9)$, $(1,3,6)$, and $(1,2,3,4)$ are 3 examples of distinct groups of **positive** integers for which the sum of the members of each group is 10. Within each group, the members must be arranged in ascending order, (a,b,c) with $a<b<c$ for example. (In other words, $(2,3,5)$ is an acceptable group, but $(3,2,5)$ would **not** meet the conditions.) Find the number of distinct groups of **positive** integers (with the **positive** integers arranged in ascending order) for which the sum of the members of each group is 25. Assume that each group must contain at least two members.

20. In the diagram, points A, E, F, and B are distinct points that lie on the circle. $\overline{AB}$ is a diameter, $\angle CEF$ and $\angle DFE$ are right angles, and points A, C, D, and B are collinear in that order. $AB = 200$, and $CE = 113$. Find the sum of all distinct integer values for the length of $\overline{EF}$ such that the length of $\overline{DF}$ is an **odd integer**.



2009 SAA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

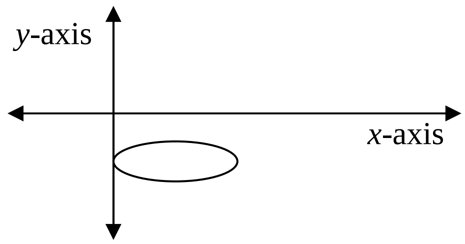
1. 3
2. 70.4
3. $(4,2)$ (Must be this ordered pair.)
4. 86
5. -6
6. 18
7. 4
8. 102
9. 180
10. 32
11. $\frac{141}{400}$ (Must be this reduced common fraction.)
12. 46
13. 157
14. D (Must be this capital letter.)
15. 698 (Pounds optional.)
16. 32
17. 21
18. 540 (Degrees optional.)
19. 141
20. 336

NO CALCULATORS

1. If $x > 0$, find the value of x such that $\log_x 1024 = 2$. Express your answer as an integer.

2. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never—**whichever is correct.

   The ellipse shown to the right has the equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. Given the center of the ellipse is in Quadrant IV, then $h > k$?



3. When $x^{24} - 1$ is factored completely with respect to the integers, how many of the factors are trinomials?

4. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   For all values of x where $\sin(x)$, $\cos(x)$, and $\cot(x)$ are all defined, $\left(\sin(x)\right)\left(\cos(x)\right)\left(\cot(x)\right) =$

   A) $\sin^2(x)$
   B) $\cos^2(x)$
   C) $\left(\sin(x)\right)\left(\cos(x)\right)$
   D) $\tan^2(x)$
   E) 1
   F) -1
   G) 0

   Note: Be certain to write the correct capital letter as your answer.

5. Let $i = \sqrt{-1}$. Expressed in trigonometric form, one of the cube roots of $4 - 4\sqrt{3}i$ is $2cis(k°)$ where $0° < k < 360°$. Find the largest value of k.

6. For the function $y = -5\cos\left(\frac{1}{4}\theta\right)$, the sum of the amplitude k and the radian period $w\pi$ is $k + w\pi$. Find the value of $(k + w)$.

7. The vector $(21,105)$ is perpendicular to the vector (k,w), and the vector (k,w) is parallel to the vector $(p,-36)$. Find the value of $\left(\frac{k}{w} + p\right)$.

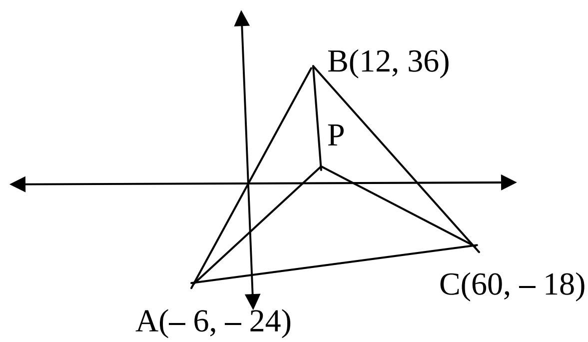
NO CALCULATORS

NO CALCULATORS

8. Find the value of $\sum_{k=1}^{\infty}\left(\frac{1}{4^k}\right)$. Express your answer as a fraction reduced to lowest terms.

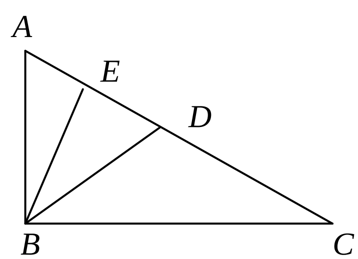
9. An ellipse has the equation $9x^2+4y^2+72x+8y=77$. The area of this ellipse can be expressed as $k\pi$. Find the value of k. Express your answer as a **decimal**.

10. In the diagram with ΔABC and coordinates as shown, the ratio of the area of ΔPAB to the area of ΔPBC to the area of ΔPAC is $1:2:3$. Find the **ordered pair** that represents point P.



11. Line L passes through the point represented by $(8,-12)$ and is parallel to the line passing through points represented by $(12,10)$ and $(15,-23)$. Line L can be represented as $\{(8+t,-12+kt)\}$. Find the value of k.

12. In the diagram, *E* and *D* lie on hypotenuse $\overline{AC}$ of right triangle *ABC*. $\overline{EC}\cong\overline{BC}$, $\overline{AD}\cong\overline{AB}$, and the length of the radius of the inscribed circle of Triangle *ABC* is $6\sqrt{6}$. Find the exact length of the radius of the circumscribed circle of Triangle *BDE*.



13. It is known that y varies directly as the square of x and inversely as the cube of r. If $y=\frac{40}{9}$ when $x=4$ and when $r=3$, find y when $x=2$ and $r=5$. Express your answer as a proper or improper fraction reduced to lowest terms, whichever is appropriate.

14. Natalie prepares for her math exam by studying, praying, or bowling. (She does only **one** of these per exam.) Six out of ten times she studies in preparation, three out of ten times she prays, and she bowls one out of ten times. She has a $\frac{4}{5}$ probability of passing the exam when she studies, a $\frac{1}{2}$ probability of passing if she prays, and a $\frac{1}{5}$ probability of passing when she bowls. If Natalie fails this exam, find the probability that she bowled in preparation. Express your answer as a common fraction reduced to lowest terms.

NO CALCULATORS

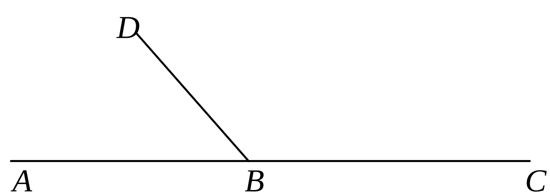
NO CALCULATORS

15. If $\lim_{x\to 2} f(x) = 4$ and $\lim_{x\to 2} g(x) = 9$, find $\lim_{x\to 2} \dfrac{2\left[\frac{1}{2} f(x) + 3g(x) + 1\right]}{\left[3f(x) + 2g(x)\right]}$.

16. A 24-hour clock has 24 hours as opposed to a 12-hour clock that has 12 hours. A 24-hour cuckoo clock strikes as many times as the hour at each hour. That is, it cuckoos once at 1:00, twice at 2:00, etc., to 24 cuckoos at 24:00. It also cuckoos exactly once at each of the three quarter hours throughout the day (for example, once at each time 1:15, 1:30, and 1:45). Find the total number of times the 24-hour cuckoo clock cuckoos during a 24-hour period.

17. Let a, b, and c each represent a single non-zero digit. Find the number of distinct **ordered triples** of the form (a,b,c) for which $1 \le \dfrac{abc}{a+b+c} \le 2$.

18. In the diagram, *A*, *B*, and *C* are collinear, and $\angle ABD$ is acute.



$\dfrac{\cos(\angle ABD)}{\cos(\angle CBD)} = k$. Find the value of k.

19. Let $A = \{2,4,6,8\}$. Let $B = \{2,4,6,y\}$. There are 2 different values for *y* such that the total population standard deviation (σ_x) of A is equal to the sample population standard deviation (S_x) of B. The larger of these two values for y, expressed in simplest radical form, is $\dfrac{k + w\sqrt{p}}{f}$ where k, w, p and f are positive integers. Find the value of $(k + w + p + f)$.

20. Let *x*, *y*, and *z* be real numbers such that $2y + 3z > 2 - x$, $x^2 + 4y^2 = 16 - 4xy$, $x^2 - 144 + 9z^2 = -6xz$, and $4y^2 + 9z^2 = 100 - 12yz$. Find the largest value of x for any ordered triple that satisfies all the given conditions.

NO CALCULATORS

2009 SAA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

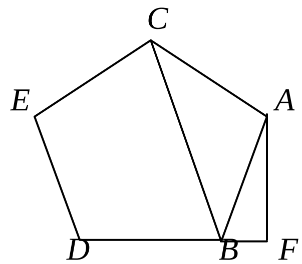
_______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 32 (Must be this integer.)
2. Always (Must be the whole word.)
3. 4
4. B (Must be this capital letter.)
5. 340 (Degrees optional.)
6. 13
7. 175
8. $\frac{1}{3}$ (Must be this reduced common fraction.)
9. 37.5 (Must be decimal answer.)
10. $(14,7)$ (Must be this ordered pair.)
11. -11
12. $12\sqrt{3}$ (Must be this exact radical.)
13. $\frac{6}{25}$ (Must be this reduced common fraction.)
14. $\frac{8}{35}$ (Must be this reduced common fraction.)
15. 2
16. 372
17. 91
18. -1
19. 38
20. 13

1. The lengths of two of the sides of a right triangle are 65 and 72. The length of the third side is also an integer. Find that integer. Express your answer as an **exact integer**. Do **not** use scientific notation.

2. $ABDEC$ is a regular pentagon with $\overline{DB}$ extended through B to F. (D, B, F are collinear.) $AF = 26$, and $\angle AFB = 91.42°$. Find the area of ΔABC.



3. When $x = 4.12$, $y = 9.888$. If y varies inversely as x, find the value of x when $y = 2.9616$.

4. Find the length of a radius of a circle whose center is at the point $(-10,10)$ if the point $(87.41, 96.13)$ lies on the circle.

5. If $\log_2(13.24^{(x+1.023)})$ is greater than 3.221 and is less than 5.118, then the set of all possibilities for x is $\{x : k < x < w\}$. Find the value of $(k+w)$.

6. In this problem, assume that the standard deviation is calculated according to the standard method of calculating the standard deviation for a set of sample proportions. Also, assume the following table of *z*-scores with the accompanying standard normal probabilities is accurate. On the table – 2(.0228) means that there is a normal probability of .0228 of obtaining a z score of -2 or less.

– 2(.0228)	– 1.9(.0287)	– 1.8(.0359)	– 1.7(.0446)	– 1.6(.0548)
– 1.5(.0668)	– 1.45(.0735)	– 1.4(.0808)	–1.35(.0885)	– 1.3(.0968)
– 1.25(.1056)	– 1.2(.1151)	– 1.1(.1357)	–1(.1587)	– 0.9(.1841)
– 0.8(.2119)	– 0.75(.2266)	– 0.7(.2420)	– 0.6(.2743)	– 0.5(.3085)
– 0.4(.3446)	– 0.3(.3821)	– 0.25(.4013)	– 0.1(.4602)	0(.5000)
0.1(.5398)	0.2(.5793)	0.25(.5987)	0.3(.6179)	0.4(.6554)
0.5(.6915)	0.6(.7257)	0.7(.7580)	0.75(.7734)	0.8(.7881)
0.9(.8159)	1.0(.8413)	1.1(.8643)	1.2(.8849)	1.25(.8944)
1.3(.9032)	1.4(.9192)	1.5(.9332)	1.6(.9452)	1.7(.9554)
1.75(.9599)	1.8(.9641)	1.9(.9713)	2.0(.9772)	2.1(.9821)
2.2(.9861)	2.25(.9878)	2.3(.9893)	2.4(.9918)	2.5(.9938)

Professor Shilenne announces to his class that anyone who receives a z-score whose absolute value is less than or equal to x will receive a grade of B on the exam. Assume the scores of the students on the exam will be normally distributed. To the nearest tenth, find the smallest possible value of x for which more than 50% of the students will receive a grade of B on the exam. Express your answer as a **decimal**.

7. If $12x^2 + 43x = 20$, find the absolute value of the difference between the smallest possible value of x and the largest possible value of x.

8. On a plane surface, two circles with radii of lengths 12 and 9 are x units apart where x is a positive integer and $x < 29$. If the length of a common external tangent segment of the two circles is $\sqrt{w}$ where w is an integral multiple of 7, find the sum of all possible distinct values of x. Express your answer as an exact **integer.**

9. If $x > 0$ and if $\log_{\left(\log_{(x+2)}(x+5)\right)}(x+8) = 26.63354547$, find the value of x.

10. The apothem of a regular hexagon has a length of 123.1. Find the perimeter of the regular hexagon.

11. Ten fair coins are tossed. Find the probability that no more than 7 coins land heads-up.

12. On May 1, Tom bought a used automobile for \$8000 for which he took out a loan at 7% annual percentage rate with monthly compounding. At the end of May and at the end of each succeeding month, Tom will make a payment of \$247.02. After he has made the twentieth payment, Tom decides to pay off the loan in 1 year (12 months). How much will his <u>new monthly payment</u> be? Round your answer to the nearest dollar and express your answer as an integer. Do **not** use scientific notation. Neglect that the final payment may not be exactly the same as the previous eleven.

13. According to Wikipedia, rifle target shooters use the term "click" to represent "one minute of an arc" in their rifle sighting system. Thus, one click moves the projectile impact point <u>*approximately*</u> 1 inch left or right when aimed at a perpendicular target at the distance of 100 yards. What is the actual distance, in inches, that one "click" moves the impact point on the target horizontally left or right?

14. If $f(x)=\dfrac{(5x-1)(x+6)(x+2)}{(x-8)(x+2)}$, then $f\left(x\right)$ is increasing in two intervals. In interval form, these two intervals are $(-\infty,k)$ and (w,∞). Find the value of $\left(k+w\right)$.

15. The area of a triangle is 630 square units. Two sides of the triangle are lengths 25 and 63. Find the sum of all possible third side length(s) for a triangle with the given information.

16. Find the value of $\sin(45°)+\sin(46°)+\cdots+\sin(n°)+\cdots+\sin(90°)$.

17. The three points $\left(1,6\right)$, $\left(4,3\right)$, and $\left(5,6\right)$ lie on a parabola whose axis of symmetry is parallel to the $y-$axis. Sam starts at $\left(4,4.5\right)$, walks to a point on the parabola, and then finishes his walk by walking from the parabola to a point that lies on the graph of $x^2-14x+y^2-6y+42=0$. Find the absolute value of the shortest possible total length of Sam's total walk.

18. If $x^7+x^6-2.112x^5+1.021x^4-x^3+x^2+4.113x-7.118$ is divided by $\left(x+1.114\right)$, find the numerical remainder.

19. In poker, a full house is a five-card hand in which three of the cards are of the same denomination and the other two cards are the same of a different rank (a pair), such as 10-10-10-3-3 or Jack-Jack-3-3-Jack. From a standard 52-card deck, five cards are drawn at random without replacement. Find the probability of getting a full house. Express your answer as a common fraction reduced to lowest terms.

20. Let $ABCD$ be a convex quadrilateral in which $AB=5$, $BC=6$, and $CD=9$. $\angle ABC$ is obtuse, $\sin(\angle ABC)=\dfrac{3}{5}$, and $\cos(\angle BCD)=-\dfrac{1}{6}$. Point E is the intersection of the diagonals $\overline{AC}$ and $\overline{BD}$. Find the length of $\overline{CE}$.

2009 SAA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

_______ **Correct X 5 pts. ea. =** ☐

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 97 **(Must be this integer.)**

2. 355.2 or 3.552×10^{2}

3. 13.76 or 1.376×10 or 1.376×10^{1}

4. 130.0 or 1.300×10^{2} **(Trailing zeroes necessary.)**

5. 0.1916 or .1916 or 1.916×10^{-1}

6. 0.7 or .7 or 7.000×10^{-1} **(Must be decimal answer, trailing zeroes necessary in sci. not.)**

7. 4.417 or 4.417×10^{0}

8. 112 **(Must be this integer.)**

9. 8.247 or 8.247×10^{0}

10. 852.9 or 8.529×10^{2}

11. 0.9453 or .9453 or 9.453×10^{-1}

12. 326 **(Must be this integer, no scientific notation, $ optional.)**

13. 1.047 or 1.047×10^{0} **(Inches optional.)**

14. 16.00 or 1.600×10 or 1.600×10^{1} **(Trailing zeroes necessary.)**

15. 132.5 or 1.325×10^{2}

16. 41.37 or 4.137×10 4.137×10^{1}

17. 1.734 or 1.734×10^{0}

18. –4.098 or -4.098×10^{0}

19. $\frac{6}{4165}$ **(Must be this reduced common fraction.)**

20. 4.997 or 4.997×10^{0}

1. Machine X produces 15 units per minute, and machine Y produces 12 units per minute. In 24 hours, machine X will produce k more units than machine Y. In ΔABC, $\angle A = 30°$, $\angle B = 60°$, and one of the sides has a length of 13. Let w be the largest possible length of the longest side. Find the value of $(k+w)$.

2. How many of the first 250 members of the Fibonacci sequence: $1,1,2,3,5,8,13,21,34,55,\cdots$ are odd integers?

3. Bob rides his motorcycle at a constant rate of w mph. for 24 miles and then at a constant rate of $4w$ mph. for 48 miles and his average rate for the 72 miles is 12 mph. Let k be the area of a triangle whose vertices are at $(-8,0)$, $(2,0)$, and $(0,4)$. Find the value of $(k+w)$.

4. Let p and q be the exact real solutions of $\dfrac{x^3}{\sqrt{2x^3-12}} = 4$. Let r be the radius of a circle inscribed in a triangle whose sides have lengths 18, 80, and 82. Find the exact value of $\left(\dfrac{2pq}{r}\right)$.

5. The measures of the exterior angles of a triangle are in the ratio of $5:6:7$. The measures of the corresponding interior angles of this triangle are in the ratio of $a:b:c$ where a, b, and c are positive integers. Find the smallest possible value of $(2a+3b+4c)$.

6. One of the edges of a rectangular solid has a length of 5 units. The number of cubic units in the volume of this rectangular solid is $\dfrac{30}{23}$ times the number of square units in the total surface area of this rectangular solid. If all edges have lengths of integral numbers of units, find the sum of all possible distinct volumes (as measured in cubic units) of this rectangular solid.

7. The reversal of 63 is 36 because the digits are reversed. The number 2662 is a palindromic number because the number and its reversal are identical. A reversal process proceeds as follows: An original two-digit number, considered to be the first number produced, is added to its reversal to produce a second number. The reversal of the second number is added to the second number to produce a third number. This reversal process continues. The fifth number produced is the first palindromic number produced in this reversal process. Find the sum of all possible distinct values of the original two-digit number.

8. Define a prime decade as a sequence of ten consecutive integers starting with a positive multiple of 10 that contains exactly four numbers that are primes. The smallest prime decade starts with the integer 10 and contains the primes 11, 13, 17, and 19. Find the **sum of the primes** in the largest prime decade less than 500.

9. Let $ABCD$ be a rectangle with $AB = 28$ and $BC = 45$. Let E be the midpoint of $\overline{AB}$, F the midpoint of $\overline{BC}$, G the midpoint of $\overline{CD}$, and H the midpoint of $\overline{AD}$. Let p be the perimeter of the quadrilateral that joins the midpoints of the sides, taken in order, of the rectangle. Let q be the sum $(EC + FD + GA + HB)$. Find the value of $|p - q|$. Write your answer as a decimal correct to four significant digits.

10. For how many distinct groups of consecutive odd positive integers will the sum of its distinct members be 576?

2009 SAA

Fr/So 2 Person Team

School ANSWERS

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. 4346	
2. 167	
3. 26	
4. $\sqrt[3]{3}$ or $3^{\frac{1}{3}}$ **(Must be an exact answer.)**	
5. 25	
6. 2400 **(Cubic units optional.)**	
7. 330	
8. 780	
9. 60.10 or 6.010×10^1 or 6.010×10 **(Must be this decimal, trailing zero necessary.)**	
10. 7	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 804

12. 512

13. 51.414 or 5.1414×10^1

14. 24

15. 42

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. One of the following four quadratic equations is selected at random: $x^2+5x+6=0$, $x^2-3x-4=0$, $x^2-8x-20=0$, and $15x-26=x^2$. Find the probability that 2 is one of the roots of the equation selected. Express your answer as a common fraction reduced to lowest terms.

2. If $P(a,b)$ is the center of the conic whose equation is $9x^2+72x-4y^2+180=0$ and $Q(c,d)$ is the focus of the conic whose equation is $y^2-6y-8x+1=0$, find the exact distance PQ.

3. What is the exact maximum value of the function $f(x)=|x-2||x-3||x-4||x-5||x-6||x-7|$ for $x \in [4,5]$? Give your answer as a reduced improper fraction.

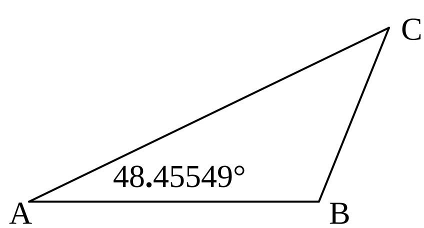
4. Let A be the period in radians of the polar graph of $r=4\cos(3\theta)$ and let B be the period in radians of the rectangular graph of $y=4\cos(\frac{3}{4}x+\pi)$. Then $A+B=\frac{k\pi}{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

5. The quartic equation $x^4+ax^3+bx^2+cx+2002=0$ has four distinct positive integral roots for x. Find the smallest possible value of a.

6. Each face of the k distinct faces of a die in the shape of a regular icosahedron bears a different positive integer from 1 through k inclusive. Such a fair die is rolled three times. Each time, exactly one face is uppermost. Find the probability that the sum of the three uppermost faces is 8. Express your answer as a common fraction reduced to lowest terms.

7. Let k be the probability that a committee of 5 randomly chosen from 5 girls and 5 boys will consist of 3 persons of one sex and 2 persons of the other. Let n be selected at random from the first 22 positive integers. Let w be the probability that $\sum_{x=1}^{n} x$ is an integral multiple of 3. Expressed as an improper fraction reduced to lowest terms, find the value of $(k+w)$.

8. Let $y_1=\frac{x+1}{x-1}$ and for all integral n such that $n \ge 2$, y_n is found by replacing x in $y_{(n-1)}$ by $\frac{x+1}{x-1}$. Let w be the value of y_6 when $x=13$. $AC=37$, $AB=13$, $\angle CAB=48.45549°$, and the area of ΔABC is K. Rounded to the nearest **integer**¸ find the numerical value of $(w+K)$.



9. Let N be the greatest common divisor of 14,040 and 202,800. Let M be the greatest common divisor of 140,448 and 428,868. Find the least common integral multiple of N and M. Express your answer as an **exact integer**. Do **not** use scientific notation.

10. Let S be the sum of all distinct positive integers that leave a remainder of 53 when divided into 7060. One of the transformations needed to produce the graph of $y = -5x^2 + 80x - 41$ from the graph of $y = x^2$ is a horizontal shift h units to the right. Find the value of $(S + h)$.

2009 SAA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer		Score (to be filled in by proctor)
1.	$\frac{1}{4}$	(Must be this reduced common fraction.)	
2.	$\sqrt{34}$		
3.	$\frac{225}{64}$	(Must be this reduced improper fraction.)	
4.	14		
5.	-153		
6.	$\frac{21}{8000}$	(Must be this reduced common fraction.)	
7.	$\frac{991}{693}$	(Must be this reduced improper fraction.)	
8.	193		
9.	326,040	(Must be this integer.)	
10.	9503		

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 10
12. -28
13. $(0.445, -1.95)$ or $(.445, -1.95)$ **(Must be this ordered pr.)**
14. -5
15. 9.5 **(Must be decimal answer.)**

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 1

ICTM 2009 DIVISION AA STATE FINALS

1. If a number k is added to the numerator of $\frac{5}{8}$ and, at the same time, twice the number k is added to the denominator of $\frac{5}{8}$, the result is $\frac{5}{9}$. Find the number k.

2. When completely simplified, $\sqrt{98}+\sqrt{450}+\sqrt{588}+\sqrt{80} = p\sqrt{2}+w\sqrt{3}+d\sqrt{5}$. Find the value of $\left(p+w+d+ANS\right)$.

3. $\Delta ABC \sim \Delta DEF$. $AB=14$, $BC=41$, and $AC=ANS$. If the smallest side of ΔDEF has a length of 70, find the perimeter of ΔDEF.

4. In right ΔABC, the ratio of lengths of sides is $8:15:17$. The area of ΔABC is ANS square units. Find the exact length of the radius of the inscribed circle in ΔABC.

ANSWERS:

1. 5

2. 45

3. 500

4. $5\sqrt{3}$ (Must be this exact radical, units optional)

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 2

ICTM 2009 DIVISION AA STATE FINALS

1. Find the positive value of k so that the product of k and $(k+3)$ is equal to twice the sum of k and $(k+3)$.

2. Find the slope of the line represented by $(5-ANS)(x+y)=(5+ANS)(x-y)$. Write your answer as a whole number or as a common or improper fraction reduced to lowest terms, whichever is correct.

3. ANS should be a common fraction reduced to lowest terms in the form $\frac{k}{w}$. ANS is the ratio of the lengths of the two legs of a right triangle. The altitude to the hypotenuse of the right triangle divides the hypotenuse into two segments, the shorter of which has length of 5. Find the exact length of the altitude to the hypotenuse. Write your answer as a common or improper fraction reduced to lowest terms, whichever is correct.

4. The area of an annulus (the region bounded by two concentric circles, outside the smaller circle, inside the larger circle) is ANS times the area of the inner circle. If the outer circle has radius whose length is 14, the length of the radius of the inner circle is k. Find the exact value of k.

ANSWERS:

1. 3

2. $\frac{3}{5}$ (Must be this reduced common fraction.)

3. $\frac{25}{3}$ (Must be this reduced improper fraction.)

4. $\sqrt{21}$ (Must be this exact radical.)

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 3

ICTM 2009 DIVISION AA STATE FINALS

1. Simplify $\dfrac{\dfrac{5}{x+3}-\dfrac{2}{3}}{\dfrac{1}{6}+\dfrac{2}{x+3}}$ when $x=7$. Write your answer as a common fraction reduced to lowest terms.

2. ANS should be a common fraction in the form $\dfrac{a}{b}$ with $a<0$ and $b>0$. Let k be an integer with $a<k<b$. What is the probability k is a solution for $x^2-6x<16$? Write your answer as a common fraction reduced to lowest terms.

3. Kite $KITE$ has $\overline{KI}\cong\overline{KE}$. $K(0,0)$, $I(8,0)$, and $E(a,b)$, with $a>0$ and $b>0$, are vertices. $\overline{KT}$ has slope ANS. Find the exact y-coordinate of point E. Write your answer as an improper fraction reduced to lowest terms.

4. ANS should be an improper fraction reduced to lowest terms in the form $\dfrac{k}{w}$. Let $d=k+w+390$. The number of distinct diagonals that can be drawn in a convex polygon is d. Find the number of sides of this convex polygon.

ANSWERS:

1. $-\dfrac{5}{11}$ or $\dfrac{-5}{11}$ or $\dfrac{5}{-11}$ (Must be this reduced common fraction.)

2. $\dfrac{3}{5}$ (Must be this reduced common fraction.)

3. $\dfrac{120}{17}$ (Must be this reduced improper fraction.)

ICTM 2009 DIVISION AA STATE FINALS
4. 34

1. Tickets to a water park cost \$7.50 for residents and \$11.50 for non-residents. If 650 people attend one day and the total amount collected is \$5715.00, then how many non-residents attended that day?

2. A computer printer can print ANS lines per second. How many **minutes** will it take the printer to print 1,000,000 lines? Give your answer as an improper fraction reduced to lowest terms.

3. A circle contains a sector subtended by a $140°$ central angle and the sector has area of $ANS\pi$. Find the exact radius of the circle. Write you answer as an improper fraction reduced to lowest terms.

4. ANS should be in the form $\frac{a}{b}$. Find the area of an obtuse triangle where 2 sides have length a and b and the included angle is $150°$.

ANSWERS:

1. 210 (Non-residents optional.)

2. $\frac{5000}{63}$ (Must be this reduced improper fraction, minutes optional.)

3. $\frac{100}{7}$ (Must be this reduced improper fraction.)

4. 175

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 5

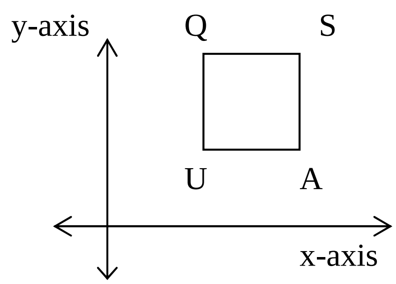
ICTM 2009 DIVISION AA STATE FINALS

1. Ryan has only dimes and quarters in his piggy bank. If the ratio of dimes to quarters is $3:2$ and the total value in the piggy bank is $3.20, how many quarters does Ryan have?

2. $25^x \bullet 5^{(4-x)} = 125^{ANS}$. Solve for x.

3. Let $k = ANS$. Two triangles are similar. The larger triangle has an area of 484; the smaller triangle has an area of k^2. Find the ratio of the length of a side of the smaller triangle to the length of a corresponding side of the larger triangle. Express your ratio as a common fraction reduced to lowest terms.

4. ANS should be a common fraction $\frac{a}{b}$ reduced to lowest terms. Let $k = \frac{1}{2}a$. Square $SQUA$ lies entirely in the first quadrant with coordinates $U(5,k)$, $A(b,5)$, and $Q(k,b)$. Q' is the reflection of Q over the y-axis and A' is the reflection of A over the x-axis. Find the area of the quadrilateral $Q'QAA'$.



ANSWERS:

1. 8 (Quarters optional.)

2. 20

3. $\frac{10}{11}$ (Must be this reduced common fraction.)

4. 110

JUNIOR-SENIOR RELAY COMPETITION ROUND 1

ICTM 2009 DIVISION AA STATE FINALS

1. $13! = \left(2^k\right)\left(3^w\right)\left(5^2\right)(7)(11)(13)$. Find the value of $(k+w)$.

2. A bag contains only red and blue marbles. This bag contains 18 red marbles and ANS blue marbles. If 2 marbles are drawn at random without replacement from this bag, find the probability that exactly 1 blue marble was drawn. Express your answer as a common fraction reduced to lowest terms.

3. ANS should be a common fraction reduced to lowest terms in the form $\frac{a}{b}$. Let $m = a - 24$. The exact distance between the vertices of the parabolas represented by $y = x^2 + 1$ and $y = x^2 + mx + 1$ can be represented in simplest form as $\frac{k\sqrt{w}}{p}$. Report the sum $(k + w + p)$ as your answer.

4. The EPA tests vehicle emissions by rotating tires on a drum system to simulate driving speed. If tires 26 inches in diameter are rotated ANS times per minute, what speed, in miles per hour, is the tread of the tire being tested? Round you answer to the nearest mile per hour.

ANSWERS:

1. 15

2. $\frac{45}{88}$ (Must be this reduced common fraction.)

3. 470

4. 36 (Miles per hour optional.)

1. Find the value of k if $(3k+2, 9-k)$ is a point on the line represented by $\frac{1}{2}x - \frac{1}{3}y - 20 = 0$.

2. Let $k = ANS$. Find the value of $\frac{(2k+1)!}{(2k-1)!}$.

3. Find the remainder when $P(x) = 2x^3 - 12x^2 + 24x - ANS$ is divided by $x+2$.

4. A charity sells exactly 1000 tickets at $1000 per ticket in a raffle. First prize is a diamond necklace worth $154,000. Second prize is a $49,000 car. There are 8 cash third prizes, and each of these third prizes is for the same amount and of lesser value than either the first or second prize. There are no other prizes, and no ticket may win more than one prize. If the expected value of each ticket is ANS dollars, find the dollar value of each of the 8 cash prizes awarded.

ANSWERS:

1. 12

2. 600

3. -712

4. 10625 ($ or dollars optional.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 3

ICTM 2009 DIVISION AA STATE FINALS

1. Using only the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 and allowing for repetition of digits, how many 3-digit numbers can be formed in which the digit 7 appears at least once?

2. Let k be an integer and let k^2 have the largest possible value less than ANS. A rhombus has one diagonal that is twice the length of the second diagonal. The area of the rhombus is k^2. Find the exact perimeter of the rhombus.

3. ANS should be in the simplified radical form $p\sqrt{w}$. Let $f\left(x\right)=28x^2-7$ and $g\left(x\right)=5x+k$. Find the positive value of k so that the graph of $\left(f\circ g\right)\left(x\right)$ intersects the y-axis at $\left(0,2p\right)$. Express your answer as a **decimal**.

4. Let $k=ANS$. Find the exact distance from the point $P\left(2,4,3\right)$ to the plane $2x+ky+6z+11=0$.

ANSWERS:

1. 217

2. $28\sqrt{5}$ (Must be this exact answer.)

3. 1.5 (Must be this decimal.)

4. 6

JUNIOR-SENIOR RELAY COMPETITION ROUND 4

ICTM 2009 DIVISION AA STATE FINALS

1. How many positive integers k leave a remainder of 5 when 65 is divided by that integer k ?

2. Let $k = ANS + 24$. Find the sum of the first k terms of the increasing sequence $1, 6, 11, 16, 21, 26, \cdots$ where each term is one more than a multiple of 5.

3. a and b are integers in polynomial $P(x)$. Applying the Rational Root Theorem, $P(x) = 19x^7 + 8x^6 + ax^5 - bx^3 + 3x^2 + 2x + ANS$ has *potential* rational roots (which may or may not be actual roots.) Find the sum of the positive *potential* rational roots that are integers.

4. Let $p = \frac{ANS}{28}$. When written in completely simplified standard form and in decreasing powers of x, the fourth term of $\left(\frac{4}{3}x + ky\right)^{10}$ has numeric coefficient p. Find the exact value of k. Write your answer as a common fraction reduced to lowest terms.

ANSWERS:

1. 7

2. 2356

3. 4480

4. $\frac{9}{16}$ (Must be this simplified common fraction.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 5

ICTM 2009 DIVISION AA STATE FINALS

1. $P(x)$ is a polynomial in x. If $\left(3x^4-7x^3+5x^2-4x+2\right)-P(x)=x^4+6x^3+x^2-5$, what is the coefficient of the degree 3 term of $P(x)$?

2. Let r and t represent the solutions to $x^2-5x+ANS=0$. Find the value of $\frac{2}{rt^2}+\frac{2}{r^2t}$. Write your answer as a common fraction reduced to lowest terms.

3. Find the value of the eccentricity of the conic $(ANS)x^2-\frac{10}{27}y^2=1$. Write your answer as an improper fraction reduced to lowest terms.

4. ANS should be an improper fraction reduced to lowest terms in the form $\frac{k}{w}$. Let $p=k+w-1$. Let $f(x)=3+7\tan(13x+p)$. Find the negative value of x closest to zero for which $f(x)=3$. Write your answer as a decimal correct to **4 decimal places.**

ANSWERS:

1. -13

2. $\frac{10}{169}$ (Must be this reduced common fraction.)

3. $\frac{14}{13}$ (Must be this reduced improper fraction.)

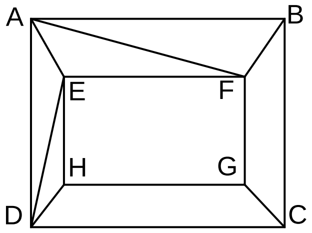
4. -0.0667 or $-.0667$ (Must be this exact decimal.)

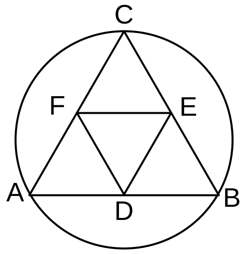
Questions for the Oral Competition – State AA Level, 2009
Ch. 1, 2, For All Practical Purposes, sixth edition

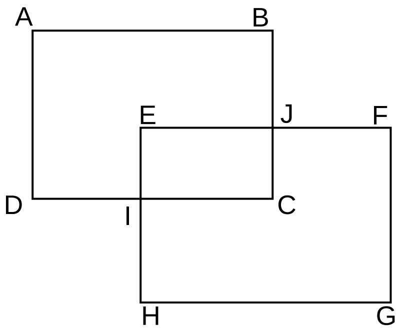
Answers for the Questions for the Oral Competition – State AA Level, 2009

1. In your own words, explain the difference between an Euler circuit and a Hamiltonian circuit.

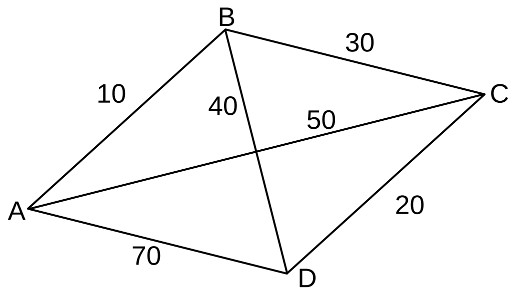
   Determine whether each of the figures below contains an Euler circuit and/or a Hamiltonian circuit.

   a)


   b)


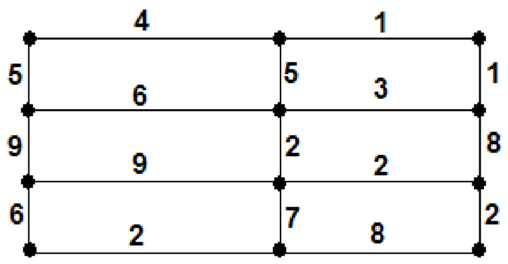
   c)


2. a. Use the algorithms listed to solve the traveling salesman problem shown in the graph below starting with vertex A. Explain the procedures that you used for each method.
   i. brute force
   ii. nearest neighbor
   iii. sorted edges

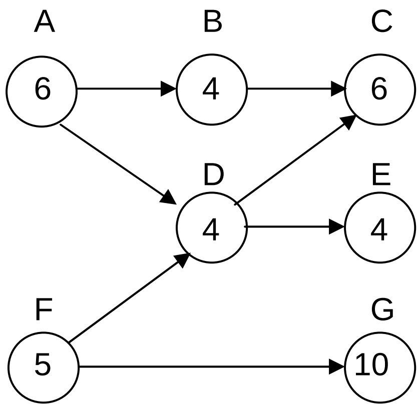


   b. For each of the methods that you used, would the length of the path be different if you started at vertex B? Explain your reasoning.

3. Use Kruskal's algorithm for determining a minimum –cost spanning tree on the graph below. What is the cost of the tree found? Explain how you determined it.



4. a. Give the minimum completion time and critical paths for the order-requirement diagraph below.



b. Reducing the time on one task would create four critical paths. Reducing the time on which task would accomplish this? What would be the new minimum completion time?

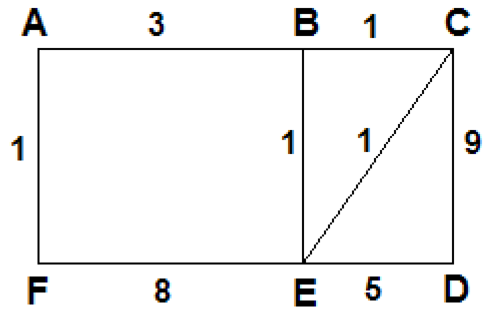
Extra Questions

1. What is the difference between a spanning tree and a Hamiltonian circuit?

2. State one advantage and one disadvantage to a heuristic algorithm.

3. There are six juniors and six seniors on your math team including you. Each person lives at a different location. You want to plan to visit each of your teammates and return home. If it takes ½ minute to compute the total length of one tour, set up (but do not evaluate) the expression you would use to determine how long it will it take to apply the brute force algorithm to find the optimal tour.

4. It is possible to have more than one spanning tree with the same minimum cost in one graph. Find two different minimum-cost spanning trees for the graph below.

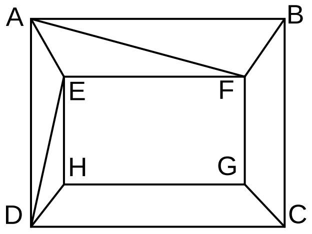


Answers for the Questions for the Oral Competition – State AA Level, 2009

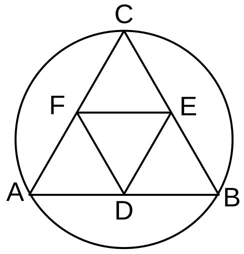
1. In your own words, explain the difference between an Euler circuit and a Hamiltonian circuit.
   An Euler circuit must cover each edge only once and return to its starting location. A Hamiltonian circuit must visit each vertex once and return to its starting location.

   Determine whether each of the figures below contains an Euler circuit and/or a Hamiltonian circuit.

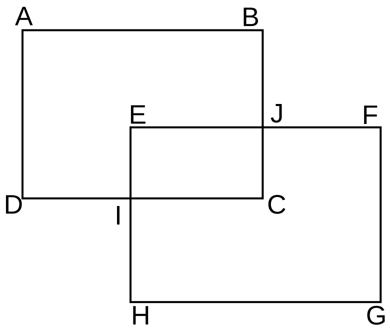
a)



b)

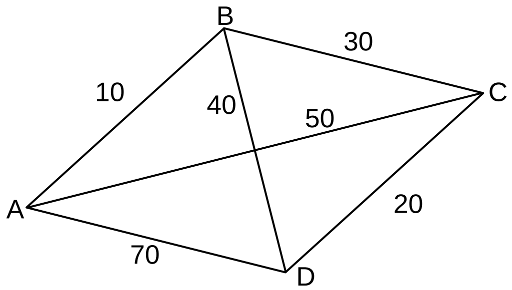


c)



a) No Euler, Yes Hamiltonian
b) Yes Euler, Yes Hamiltonian
c) Yes Euler, No Hamiltonian

2. a. Use the algorithms listed to solve the traveling salesman problem shown in the graph below starting with vertex A. Explain the procedures that you used for each method.
   i. brute force
   ii. nearest neighbor
   iii. sorted edges



i) ABCDA = 10 + 30 + 20 + 70 = 130
ABDCA = 10 + 40 + 20 + 50 = 120 optimal route
ACBDA = 50 + 30 + 40 + 70 = 190

ii) At each vertex choose the path that has the least cost. This gives us path ABCDA = 130.

iii) Sort the edges from least cost to most cost. This gives us AB = 10, CD = 20, BC = 30, BD = 40, AC = 50, AD = 70. Use AB, CD, BC. That leaves us with having to use AD to complete the circuit, so the path is ABCDA = 130

b. For each of the methods that you used, would the length of the path be different if you started at vertex B? Explain your reasoning.

Brute force – No. We already have an optimal route.

BACDB = 120 optimal BCDAB = 130 BDACB = 190

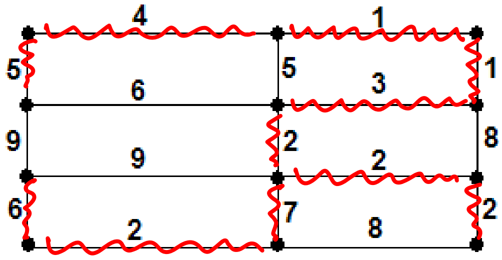
(or explain why the length would not change)

Nearest Neighbor – Yes, if we started at vertex B, we would have an optimal route of 120.

Sorted edges – No. We have already used the three shortest edges.

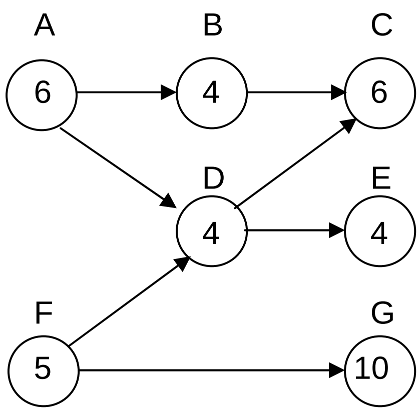
3. Use Kruskal's algorithm for determining a minimum –cost spanning tree on the graph below. What is the cost of the tree found? Explain how you determined it.

Use the least costs first. Continue using the smallest costs unless a circuit is formed. The costs are 1, 1, 2, 2, 2, 2, 3, 4, 5, 5, 6, 6, 7, 8, 8, 9, 9. The squiggly lines show the minimum-cost spanning tree with a cost of 35.



4 + 1 + 5 + 3 + 1 + 2 + 2 + 6 + 2 + 7 + 2 = 35

4. a. Give the minimum completion time and critical paths for the order-requirement diagraph below.



Possible paths are:
ABC – length 16
ADC – length 16
ADE – length 14
FDC – length 15
FDE – length 13
FG – length 15

Minimum completion time is 16 with critical paths ABC and ADC.

b. Reducing the time on one task would create four critical paths. Reducing the time on which task would accomplish this? What would be the new minimum completion time?

Reducing Task A by one (to a value of 5) would create a length of 15 for ABC and ADC. Thus, the new minimum completion time would be 15 with four critical paths (ABC, ADC, FDC, and FG)

Answers for the Extra Questions

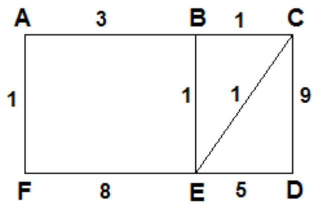
1. What is the difference between a spanning tree and a Hamiltonian circuit?
Both a spanning tree and a Hamiltonian circuit visit every vertex, but a Hamiltonian circuit must complete a closed circuit and end up where it started.

2. State one advantage and one disadvantage to a heuristic algorithm.
Advantage – It is fast. Disadvantage – It may not give an optimal solution

3. There are six juniors and six seniors on your math team including you. Each person lives at a different location. You want to plan to visit each of your teammates and return home. If it takes ½ minute to compute the total length of one tour, set up (but do not evaluate) the expression you would use to determine how long it will it take to apply the brute force algorithm to find the optimal tour.

$$\left(\frac{11!}{2}\right)\left(\frac{1}{2}\right)$$

4. It is possible to have more than one spanning tree with the same minimum cost in one graph. Find two different minimum-cost spanning trees for the graph below.



There are three possibilities, all with length 11. Students can give any two of the three possibilities.