1. One of the whole numbers greater than 150.2 and less than 157.2 is selected at random. Find the probability that the number selected is a prime number. Express your answer as a common fraction reduced to lowest terms.

2. If 25% of a number is 42, find the number.

3. $3a - 2b + a + 3(a+b) - (2a-b) = ka + wb$ where k and w are integers. Find the value of $(2k+3w)$.

4. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

   If the average of four different positive integers is 3, then no two of the numbers have a sum that is less than 3.

5. Find the sum of all distinct integers greater than 10 and less than 100 that are increased by 9 when their digits are reversed.

6. If $\sqrt{x} = \sqrt{992 + \sqrt{992 + \sqrt{992 + \sqrt{992 + \cdots}}}}$, find the value of x.

7. From a penny, a nickel, and a dime, two different coins are selected at random. Find the probability that the total monetary value of the two coins selected is an integral multiple of five cents. Express your answer as a common fraction reduced to lowest terms.

8. A number n is increased by 5, and the result is multiplied by 10. This result is decreased by 10. Finally, that result is divided by 5. In terms of n, the final result is $kn+w$. Find the value of $(2k+w)$.

9. The solution set for the inequality $\frac{5}{x+1}<4$ is $\{x : x>k \text{ or } x<w\}$. Find the value of $(k-w)$. Express your answer as a reduced common or improper fraction, whichever is appropriate.

10. All ages in this problem are in whole number of years. Cindy is now 3 times as old as Jeffrey was when Cindy was 5 times as old as Jeffrey had been when Jeffrey was half as old as he is now. The sum of their present ages is 58 years. Find the number of years in Cindy's age now.

11. If A and B are real numbers such that $\frac{25x-115}{x^2-8x-9}=\frac{A}{x-9}+\frac{B}{x+1}$, find the value of $(A-B)$.

12. Kay can run 600 feet at a constant rate in 40 seconds, and Tom can run 120 feet at a constant rate in 15 seconds. If Tom has a head start of 7 **minutes**, how many **minutes** will it take Kay to catch Tom if one assumes that each continues to run at their given constant rates?

13. Let a, b, and c be integers. Let the graph of $y=ax^2+bx+c$ contain the points $(-3,299)$ and $(3,17)$. The graph also contains the point $(k,0)$ where k is an integer. Find the y-coordinate of a point on this graph that has an x-coordinate of 25.

14. In a group of 60 sophomores, 10 read "Newsweek" and "Time", 20 read "Newsweek", and 15 read "Time." Find the number of sophomores in this group that read neither magazine.

15. There are two different numbers formed using the same two non-zero digits, one number with the digits in reverse order from the other. The sum of the numbers is 7 less than 28 times the absolute difference between the two digits. Find the absolute value of the difference between the squares of the two numbers.

16. If x and y are consecutive integers, find the value of k for the system

$$\begin{cases} 3(x-4)+2(y+3)=81 \\ 5(2x-y)+ky=206 \end{cases}$$

17. How many of the first million positive odd integers are integral multiples of all five of the numbers 3, 5, 9, 11, and 63?

18. If the amount that $66\frac{2}{3}\%$ of 120 exceeds $37\frac{1}{2}\%$ of 88 by is equal to $3x+2$, find the value of x.

19. If x and y are real numbers such that $x^2+y^2+x+y=254$ and $x+y+xy=125$, find the largest possible value of (xy).

20. Let k be an integer such that both roots for x for the equation $x^2-3kx+2k^2=3-2x+5k$ are greater than -5 but less than 15. Find the sum of all distinct values of k.

2010 SA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{7}$ (Must be this reduced common fraction.)

2. 168

3. 16

4. Always (Must be this whole word.)

5. 404

6. 1024

7. $\frac{1}{3}$ (Must be this reduced common fraction.)

8. 12

9. $\frac{5}{4}$ (Must be this reduced improper fraction.)

10. 42 (Years optional.)

11. –3

12. 8 (Minutes optional.)

13. 215

14. 35

15. 2079

16. 7

17. 289

18. 15

19. 104

20. 20

ITEM ANALYSIS
Div 1A – 113 papers
% correct

1. 70%	11. 7%
2. 95%	12. 32%
3. 42%	13. 1%
4. 41%	14. 32%
5. 44%	15. 46%
6. 11%	16. 33%
7. 67%	17. 0%
8. 29%	18. 40%
9. 65%	19. 27%
10. 7%	20. 6%

ITEM ANALYSIS
Div 2A – 130 papers
% correct

1. 68%	11. 6%
2. 98%	12. 43%
3. 54%	13. 1%
4. 54%	14. 40%
5. 59%	15. 6%
6. 17%	16. 34%
7. 76%	17. 0%
8. 57%	18. 58%
9. 13%	19. 23%
10. 6%	20. 3%

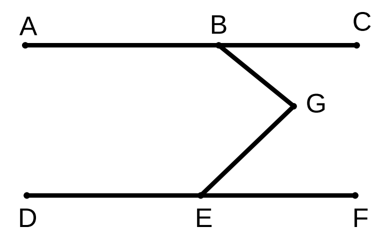
1. From the 3 ordered triples $(3,4,5)$, $(9,40,41)$, and $(5,8,9)$, one ordered triple is selected at random. Find the probability that the ordered triple selected can be the lengths of the sides of a right triangle. Express your answer as a common fraction reduced to lowest terms.

2. Let k be the number of sides in a polygon. Find the smallest possible value of k.

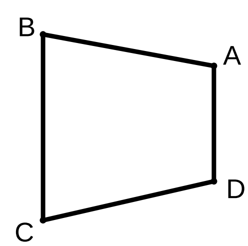
3. A rectangle whose side-lengths are $\sqrt{854}$ and $\frac{68}{3}$ is inscribed in a circle. The ratio of the length of a diagonal of the rectangle to the length of a radius of the circle is $k:w$ where k and w are positive integers. Find the smallest possible value of $(2k+3w)$.

4. Two vertices of an isosceles right triangle are $A(0,0)$ and $B(12,0)$. If $\overline{AB}$ is a leg, and the other leg lies in Quadrant IV, find the **ordered pair** that represents the third vertex of the triangle.

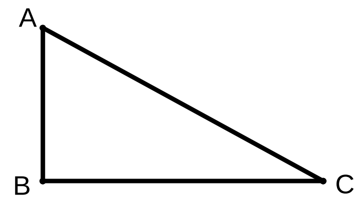
5. In the diagram, the line containing A, B, and C is parallel to the line containing D, E, and F. If $\angle BGE = 108°$ and $\angle FEG = 42°$, find the degree measure of $\angle ABG$.



6. In the diagram, $ABCD$ is an isosceles trapezoid with $\overline{BC}$ as one of the bases. $AD = kx + 28$, $DC = 6.8x - 21.1$, $BC = 15.1x$, and $AB = 4.3x + 6.9$. If k is a positive integer, find the smallest possible value of k.



7. In the triangle, $AB = \sqrt{19}$, $BC = 9$, and $AC = 10$. A point is selected at random on segment $\overline{BC}$. Find the probability that the distance from A to the point selected is more than $\sqrt{23}$. Express your answer as a common fraction reduced to lowest terms.



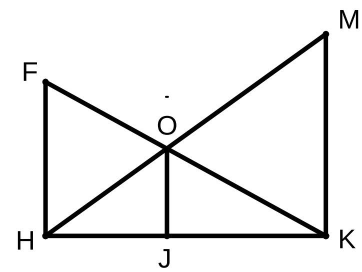
8. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never—**whichever is correct.

If, given two congruent circles, one lies completely in Quadrant I and the other lies completely in Quadrant II, then the midpoint of the line segment joining the centers of these two circles lies on the *y*-axis.

9. From the four members of the set $\{2, 3, 4.52, 6\}$, one member is selected at random. If that member selected represents the length of a radius of a sphere, find the probability that the sphere has a volume greater than 110. Express your answer as a common fraction reduced to lowest terms.

10. In rhombus $ABCD$, $\angle DAB = 60°$. A circle passes through vertices A, B, and D, and intersects diagonal $\overline{AC}$ at E. If $EC = 10$, then the area of the circle passing through A, B, D, and E is $k\pi$. Find the value of k.

11. In the diagram, $\overline{FH} \perp \overline{HK}$, $\overline{OJ} \perp \overline{HK}$, and $\overline{MK} \perp \overline{HK}$. Point O lies on $\overline{FK}$ and $\overline{HM}$, and point J lies on $\overline{HK}$. If $FH = 14$, $MK = 16$, and $HK = 48$, find OJ. Express your answer as an improper fraction reduced to lowest terms.

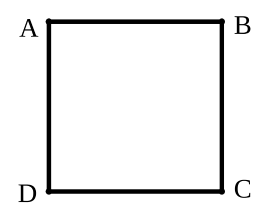


12. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never—**whichever is correct.

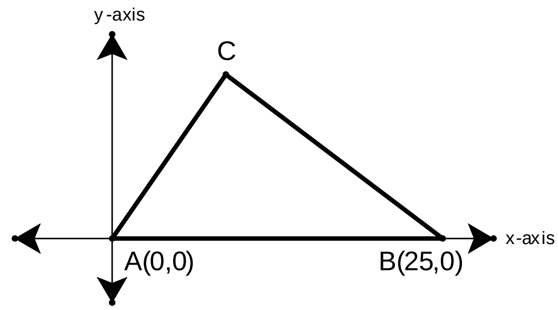
If a line segment connects the midpoints of two sides of a given triangle, then that line segment divides the original triangle into a new triangle and a trapezoid such that the area of the trapezoid is 7 times the area of the new triangle.

13. The numerical values of the area and perimeter of a triangle are both positive integers. If two of the sides of the triangle have respective lengths of 41 and 50, find the sum of all distinct possibilities for the length of the third side.

14. In the rectangle shown, $AB = 2$ and $CB = 3$. A light beam is fired from B toward $\overline{AD}$ at a $45°$ angle with $\overline{AB}$ and is reflected from each side of the rectangle until it reaches one of the vertices. Which vertex will it reach?

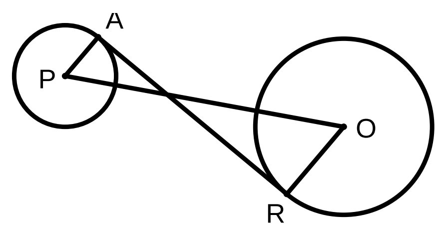


15. In the diagram with coordinates as shown, $AC = 29$, and $BC = 36$. Find the **ordered pair** that represents point C. Express your answer as an **ordered pair** with **each member** of the ordered pair expressed as an improper fraction reduced to lowest terms.



16. A star is formed by extending the sides of a regular dodecagon until they reach their initial point of intersection (i.e., regular polygon $ABCDEFGHIJKL$ will have $\overline{AB}$ extended to meet $\overline{CD}$ extended, $\overline{BC}$ extended to meet $\overline{DE}$ extended, etc. The star is then a non-convex polygon with 24 sides.). Find the sum of the degree measures of the angles at the12 vertices of the star.

17. In the diagram, P and O are centers of the circles. $\overline{AR}$ is a common internal tangent segment with points of tangency at A and R. $PA = 5$ and $OR = 6$. Let the length of $\overline{PO}$ be a number chosen from the set $\{12,13,14,15,16,17,18,19,20\}$. Find the sum of all distinct numbers for the length $\overline{PO}$ such that, when expressed in simplest radical form, $AR = k\sqrt{3}$ or $AR = 3\sqrt{w}$ where k and w are positive integers.



18. The length of the hypotenuse of a right triangle is 5, and the area of the right triangle is 6. Find the sum of the lengths of the two legs of the right triangle.

19. The point $A(1,6)$ is rotated $90°$ clockwise about the point $(4,2)$ to point B. Find the absolute value of the distance from point B to the line represented by $5x - 12y + 137 = 0$.

20. There are four spherical balls of four different sizes. Each of these balls lies on the floor in one of the four corners of a rectangular room (one ball per corner, tangent to two walls). On two of the balls, there is a point on each of the two balls which is 6 inches from each wall which that ball touches and 8 inches from the floor. On the remaining two balls, there is a point on each of the two balls which is 5 inches from each wall which that ball touches and 12 inches from the floor. Find the number of inches in the sum of the diameters of the balls (one diameter per ball).

2010 SA Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced common fraction.)

2. 3

3. 7

4. $(12, -12)$ (Must be this ordered pair.)

5. 114 (Degrees optional.)

6. 3

7. $\frac{7}{9}$ (Must be this reduced common fraction.)

8. Sometimes (Must be the whole word.)

9. $\frac{3}{4}$ (Must be this reduced common fraction.)

10. 100

11. $\frac{112}{15}$ (Must be this reduced improper fraction.)

12. Never (Must be the whole word.)

13. 222

14. A OR "Vertex A"

15. $\left(\frac{17}{5}, \frac{144}{5}\right)$ (Must be this ordered pair with reduced improper fractions.)

16. 1440 (Degrees optional.)

17. 63

18. 7

19. 9

20. 84 (Inches optional.)

ITEM ANALYSIS
Div 1A – 133 papers
% correct

1. 75%	11. 9%
2. 76%	12. 63%
3. 35%	13. 6%
4. 23%	14. 31%
5. 46%	15. 7%
6. 4%	16. 8%
7. 25%	17. 5%
8. 65%	18. 76%
9. 40%	19. 3%
10. 23%	20. 0%

ITEM ANALYSIS
Div 2A – 135 papers
% correct

1. 91%	11. 13%
2. 90%	12. 63%
3. 50%	13. 2%
4. 40%	14. 51%
5. 74%	15. 23%
6. 13%	16. 19%
7. 38%	17. 6%
8. 84%	18. 85%
9. 62%	19. 13%
10. 22%	20. 0%

1. From the set $\{5,10,15,\cdots,5n,\cdots,45\}$, one member is selected at random. Find the probability that the member selected is an **even** integer. Express your answer as a common fraction reduced to lowest terms.

2. Find the product of the five prime positive integral factors of 3570.

3. Find the sum of the first ten positive integers.

4. When $(2y+x)^5$ is expanded and completely simplified, what is the numerical coefficient of the y^2x^3 term?

5. Let $i=\sqrt{-1}$ and let k represent a real number. If $\frac{3-i}{k+4i}=0.2-0.15i$, find the value of k.

6. The parabola which is the graph of the equation $y=ax^2+bx+c$ passes through the points $(-16,-226)$, $(4,64)$, and $(12,208)$. Find the focus of this parabola. Express your answer as an **ordered pair** of the form (x,y).

7. If the letters of "ILLINOIS" are jumbled, selected randomly one at a time without replacement, and written on a straight line in the order of the draw, find the probability that the three "I" letters are together, in adjacent positions in the random re-arrangement of letters. Express your answer as a common fraction reduced to lowest terms.

8. In a certain variation, z varies jointly as x and y and inversely as w^3. In this variation, $z=800$ when $x=10$, $y=64$, and $w=2$. Find the value of z when $x=5$, $y=54$, and $w=3$.

9. If the multiplicative inverse of $\begin{bmatrix} 1 & 3 \\ 2 & k \end{bmatrix}$ is $\begin{bmatrix} 4 & -1.5 \\ -1 & 0.5 \end{bmatrix}$, find the value of k.

10. The three real roots for x of $27x^3+ax^2+bx-125=0$ are in geometric progression, and a and b are both integers. If the smallest root for x were decreased by $\frac{20}{9}$, then this resulting number and the other two roots for x would be in arithmetic progression. Find the value of a.

11. The sum of two numbers is 14. If $\frac{1}{3}$ of the square of the smaller is added to the larger, find the smallest possible result that can be obtained. Express your answer as an improper fraction reduced to lowest terms.

12. If the system $\begin{cases} 2x-3y+z=4 \\ 4x-2y+3z=11 \\ x+5y-7z=9 \end{cases}$ is solved by the determinant method for y, then:

$$y=\frac{\begin{vmatrix} 2 & a & b \\ c & d & e \\ 1 & f & g \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 4 & -2 & 3 \\ 1 & 5 & -7 \end{vmatrix}}$$

. Find the value of $(2c+3d+4g)$.

13. The lines f and k are symmetric to each other with respect to the line $y=2x$. The equation of line f is $y=\frac{1}{8}x+\frac{1}{2}$. The equation of line k can be written as $y=mx+b$. Find the value of $(m+b)$. Express your answer as a decimal.

14. Let $i=\sqrt{-1}$. One of the three distinct cube roots of -8 can be expressed in the form $k+i\sqrt{w}$, where k and w are positive integers. Find the value of $(2k+3w)$.

15. By substituting 1, 2, 3, and 4 for n in a quadratic expression for n, the first four terms are respectively 2, 6, 12, and 20. The $(n+1)^{st}$ term of this sequence is equal to $n^k + wn + p$ where k, w, and p are positive integers. Find the value of $(2k+3w+4p)$.

16. When completely simplified, one of the terms of the expansion of $(x+\frac{y}{2})^9$ is $\frac{k}{w}x^3y^p$ where k, w, and p are positive integers. Find the minimum value of $(k+w+p)$.

17. In a previous year, 8412 students participated in the ICTM State Math Contest. Let each of those students receive a number, and let the numbers be 8412 consecutive integers starting with 1 and ending with 8412. If 5 of those numbers were drawn at random (one at a time without replacement), find the probability that the 5 numbers drawn were in descending order. Express your answer as a common fraction reduced to lowest terms.

18. How many distinct integral multiples of 6 are there between 1 and 207?

19. The terms of an arithmetic sequence are $175, 185, 195, \cdots$. The terms of the triangular sequence whose n^{th} term is $\frac{n^2+n}{2}$ are $1, 3, 6, 10, \cdots$. Find the value of the first term of the triangular sequence that is greater than the corresponding term of the arithmetic sequence.

20. By substituting 1, 2, 3, 4, 5, and 6 for x in a polynomial expression for x, the first six terms are respectively 2, 24, 72, 158, 294, and 492. If $P(x)$ is the polynomial expression of lowest degree satisfying the given, find $P(37)$.

2010 SA Algebra II

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{4}{9}$ (Must be this reduced common fraction.)
2. 3570
3. 55
4. 40
5. 12
6. $(-64, -512)$ (Must be this ordered pair.)
7. $\frac{3}{28}$ (Must be this reduced common fraction.)
8. 100
9. 8
10. –195
11. $\frac{53}{4}$ (Must be this reduced improper fraction.)
12. 13
13. –0.75 OR –.75
14. 11
15. 21
16. 43
17. $\frac{1}{120}$ (Must be this reduced common fraction.)
18. 34
19. 496
20. 102854

ITEM ANALYSIS
Div 1A – 140 papers
% correct

1.	73%	11.	5%
2.	54%	12.	6%
3.	82%	13.	1%
4.	44%	14.	9%
5.	22%	15.	5%
6.	7%	16.	20%
7.	7%	17.	2%
8.	32%	18.	79%
9.	29%	19.	19%
10.	2%	20.	6%

ITEM ANALYSIS
Div 2A – 145 papers
% correct

1.	91%	11.	15%
2.	68%	12.	9%
3.	89%	13.	2%
4.	70%	14.	15%
5.	55%	15.	18%
6.	13%	16.	43%
7.	6%	17.	2%
8.	37%	18.	81%
9.	64%	19.	41%
10.	3%	20.	26%

1. In quadrilateral $ABCD$, $\angle ABC = 102°$. From the set of integral degree values $\{1°, 2°3°, \cdots, n°, \cdots, 84°\}$ one member is selected at random as the measure of $\angle DAB$. Find the probability that $\angle ABC$ is supplementary to $\angle DAB$. Express your answer as a common fraction reduced to lowest terms.

2. Let vector $\vec{c} = (1, -4, k)$ and let vector $\vec{d} = (4, 6, 1)$. If the two given vectors are perpendicular to each other, find the value of k.

3. For all positive integers n such that $n > 1$, $t_n = 2t_{(n-1)}$. If $t_1 = 3$, find the value of t_3.

4. Find the sum of the next four terms after 13 in the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, $\cdots$

5. If $\tan(x°) = \dfrac{1}{3}$, and if $x - y = 45°$, find $\tan(y°)$. Express your answer as a common fraction reduced to lowest terms.

6. Let $a_n = \dfrac{2n}{n^2+2}$. Let $L = \lim\limits_{n\to\infty}(\dfrac{2n}{n^2+2})$. Find the smallest **positive** integral value of n for which $|L - a_n| < \dfrac{1}{12}$.

7. The equation of a parabola whose vertex is at $(0, -2)$ and whose focus is $(0, 11)$ can be written as $x^2 = k(y + w)$. Find the value of $(k + w)$.

8. If a normal, fair cubical die is rolled 7 times, find the probability that the uppermost face will be either a 2 or a 4 exactly 4 times, either a 1, 3 or 5 exactly twice, and a 6 exactly once. Express your answer as a decimal rounded to 4 significant digits.

9. The coordinate axes of the graph of the equation $7x^2 + 6\sqrt{7}\ xy - 6y^2 = 35$ are rotated through a positive angle (θ) such that $\theta > 180°$ so as to eliminate the xy term. Find the smallest possible value of θ. Express the degree measure of θ as a **decimal** rounded to the nearest hundredth of a degree.

10. The graph of $y = \dfrac{12x^2 + 21x - 2}{2x^2 - x - 8}$ has a horizontal asymptote of $y = k$. Find the value of k.

11. A rectangle that is **not** a square has both diagonals drawn. From the four sides and the two diagonals, two segments are selected at random without replacement. Find the probability that the two segments are congruent. Express your answer as a common fraction reduced to lowest terms.

12. Let k represent a positive integer. If $\dfrac{k\pi}{12}$ is a radian measure, find the smallest possible value of k such that $\tan(\dfrac{k\pi}{12}) = 2 + \sqrt{3}$.

13. Tiger tees off on the 16^{th} hole of the Illini golf course. His ball leaves the tee (assume at ground level) with an initial velocity of 186.4 feet per second and at an initial angle with the horizontal of 28.46°. Assuming the ground is level and that the effect-on-gravity vector is $\left(0, -16t^2\right)$ where t is measured in seconds, find the horizontal distance in feet that the ball will have traveled when it first hits the ground after launch. Express your answer as a **decimal** rounded to 4 significant digits.

14. $S = \{3100,4000,3693,2800,2100,1300,4700,3000,2595,1302,4200,5400,2605,2900,2400\}$
Set S contains 15 distinct elements. The six-number summary—that is, the minimum data value; the first quartile; the median; the third quartile; the maximum data value; and the arithmetic mean are respectively 1300, 2400, 2900, 4000, 5400, and 3073

Let x be an integral multiple of 1001 such that $1000 < x < 9999$. Let a second set consist of the 15 distinct elements of the above given set S, and let the sixteenth element of this second set be x. If at least 5 values of the six-number summary of the second set are different than their respective values of the six-number summary of the original 15 element set, find the sum of all possible distinct values of x.

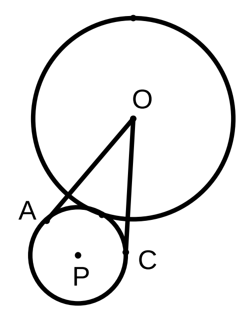
15. In a room in a shape of a rectangular solid with a high ceiling, a cube with an edge of 3 is placed so that one face is flat against a wall and an adjacent face is flat against the floor. A ladder with a length of 16 is placed in such a way that the top is flat against the wall and the ladder rests completely against the free horizontal edge of the cube. Find the maximum number of units in the height above the floor that is touched by the top of the ladder. Express your answer as a **decimal** rounded to the nearest hundredth.

16. The first term of a geometric sequence is a negative number, the second term is 2, and the fourth term is 4. Find the third term.

17. Point A has a coordinate of 0 and Point E has a coordinate of 100 on line segment $\overline{AE}$. B and C are two points between A and E with C between B and E. Let point B be chosen at random such that $BC = 10$. A point D is picked at random between A and E. If the probability that the coordinate of D is at least k more than the coordinate of B is 0.4675, find the value of k. Express your answer as an **exact decimal**.

18. One of the vertical asymptotes of the graph of $f(x) = \dfrac{x-7}{x^2-5x+6}$ is $x = 3$. The other vertical asymptote is $x = k$. Find the value of k.

19. In the diagram, a circle with center at P is externally tangent to the circle with center at O. Circle P is also tangent to $\overleftrightarrow{OA}$ at A and to $\overleftrightarrow{OC}$ at C. $\angle AOC = 28°$, and the length of a radius of the circle with center at P is 9.438. Find the length of a radius of the circle with center at O. Express your answer as a **decimal** rounded to the nearest hundredth.



20. The Polar Coordinates of points A, B, and C are $A(12, 49°)$, $B(77, 77°)$, and $C(48, 2°)$. Find the radius of the **inscribed** circle of ΔABC. Round your answer for the radius to the nearest integer and express your answer as that **integer**.

2010 SA

Pre-Calculus

Name ANSWERS

School ______

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{84}$ (Must be this reduced common fraction.)

2. 20

3. 12

4. 199

5. $-\frac{1}{2}$ OR $\frac{-1}{2}$ OR $\frac{1}{-2}$ (Must be this reduced common fraction.)

6. 24

7. 54

8. 0.05401 OR .05401 (Must be this decimal.)

9. 205.34 (Must be this decimal.)

10. 6

11. $\frac{1}{5}$ (Must be this reduced common fraction.)

12. 5

13. 909.8 (Must be this decimal.)

14. 31031

15. 15.56 (Must be this decimal.)

16. $-2\sqrt{2}$ (Must be this exact answer.)

17. 8.25 (Must be this decimal.)

18. 2

19. 29.57 (Must be this decimal.)

20. 15 (Must be this integer.)

ITEM ANALYSIS
Div 1A – 111 papers
% correct

1. 68%	11. 23%
2. 13%	12. 74%
3. 64%	13. 3%
4. 73%	14. 7%
5. 54%	15. 1%
6. 24%	16. 31%
7. 11%	17. 1%
8. 3%	18. 89%
9. 0%	19. 15%
10. 68%	20. 4%

ITEM ANALYSIS
Div 2A – 134 papers
% correct

1. 90%	11. 42%
2. 29%	12. 78%
3. 82%	13. 8%
4. 81%	14. 17%
5. 68%	15. 2%
6. 42%	16. 52%
7. 15%	17. 1%
8. 3%	18. 95%
9. 1%	19. 21%
10. 78%	20. 8%

NO CALCULATORS

1. A five person committee is to be chosen at random from a group that consists of 11 women and 10 men. Find the probability that the first person chosen for this committee will be a woman. Express your answer as a common fraction reduced to lowest terms.

2. Find the sum of all distinct **integers** in the solution set of the equation $(x-5)(x+6.4)(4.3x+12.9)(2x-1)(5x-60)=0$

3. Find the value of x such that $8^{(x+5)}\left(\frac{1}{4}\right)^{(x-1)}=\left(2^5\right)\left(2^2\right)\left(8^{(-x+2)}\right)$.

4. The length of a side of a square is the same as the length of a side of a regular pentagon. The regular pentagon has the same perimeter as the perimeter of a regular hexagon. The four sides of the square, the five sides of the pentagon, and the six sides of the hexagon are then laid end to end in a straight line segment which has a length that is the sum of the 15 line segments. If a point is selected at random in the interior of this line segment, find the probability that the point selected was on one of the original sides of the regular hexagon. Express your answer as a common fraction reduced to lowest terms.

5. A square has a side with a length of 48. By connecting the consecutive midpoints of this square, a quadrilateral is formed. By connecting the consecutive midpoints of the quadrilateral, a second quadrilateral is formed. This process is continued for a third, fourth, fifth, and sixth quadrilateral. Find the sum of the **areas** of the six quadrilaterals. Note: the original square is **not** to be considered as one of the six quadrilaterals.

6. One day Bob was observing two female monkeys at the zoo. He noticed that Michelle ate 3 more bananas than Patty did. If Michelle and Patty ate a total of 13 bananas, how many bananas did Michelle eat?

7. Karen rides her bicycle from A to B at the constant rate of 15 mph. She then walks her bicycle from B toward A at the constant rate of 4 mph. for a time that is 25% of the time it took her to ride from A to B. After walking, she immediately gets on her bicycle and finishes the journey to A at a constant rate of 12 mph. Find the number of miles per hour in Karen's average rate for the round trip from A to B and back to A. Express your answer as an improper fraction reduced to lowest terms.

NO CALCULATORS

8. On a circle whose radius has a length of 600, find the exact **length** of an arc of the circle if the arc is intercepted by a central angle of $90°$.

9. If three times the degree measure of the complement of an angle is subtracted from the degree measure of the supplement of the angle, the answer is $20°$. Find the degree measure of the supplement of the angle.

10. A car and a boat were sold for $12,000 each. The car was sold at a loss of 20% of its cost, and the boat was sold at a gain of 20% of its cost. Find the number of dollars in the net gain or net loss of the two transactions. For your answer, write the number of dollars and then attach **gain** or **loss**, whichever is correct.

11. Marnie has only nickels, dimes, and quarters in her purse. She has exactly the same number of each coin in her purse, and the total value of those coins is $6.80. Find the total number of coins that Marnie has in her purse.

12. The sides of a triangle have lengths of 7, 15, and 20. The area of the circumscribed circle of this triangle can be expressed as $\frac{k\pi}{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

13. For all real numbers a and b, $a \otimes b = a^2 b$ and $a \oplus b = \frac{a}{b} - b$. Find the value of $(2 \otimes 3) \oplus 2$.

14. Two fair, standard cubical dice are rolled. Find the probability that the dots on the two uppermost faces add up to less than five. Express your answer as a common fraction reduced to lowest terms.

15. One root for x of the equation $x^2 + kx + 1 = 0$ is twice the other root for x. If $k > 0$, then k can be expressed in the form $\frac{p\sqrt{w}}{k}$ where p, w, and k are positive integers. Find the smallest possible value of $(k + w + p)$.

NO CALCULATORS

NO CALCULATORS

16. If a number whose base five representation is 423_{five} is expressed in base nine, find that base nine representation and attach base nine subscript in your answer.

17. $\sqrt{67+42\sqrt{2}} = a+b\sqrt{c}$ where $a, b,$ and c are integers. Find the value of $(a+b+c)$.

18. Let $A = \{(18,-5),(27,3),(-14,5),(0,10),(24,11),(-5,-2)\}$ Find the sum of the largest number and the smallest number in the range of A.

19. The solutions to the equation $3x^2 - 10x + 8 = 0$ are the lengths of the legs of a right triangle. The length of a radius of a circle that is inscribed in that right triangle can be expressed as $\frac{k-\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.

20. In the addition alphametic shown to the right, each letter stands for the same digit throughout the puzzle. No digit is represented by more than one letter, and $P \neq 0$. For your answer write the 6 digits (from left to right in order) that represent $DELAYS$.

$$\begin{array}{cccccccc} & P & L & E & A & S & E \\ + & P & A & R & D & O & N \\ \hline & D & E & L & A & Y & S \end{array}$$

2010 SA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{11}{21}$ (Must be this reduced common fraction.)

2. 14

3. -1

4. $\frac{5}{14}$ (Must be this reduced common fraction.)

5. 2268

6. 8 (Bananas optional.)

7. $\frac{360}{29}$ (Must be this reduced improper fraction.)

8. 300π (Must be this exact answer.)

9. 125 (Degrees optional.)

10. 1000 Loss (Must have "loss", optional.)

11. 51

12. 629

13. 4

14. $\frac{1}{6}$ (Must be this reduced common fraction.)

15. 7

16. 135_9 OR 135_{nine} (Must have subscript.)

17. 12

18. 6

19. 21

20. 915608

ITEM ANALYSIS
Div 1A – 22 papers
% correct

1. 86%	11. 81%
2. 14%	12. 10%
3. 10%	13. 67%
4. 14%	14. 33%
5. 14%	15. 10%
6. 95%	16. 14%
7. 5%	17. 10%
8. 48%	18. 10%
9. 52%	19. 0%
10. 24%	20. 0%

ITEM ANALYSIS
Div 2A – 23 papers
% correct

1. 96%	11. 83%
2. 35%	12. 0%
3. 43%	13. 96%
4. 43%	14. 30%
5. 26%	15. 17%
6. 100%	16. 52%
7. 4%	17. 4%
8. 70%	18. 48%
9. 61%	19. 9%
10. 52%	20. 0%

NO CALCULATORS

1. Let $C = \{1, 2, 3, 4, 5, 20, 365, 980, 861, 924, 18740, 864555\}$. If one of the members of C is selected at random, find the probability that the number selected is an integral multiple of five. Express your answer as a common fraction reduced to lowest terms.

2. In which quadrant is secant positive and cotangent negative? Express your answer as a **Roman numeral**.

3. Find the value of x such that $\log_3 x - \log_3 4 + \log_3 7 = 3$. Express your answer as an improper fraction reduced to lowest terms.

4. There are 38 members of the Math Department at Large High School. If each member of the Math Department shakes the right hand of every other Math Department member exactly once, find the total number of hand shakes that will take place.

5. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   The graph of $x^2 - 23y^2 = 0$ is:

   A) a parabola B) a hyperbola C) a circle that is **not** an ellipse
   D) an ellipse E) a pair of straight lines F) a point

   Note: Be certain to write the correct capital letter as your answer.

6. Let $i = \sqrt{-1}$. If $3x + 2yi = 60 - 18i$ and x and y are real numbers, find the value of $(2x + 3y)$.

7. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   Define the difference set $A - B$ as the set $\{x \in A : x \notin B\}$. $(A \cap B) \cup (A - B) =$
   (Note: In set notation, S' is one representation for the complement of set S.)

   A) A B) B C) $A \cup B$ D) $A' \cup B'$ E) none of the previous

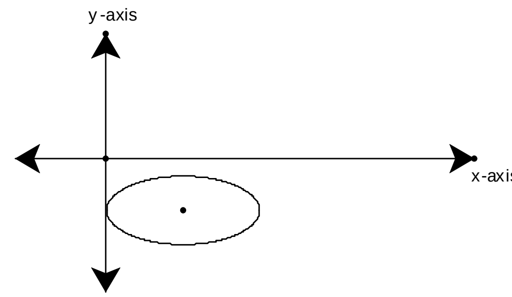
   Note: Be sure to write the correct capital letter as your answer.

8. The graph of f consists of the union of two line segments. The first line segment connects $(0, -1)$ to $(3, 2)$, and the second line segment connects $(3, 2)$ to $(5, 0)$. The graph of $y = -2f(x - 2) + 6$ also consists of the union of two line segments. One line segment connects $(5, 2)$ to $(7, 6)$, and the other line segment connects $(5, 2)$ to (x, y). Find the ordered pair (x, y).

NO CALCULATORS

9. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

The ellipse shown to the right has the equation $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$. What are the respective signs of h and k?



A) $+,+$ B) $+,-$ C) $-,+$ D) $-,-$ E) Both h and k are zero

Note: Be certain to write the correct capital letter as your answer.

10. If x and y are real, and $x^2+y^2=1$, find the maximum value of $(x+y)^3$.

11. The equation of one of the asymptotes of the graph of $xy=-16$ is $x=0$. Give the equation of the other asymptote.

12. Let a, b, and c be integers and let $x^3+ax^2+bx+c=0$ have $\sqrt[3]{11}+\sqrt[3]{121}$ as one of the roots for x. Find the value of $(a+b+c)$.

13. The arithmetic mean of 6 and n is 12 more than the positive geometric mean of 6 and n. Find the value of n.

14. Without replacement, three of the nine letters of "TENNESSEE" are selected at random. Find the probability that all three letters selected were the letter E. Express your answer as a common fraction reduced to lowest terms.

15. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

The graph of $\begin{cases} x=8t-4 \\ y=8t^2 \end{cases}$ in the $xy-plane$ is:

A) a circle B) a parabola C) an ellipse that is **not** a circle
D) a hyperbola E) a straight line

Note: Be sure to write the correct capital letter as your answer.

NO CALCULATORS

16. Find the value of x such that $\log x - 6\log 3 = -2$. Express your answer as a **decimal**.

17. In the game of Yahtzee there are five standard 6-sided dice. A full house consists of 3 numbers of one kind (sixes, for example) and 2 of another (fours, for example). On the first roll of these 5 dice, Jefferson gets 3 numbers of one kind, 1 number of a second kind, and 1 number of a third kind. Jackson suggests keeping the 3 numbers of one kind, picking up one of the other dice and rolling that die again. Madison suggests keeping the 3 numbers of one kind, picking up both of the other dice, and rolling those 2 dice again. Considering these two suggestions, find the absolute value of the difference of the two probabilities of obtaining a full house. Express your answer as a common fraction reduced to lowest terms.

18. Find the value of $10^{(\log_{10} 17)}$.

19. The roots for x of $x^3 - 55x^2 + 887x + k = 0$ are three real numbers, and k is an integer. The roots for x of $x^3 + ax^2 + cx + d = 0$ are each 2 more than the respective roots for x of the cubic equation in the first sentence of this problem. Find the value of c if a, c, and d are integers.

20. Let a, b, and c be the lengths of the sides of Triangle ABC (with a opposite $\angle A$, etc.). If $3a - 2b = 0$ and $4a^2 - 4ac + c^2 = 0$, then, in simplest radical form, $\sin(B) = \dfrac{k\sqrt{w}}{16}$ where k and w are positive integers. Find the value of $(k + w)$.

2010 SA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)
2. IV (Must be this Roman numeral.)
3. $\frac{108}{7}$ (Must be this reduced improper fraction.)
4. 703
5. E (Must be this capital letter.)
6. 13
7. A (Must be this capital letter.)
8. $(2,8)$ (Must be this ordered pair.)
9. B (Must be this capital letter.)
10. $2\sqrt{2}$ (Must be this exact answer.)
11. $y = 0$ (Must be this equation.)
12. -165
13. 54
14. $\frac{1}{21}$ (Must be this reduced common fraction.)
15. B (Must be this capital letter.)
16. 7.29 (Must be this Decimal.)
17. $\frac{1}{36}$ (Must be this reduced common fraction.)
18. 17
19. 1119
20. 18

ITEM ANALYSIS
Div 1A – 23 papers
% correct

1. 100%	11. 83%
2. 70%	12. 0%
3. 48%	13. 9%
4. 35%	14. 39%
5. 30%	15. 78%
6. 52%	16. 43%
7. 22%	17. 17%
8. 4%	18. 91%
9. 52%	19. 4%
10. 17%	20. 4%

ITEM ANALYSIS
Div 2A – 23 papers
% correct

1. 100%	11. 91%
2. 87%	12. 0%
3. 57%	13. 17%
4. 57%	14. 65%
5. 48%	15. 96%
6. 87%	16. 65%
7. 30%	17. 26%
8. 13%	18. 96%
9. 83%	19. 0%
10.39%	20. 4%

Answers should be expressed either in **scientific notation OR** as a **decimal,** and answers should be rounded to four **significant** digits. **However**, specific instructions in a given problem take precedence. For example, if instructions ask for the answer to be expressed as a **decimal** or as an **integer**, you may NOT use scientific notation for that answer.

1. If leaves are falling at the constant rate of 7 leaves every 29 seconds, how many leaves will fall during a 2 day period? Round your answer to the nearest integer, and express your answer as that **integer**.

2. If $3.002^{(x+2.987)} = 135.1$, find the value of x.

3. By how much does the perimeter of a square whose side has a length of π exceed the circumference of a circle whose radius has a length of 1?

4. In a circle, chords $\overline{AD}$ and $\overline{BC}$ intersect at E. If $AE = 12.78$ and $DE = 78.03$, find the value of $(BE)(CE)$.

5. Find the degree measure of the positive acute angle formed by the lines $2x + y = 7$ and $6x - 3y = 4$.

6. A regular hexagon is inscribed in a circle with a radius whose length is 17.14. An equilateral triangle is inscribed in a circle with a radius whose length is 12.39. Find the absolute value of the difference between the area of the regular hexagon and the area of the equilateral triangle.

7. In ΔABC with $A(403,14)$, $B(23,-489)$, and $C(57,79)$, the median is drawn from C to $\overline{AB}$. Find the length of this median.

8. The lengths of the sides of a triangle are respectively 31, 35, and x. There are two distinct irrational values of x for which one of the angles of the triangle is $60°$. Find the product of these two irrational values of x.

9. If $\log_{2.223}(x-3.017)=5.114$, find the value of $\log_{9.117}(x-3.124)$

10. As viewed from a window that is 189**.**1 feet above a level surface, the angle of depression of an ant on that level surface is $28.14°$. Find the distance from the ant to the viewing point at the window.

11. Assume the earth is a sphere with radius 6001 kilometers. Assume Miami, Florida and Erie, Pennsylvania are on the same meridian. The latitude of Miami is $25°46'37''N$ and the latitude of Erie is $42°7'15''N$. Find the number of kilometers between the two cities traveling across the surface of the sphere.

12. Each face of the k distinct faces of a die in the shape of a regular icosahedron bears a different positive integer from 1 through k inclusive. Such a fair die is rolled twice. Each time, exactly one face is uppermost. Find the probability that the sum of the two uppermost faces is less than 15. Express your answer as a common fraction reduced to lowest terms.

13. A student received a grade of 83 on a final examination in math for which the mean grade was 78 and the standard deviation was 8. Find the “standard score” on this exam for this student. Express your answer as an **exact decimal**.

14. One of the four interior angles of a parallelogram has a degree measure of $66.58°$. Find the sum of the degree measures of the two largest interior angles of this parallelogram. Express your answer as an **exact decimal**.

15. Find the sum of all possible distinct lengths of the radius of the circumscribed circle of a right triangle if one leg has a length of 406.5 and if one of the acute angles of the right triangle has a measure of $28°$.

16. When $(1.42x-3.57y)^7$ is expanded and completely simplified, what is the numerical coefficient of the x^5y^2 term?

17. Every night Bob goes to Point A between 9:30 P. M. and 11:30 P. M. to watch the stars. Every night, Sandy goes to Point A between 9:00 P. M. and 1:00 A. M. to watch the stars. Bob stays for exactly 10 minutes and then leaves, while Sandy stays for exactly 20 minutes and then leaves. If the arrival times are randomly chosen within the respective intervals, find the probability that on a given night the two will be together at Point A. Express your answer as a common fraction reduced to lowest terms.

18. If $i=\sqrt{-1}$, then $\left(4.049i^{23}\right)^3 = ki$. Find the value of k.

19. An observer on a tower that rises vertically notes that two objects on a horizontal road below have respective angles of depression of 46° and 29° respectively. If the eye of the observer is 100 feet above the horizontal road and if the road runs directly away from the observer, find the number of feet in the distance between the objects.

20. A circle has a center at $P(-2,6)$. Points $R(13,26)$ and $T(-9,30)$ lie on the circle. A line with equation $y=4x-89$ intersects the circle at points F and G. Find the length of the minor arc of the circle from F to G.

2010 SA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** ☐

Note: **All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.**

1. 41710 (Must be this integer.)

2. 1.476 OR 1.476×10^{0}

3. 6.283 OR 6.283×10^{0}

4. 997.2 OR 9.972×10^{2}

5. 53.13 OR 5.313×10^{1} OR 5.313×10

6. 563.8 OR 5.638×10^{2}

7. 352.9 OR 3.529×10^{2}

8. 1259 OR 1.259×10^{3}

9. 1.848 OR 1.848×10^{0}

10. 401.0 OR 4.010×10^{2}

11. 1712 OR 1.712×10^{3}

12. $\frac{91}{400}$ (Must be this reduced common fraction.)

13. 0.625 OR .625 OR 0.6250 OR .6250

14. 226.8 (Must be this exact decimal.)

15. 663.1 OR 6.631×10^{2}

16. 1545 OR 1.545×10^{3}

17. $\frac{1}{8}$ (Must be this reduced common fraction.)

18. 66.38 OR 6.638×10^{1} OR 6.638×10

19. 83.84 OR 8.384×10^{1} OR 8.384×10

20. 1.941 OR 1.941×10^{0}

ITEM ANALYSIS
Div 1A – 22 papers
% correct

1.	59%	11.	9%
2.	91%	12.	0%
3.	45%	13.	18%
4.	86%	14.	41%
5.	9%	15.	5%
6.	27%	16.	36%
7.	41%	17.	0%
8.	9%	18.	54%
9.	59%	19.	23%
10.	18%	20.	0%

ITEM ANALYSIS
Div 2A – 29 papers
% correct

1.	69%	11.	45%
2.	100%	12.	3%
3.	83%	13.	21%
4.	69%	14.	69%
5.	31%	15.	24%
6.	28%	16.	59%
7.	59%	17.	7%
8.	24%	18.	86%
9.	79%	19.	69%
10.	45%	20.	10%

1. Given the set of prime integers $\{2,3,5,7,11,\cdots,67\}$, if one of the integers in the set is selected at random, find the probability that the integer selected is odd. Express your answer as a common fraction reduced to lowest terms.

2. The perimeter of an isosceles triangle is 36, and the length of one side of the triangle is known to be 10. Find the sum of the two distinct possible areas of the triangle.

3. Find the smallest 3 digit number that is the square of an integer and is also the cube of some other integer.

4. Find the value of $10110_{two} + 2102_{three} + 3112_{four} + 534_{six}$. Express your answer in **base ten.**

5. Let p be the product of the two distinct roots of $|x+5| = 18$. Let k be the number of distinct diagonals that can be drawn from **one vertex** of a convex pentadecagon. Find the value of $(k+p)$.

6. Let k be the smallest integer greater than 54 that is **not** the sum of two or more consecutive **odd** natural numbers. Quadrilateral $ABCD$ is inscribed in a circle. $\angle ABC = 80°$, $\angle BCD = x°$, $\angle BAD = (x+40)°$. Find the value of $(k+x)$.

7. Let k be the area of a circle whose diameter has a length of 78.33. Let w be the area of a square whose diagonal has a length of 28.86. Find the value of $(k+w)$. Express your answer as a **decimal** rounded to the nearest **tenth.**

8. The lines whose equations are $2x+5y=21$, $3x=7y+75$, and $kx+(k+3)y=6$ all intersect at the same point. Find the product of the slopes of the three lines. Express your answer as a common fraction reduced to lowest terms.

9. The expression: $8x^2-288$ can be factored over the integers as $k(x+w)(x-w)$ where k and w are positive integers. Let p be the perimeter of a rectangle whose diagonal has a length of 65 and one of whose sides has a length of 33. Find the value of $(k+w+p)$.

10. Find the smallest positive integer k such that $\frac{k!}{160^{11}}$ is an integer.

2010 SA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. $\frac{18}{19}$ (Must be this reduced common fraction.)	______
2. 108	______
3. 729	______
4. 503 OR 503_{10} OR 503_{ten}	______
5. –287	______
6. 128	______
7. 5235.3 (Must be this decimal.)	______
8. $\frac{3}{70}$ (Must be this reduced common fraction.)	______
9. 192	______
10. 60	______

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. ______

12. ______

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let $D = \{143, 168, 179, 2112, 568733, 4688\}$. If one of the six members of D is selected at random, find the probability that the member selected is an integral multiple of 11. Express your answer as a common fraction reduced to lowest terms.

2. Name all quadrants in which **no points** of the hyperbola $xy = -16$ lie? Express any and all such quadrants as **Roman numerals**.

3. If $10^p = 2$, $\log(k) = w$, and $\log(f) = g$, then $\log(4k^2) + \log(125f^3)$ is equal to $a + bp + cw + dg$. Find the numerical value of $(a + b + c + d)$.

4. Let the three terms $2x+5$, $3x-7$, and $8x-40$ taken in that order form an arithmetic sequence. Let k be the number of terms in an arithmetic sequence whose sum is 400, whose 1st term is 10, and whose last term is 15. Find the value of $(x+k)$. Express your answer as a **decimal**.

5. If $(x+2y)^8$ is expanded and completely simplified, one of the terms is kx^3y^5. Find the value of k.

6. The vertices of ΔABC are at $A(4,3)$, $B(14,5)$, and $C(-2,1)$. Let k be the length of the median from C to $\overline{AB}$. Let w be the length of the altitude from C to $\overline{AB}$. Find the value of $(k+w)$. Express your answer as a **decimal** rounded to the nearest thousandth.

7. A side of equilateral triangle A is 2, a side of equilateral triangle B is 3, square C has a side of 5, and square D has a side of 4. Let k be a side of an equilateral triangle whose area is the sum of A and B, and let w be a side of a square whose area is the sum of C and D. Expressed as a **decimal rounded to 4 significant digits**, find the value of $(k+w)$.

8. Let $i = \sqrt{-1}$. The quartic equation $x^4 - px^3 + kx^2 + wx - 500 = 0$ where k, w, and p are integers has $3-4i$ and -2 as two of its roots for x. The fourth term of an arithmetic sequence of 7 positive integers is 14, another term (not the last) is 20, the common difference is d with $d > 0$, and the sum is 98. Find the value of $(p+d)$.

9. Let k be the absolute value of the difference between the sum of the roots of $x^3 - 13x^2 - 54x + 360 = 0$ and the sum of the **squares** of the roots of $y^2 + 6y + 2 = 0$. Let θ be an integer such that $90 < \theta < 647$. Let S be the sum of all distinct values of θ such that $\sin(47°) = \sin(\theta°)$. Find the value of $(k + S)$.

10. Let $y = \dfrac{3x+2}{6x-5}$ where x can be any real number except $\dfrac{5}{6}$. Let k be the one real value that is **not** in the range of values for y. If 5 cards are selected at random from 5 hearts and 5 spades, let p be the probability that at least 4 of the cards selected are hearts. Find the value of $(k + p)$.

2010 SA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. $\frac{1}{2}$ (Must be this reduced common fraction.)	______
2. I, III (Either order, must be these Roman numerals.)	______
3. 7	______
4. 37.25 (Must be this decimal.)	______
5. 1792	______
6. 12.186 (Must be this decimal.)	______
7. 10.01 (Must be this decimal.)	______
8. 17	______
9. 1052	______
10. $\frac{38}{63}$ OR $0.\overline{603174}$ OR $.\overline{603174}$ (Must be exact answer.)	______

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. ______

12. ______

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

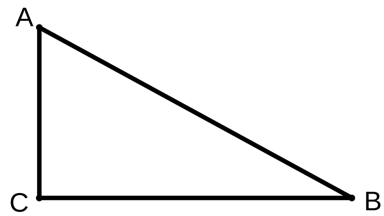
1. How many of the following four equations have a **positive** integer for the solution? For your answer, write: 0, 1, 2, 3, or 4, whichever is correct.

A) $2(x+5)=2$ B) $3-(y+7)=1$ C) $\dfrac{x+14}{3x+18}=\dfrac{3}{5}$

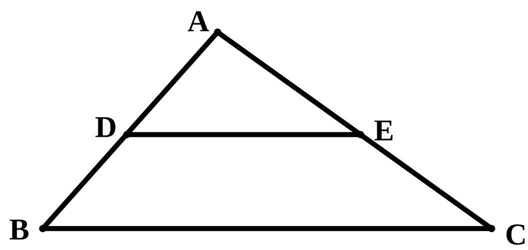
D) $5(k+8.14)=3.12-k$

2. Red left point A at 1:00 and traveled due north at a constant speed of 50 mph. ANS hour(s) later, Bud left point A and traveled due north at a constant speed of 60 mph. How many hours will Bud have traveled when Bud catches Red?

3. In ΔACB, $\angle ACB$ is a right angle. If $AC = 2$ and $AB = ANS$, find CB. Express your answer as a **decimal** rounded to the nearest tenth.



4. ΔADE is similar to ΔABC. $BC = 12.6$, $DB = 11.97$, $AC = 7.6$, $DE = ANS$. Find AD. Express your answer as a **decimal**, rounded to the nearest tenth if necessary.



1. Find the largest integer k for which 7^k is a factor of 14406.

2. June went to the store and purchased 7 bananas at 29 cents apiece, ANS apples at 46 cents apiece, and 9 oranges at 37 cents apiece. Find the total cost of all these fruit items and express your answer in cents.

3. The area of a rectangle is ANS, and the base of the rectangle has a length of 40. Find the perimeter of this rectangle.

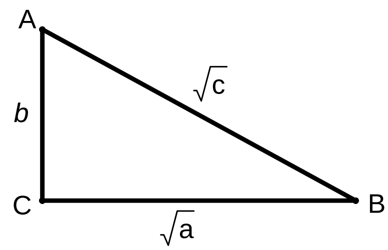
4. Two angles are supplementary, and one of the angles has a degree measure that is *ANS* degrees more than the other. Find the degree measure of the complement of the smaller of the two angles.

1. A mathtingle is a weasel whose length is 12 inches plus $\frac{3}{4}$ of its own length. Find the number of inches in the length of a mathtingle.

2. Find the result when 21 is subtracted from the sum of all prime positive integers that are greater than 4 and less than ANS .

3. A positive integer is selected at random from all positive integers that are greater than ANS and less than 2011. Find the probability that the integer selected represents the number of degrees in the sum of the measures of the angles of a regular polygon. Express your answer as a common fraction reduced to lowest terms.

4. Let $\angle ACB = 90°$, $AC = b$, $BC = \sqrt{a}$, and $AB = \sqrt{c}$. ANS should be a common fraction in the form of $\frac{k}{w}$ where k and w are relative prime positive integers. Let $c = k + w$. If a, b, and c are positive integers such that $a > 732$, find the sum of all possible distinct values of b.

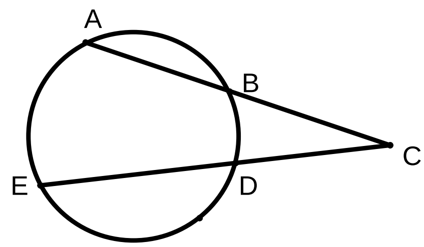


1. Given the 5 column arrangement of integers as shown. In the first row, 2 falls in column 2, 3 falls in column 3, 4 falls in column 4, and 5 falls in column 5. If the pattern continues, find the **number** of the column in which 37 will fall.

	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	

2. If $\left(\frac{1}{16}\right)^{ANS} = 32^{(x-11)}$, find the value of 2^x.

3. In the figure, points A, B, D, and E lie on the circle. Points A, B, and C are collinear, and points C, D, and E are collinear. If $\overset{\frown}{AE} = (ANS)°$ and $\overset{\frown}{BD} = 104°$, find the number of degrees in $\angle ACE$.



4. Two coplanar circles are concentric. A chord of the larger circle is tangent to the smaller circle. The length of that chord is ANS. A radius of the larger circle has a length of 10. The area of the region bounded between the two circles is $k\pi$. Find the value of k.

1. Edward starts at point A and walks due north at 3 mph. Two hours later, Ville starts at point A and walks due north at 4 mph. Find the number of <u>**minutes**</u> that Ville will walk before she catches Edward.

2. Five times a number added to twenty more than twelve times the number is ANS. Find the number.

3. The lengths of the diagonals of a rhombus are respectively ANS and 48. Find the area of the rhombus.

4. The perimeter of a regular hexagon is ANS. Find the positive difference between the radius of a circumscribed circle of this hexagon and the radius of an inscribed circle of this hexagon. Express your answer as a **decimal** rounded to the nearest hundredth.

2010 SA FR/SO RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. 1
2. 5
3. 4.6 (Must be this decimal.)
4. 6.9 (Must be this decimal.)

ROUND 2

1. 4
2. 720 (Cents optional.)
3. 116
4. 58 (Degrees optional.)

ROUND 3

1. 48 (Inches optional.)
2. 302
3. $\frac{5}{854}$ (Must be this reduced common fraction.)
4. 66

EXTRA ROUND 4

1. 5
2. 128
3. 12 (Degrees optional.)
4. 36

EXTRA ROUND 5

1. 360 (Minutes optional.)
2. 20
3. 480
4. 10.72 (Must be this decimal.)

1. Edward starts at point A and walks due north at 3 mph. Two hours later, Ville starts at point A and walks due north at 4 mph. Find the number of **minutes** that Ville will walk before she catches Edward.

2. Five times a number added to twenty more than twelve times the number is ANS. Find the number.

3. The lengths of the diagonals of a rhombus are respectively ANS and 48. Find the area of the rhombus.

4. The perimeter of a regular hexagon is ANS. Find the positive difference between the radius of a circumscribed circle of this hexagon and the radius of an inscribed circle of this hexagon. Express your answer as a **decimal** rounded to the nearest hundredth.

1. If 13 is the first number in an arithmetic sequence and 22 is the third number in this arithmetic sequence, find the eleventh number in this arithmetic sequence.

2. Find the length of a radius of a circle whose equation is $x^2 + (ANS)x + y^2 = 183$.

3. Let $0° < k° < 540°$. Find the sum of all distinct values of k such that $\cos(k)° = \sin(ANS)°$.

4. Let $f(x) = 3x + 7$ and $g(x) = 3.2x + 34.1$. Find the value of $f(g(ANS))$. Express your answer as an **exact decimal**.

1. Find the value of the difference of the determinants as shown:

$$\begin{vmatrix} 8 & 1 \\ 2 & 16.5 \end{vmatrix} - \begin{vmatrix} \frac{464}{9} & 0 \\ 403.1 & 2.25 \end{vmatrix}$$

2. The trinomial $x^2 - px + ANS$ factors over the integers into $(x-k)(x-w)$ where k and w are **positive** integers. Find the sum of all distinct possibilities for p.

3. If $C(n,r)$ represents a symbol for the combination of n things taken r at a time, find the smallest possible positive integer n such that $C(n,2) > ANS$.

4. In a random roll of two normal and fair cubical dice, find the probability that the sum of the number of dots showing on the upper faces is greater than ANS. Express your answer as a common fraction reduced to lowest terms.

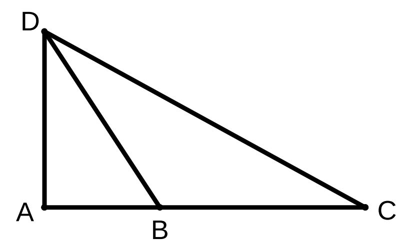
1. Let $f(x) = 17x + 8$. By how much does $3f(2)$ exceed $2f(3)$?

2. Let k, w, and p represent integers. The quartic equation $x^4 + kx^3 + wx^2 + px - 144 = 0$ factors into $(x^2 + (ANS)x - 48)(x^2 + x + 3) = 0$. One of the roots of that quartic equation is a positive integer. That integer is the length of a side of an equilateral triangle. Find the area of that equilateral triangle.

3. The points $(\sqrt{45}, -8\sqrt{3})$ and $(3\sqrt{5}, ANS)$ are the endpoints of a diameter of a circle. Find the *y*-coordinate of the center of this circle.

4. When simplified, ANS should be in the form $k\sqrt{w}$. Find the real value of x which is a solution of the equation $\sqrt{\left(\frac{1}{x}+1\right)}+\frac{1}{x}+1+kw=0$.

1. In right triangle DAC, $\angle DAC$ is a right angle. Point B lies on $\overline{AC}$ as shown. If $DA = 8$, $CD = 17$, and $DB = 10$, find BC.



2. ANS is the length of a leg in each of two right triangles. The hypotenuse of one right triangle has a length of 41, and the hypotenuse of the other right triangle has a length of 15. By how much does the length of the remaining leg of the larger right triangle exceed the length of the remaining leg of the smaller right triangle?

3. *ANS* is the sum of the terms of an infinite geometric sequence whose first term is 20. Find the ratio of the fourth term to the third term. Express your answer as a common fraction reduced to lowest terms.

4. The cosine of the largest **acute** angle of a right triangle is ANS . Find the sum of the sines of the other two angles of this right triangle. Express your answer as an improper fraction reduced to lowest terms.

2010 SA JR/SR RELAY COMPETITION
PROCTOR ANSWER SHEET

ROUND 1

1. 8
2. 80
3. -50
4. 9

ROUND 2

1. 58
2. 32
3. 778
4. 7578.1 (Must be this decimal.)

ROUND 3

1. 14
2. 24
3. 8
4. $\frac{5}{18}$ (Must be this reduced common fraction.)

EXTRA ROUND 4

1. 8
2. $4\sqrt{3}$
3. $-2\sqrt{3}$
4. $\frac{1}{3}$ OR $.\overline{3}$ OR $0.\overline{3}$

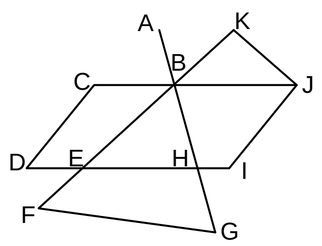
EXTRA ROUND 5

1. 9
2. 28
3. $\frac{2}{7}$ (Must be this reduced common fraction.)
4. $\frac{9}{7}$ (Must be this reduced improper fraction.)

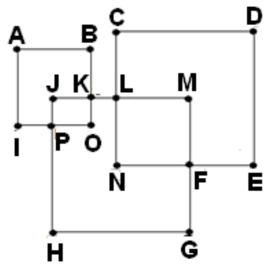
Questions for the Oral Competition – Division A, State Finals 2010

1. a) Explain the difference between an Euler circuit and a Hamiltonian circuit.
   b) Determine whether each of the figures below is an Euler circuit and/or a Hamiltonian circuit.

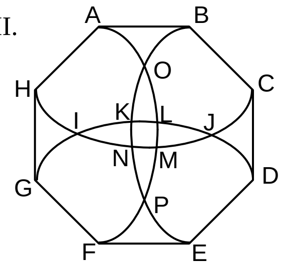
I.



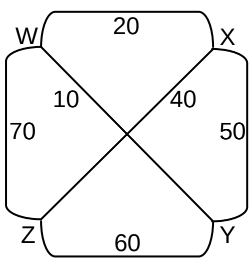
II.



III.



2. Use the algorithms listed to solve the traveling salesman problem with the given graph below. Start at vertex W. Explain the procedures that you used for each method.
   a. brute force
   b. nearest neighbor
   c. sorted edges



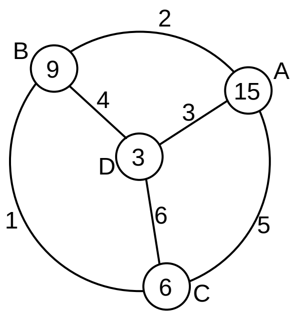
3. A manufacturing process consists of 6 tasks, each of which takes the given amount of time:

   Task A: 4 minutes
   Task B: 6 minutes
   Task C: 4 minutes
   Task D: 5 minutes
   Task E: 6 minutes
   Task F: 3 minutes

   Task A must be completed before either Task B or C can be started. Tasks A and B must be completed before Task D can be started**.** Tasks A and C must be completed before Task E can be started. Task F must be completed before Task C can be started.

   a. Draw an order requirement diagraph for this manufacturing process
   b. Find the critical path that gives the earliest completion time for the process.

4. In the graph below, the number in the circle at each vertex is the cost of relaying a message through that vertex. The number on an edge indicates the cost of providing service between the endpoints of the edge. A relay can send a message to only one vertex at a time. There is no cost to initiate or receive a message at a vertex, only to relay the message through the vertex. Begin at vertex A and find the minimum cost for sending a message to all vertices, including the relay costs.

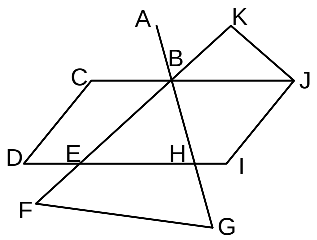


Solutions for the Oral Competition – Division A, State Finals 2010

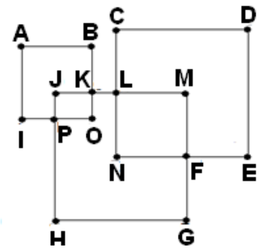
1. a. Explain the difference between an Euler circuit and a Hamiltonian circuit.
   An Euler circuit must cover each edge only once and return to its starting location. A Hamiltonian circuit must visit each vertex only once and return to its starting location.

   b. Determine whether each of the figures below is an Euler circuit and/or a Hamiltonian circuit.

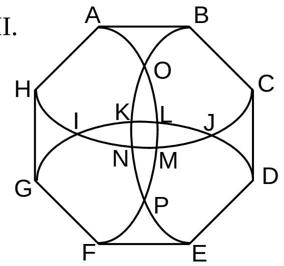
I.



II.

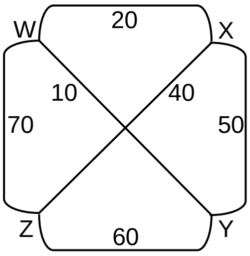


III.



I) No Euler, No Hamiltonian II) Yes Euler, No Hamiltonian III) No Euler, Yes Hamiltonian

2. Use the algorithms listed to solve the traveling salesman problem with the given graph below. Start at vertex W. Explain the procedures that you used for each method.
   a. brute force
   b. nearest neighbor
   c. sorted edges



a. WXYZW = 20 + 50 + 60 + 70 = 200
   WXZYW = 20 + 40 + 60 + 10 = 130 (optimal)
   WYXZW = 10 + 50 + 40 + 70 = 170

b. At each vertex, choose least cost. Therefore, WYXZW = 170

c. Sort edges from least to most: WY = 10, WX = 20, XZ = 40 plus connector ZY = 60 for total circuit WXZYW = 130

3. A manufacturing process consists of 6 tasks, each of which takes the given amount of time:

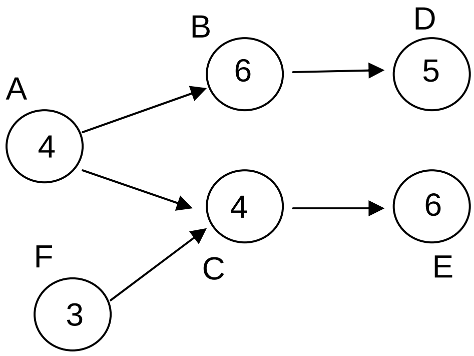
Task A:	4 minutes
Task B:	6 minutes
Task C:	4 minutes
Task D:	5 minutes
Task E:	6 minutes
Task F:	3 minutes

Task A must be completed before either Task B or C can be started. Tasks A and B must be completed before Task D can be started. Tasks A and C must be completed before Task E can be started. Task F must be completed before Task C can be started.

a. Draw an order requirement diagraph for this manufacturing process

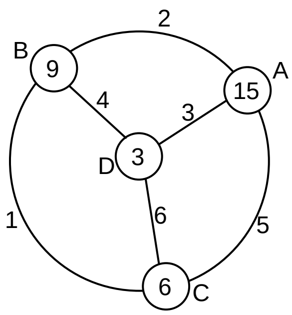
b. Find the critical path that gives the earliest completion time for the process.

a. The order requirement diagraph is:



b. The possible paths are ABD with length 15, ACE with length 14 and FCE with length 13. The critical path is FCE with completion time 13 minutes.

4. In the graph below, the number in the circle at each vertex is the cost of relaying a message through that vertex. The number on an edge indicates the cost of providing service between the endpoints of the edge. A relay can send a message to only one vertex at a time. There is no cost to initiate or receive a message at a vertex, only to relay the message through the vertex. Begin at vertex A and find the minimum cost for sending a message to all vertices, including the relay costs.



ABCD = 2 + 9 + 1 + 6 + 6 = 24

ABDC = 2 + 9 + 4 + 3 + 6 = 24

ADCB = 3 + 3 + 6 + 6 + 1 = 19 (this is the least cost)

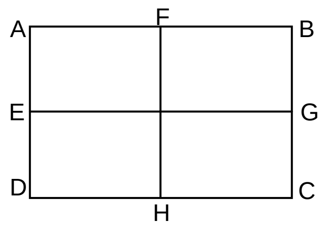
ADBC = 3 + 3 + 4 + 9 + 1 = 20

ACDB = 5 + 6 + 6 + 3 + 4 = 24

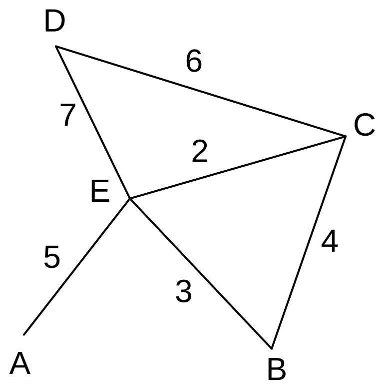
ACBD = 5 + 6 + 1 + 9 + 4 = 25

Extemporaneous Questions for the Oral Competition – Division A, State Finals 2010

1. Define Eulerization and find a best Eulerization for the graph below.



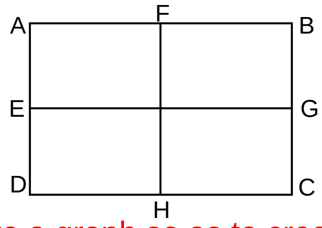
2. Use Kruskal's algorithm to find the minimum cost spanning tree for the graph below:



3. Suppose it takes 0.5 minutes to generate one Hamiltonian circuit tour of cities. What is the maximum number of cities on the tour that you can find in 1 day (1440 minutes) by using the brute force algorithm?

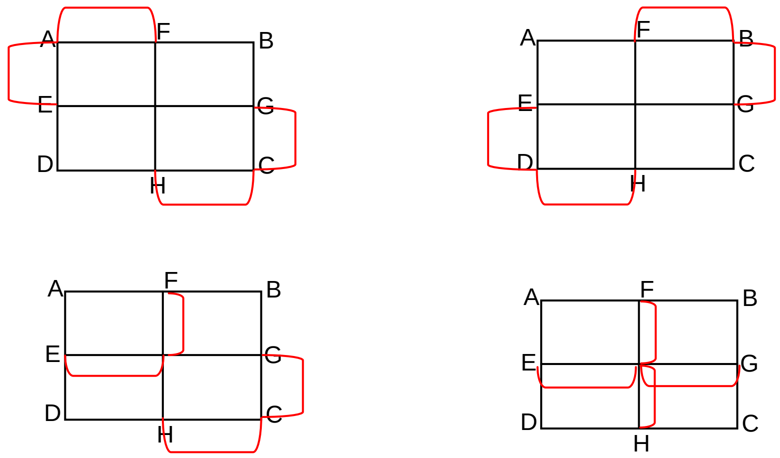
Extemporaneous Solutions for the Oral Competition – Division A, State Finals 2010

1. Define Eulerization and find a best Eulerization for the graph below.

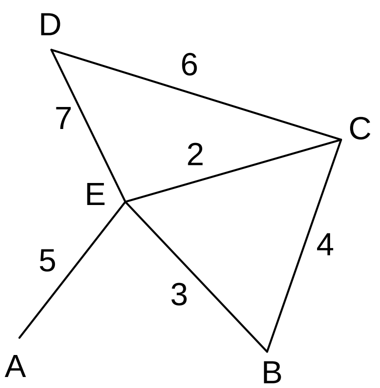


Eularization – adding new edges to a graph so as to create a graph that possesses an Euler circuit.

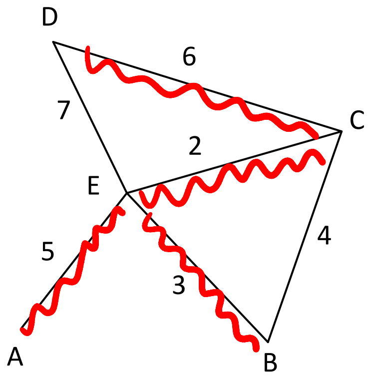
Some possibilities are:



2. Use Kruskal's algorithm to find the minimum cost spanning tree for the graph below:



Include edges in order so that no circuits are formed. Edges included would be:
EC (2), EB (3), AE (5) and CD (6) for minimum cost 16.



3. Suppose it takes 0.5 minutes to generate one Hamiltonian circuit tour of cities. What is the maximum number of cities on the tour that you can find in 1 day (1440 minutes) by using the brute force algorithm?

$$\frac{(n-1)!}{2}(0.5) \le 1440$$

$$(n-1)! \le 5760$$

$$n = 8$$