1. For what value of x does $3^x = 4^x = 5^x$?

2. One marble is drawn at random from a bag containing 9 orange, 18 blue, and 12 green marbles. Find the probability the marble drawn was orange. Express your answer as a common fraction reduced to lowest terms.

3. $k = 0.3\overline{18}$ where only the grouping $\overline{18}$ repeats. Write k as a reduced common fraction.

4. Find the **ordered pair** that represents the point at which the line with equation of $2y - 3x = 14$ intersects the y-axis.

5. Working at a constant rate, Christie finished $\frac{2}{3}$ of a job in $\frac{3}{5}$ of a day. Working at this same constant rate, what fraction of a day will Christie need to work one whole job? Express your answer as a fraction reduced to lowest terms.

6. In a certain community there are 1060 married couples (male married to female). Two-thirds of the husbands who are taller than their wives are also heavier, and one-half of the husbands who are heavier than their wives are also taller. If there are 60 wives who are at least as tall and at least as heavy as their husbands, find the number of husbands who are taller and heavier than their wives.

7. When solved over all real numbers, $\sqrt{2x-20} \le 10$ has a solution set $\{x : k \le x \le w\}$. Find the value of $(k + w)$.

8. Find the value of x such that $2\left(\left(\frac{1}{4}\right)^{(2x-3)}\right)=\frac{1}{32}$

9. Solve for x if $\frac{10x^2+41x+4}{x^2-4x} \div \frac{2x+7}{3x^3-11x^2-4x}=15x^2+11x+2$.

10. $\frac{2}{3-\frac{2}{3-x}}=\frac{5}{8}$. Find the value of x.

11. A certain woman had a considerable number of grandchildren. One Christmas, she decided to give each of her grandchildren \$150. However, she found she would be \$17 short. So she decided to give each of her grandchildren \$148 instead. By so doing, she had \$81 left over. How many grandchildren did the woman have?

12. Find the sum of all distinct 5 digit numbers such that the 5 digit number is a cube of a positive integer and the square of another positive integer.

13. $\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}}=\frac{1+\sqrt{157}}{2}$. Find the value of x.

14. Working at a constant rate, Tom would need 20 hours to build a certain brick wall. Working at a constant rate, Kay would need 15 hours to build the same wall. If they work together, a total of 5 fewer bricks will be laid per hour. By working together, they took exactly 8 hours, 53 minutes, and 20 seconds to build the wall. Find the number of bricks that the wall contained.

15. Find the value of k if $\left(\sqrt[3]{2}\right)\left(\sqrt[5]{3}\right)=\sqrt[15]{k}$.

16. When x^3+x+k is divided by $(x+3)$, the remainder is 32. Find the value of k.

17. The quadratic equation $x^2+ax+b=0$ has non-zero roots of p and q, and the quadratic equation $x^2+px+q=0$ has non-zero roots of a and $2b$. Find the value of $(p+q)$.

18. One hour out of the station, a train develops engine trouble that slows its speed to 80% of its normal average speed out of the station. Continuing at this constant reduced rate, the train arrives at its destination 2 hours and 15 minutes later than it would have by traveling constantly at its normal average speed. Had the engine trouble developed 50 miles beyond where it actually developed, the train would have arrived at its destination 2 hours and 5 minutes later than normal by traveling the remainder of its distance at the constant 80% rate. Find the number of miles from the station to the destination.

19. If $\frac{3}{4}x$ is 20% of 150, find the value of 20% of $\frac{1}{5}x$. Express your answer as a decimal.

20. Find the largest integer that divides 513, 786, and 1433 with remainders R_1, R_2, and R_3, respectively such that $R_2=R_1+1$ and $R_3=R_2+1$.

2011 SA
Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 0
2. $\frac{3}{13}$ (Must be this reduced common fraction.
3. $\frac{7}{22}$ (Must be this reduced common fraction.)
4. $(0,7)$ (Must be this ordered pair.)
5. $\frac{9}{10}$ (Must be this reduced common fraction, day optional.)
6. 400 (Husbands optional.)
7. 70
8. 3
9. 5
10. 13
11. 49 (Grandchildren optional.)
12. 62281
13. 39
14. 1200 (Bricks optional.)
15. 864
16. 62
17. -1
18. 750 (Miles optional.)
19. 1.6 (Must be this decimal.)
20. 34 (Must be this integer.)

1. The reflection of the point $(10,-24)$ over the y-axis is the point (a,b). Find the ordered pair (a,b).

2. A square inscribed in a circle has area 60. Find the area of the circle. Express your answer as a decimal rounded to the nearest hundredth.

3. A triangle has integer length sides of 17, 24, and k. Find the smallest possible k so the triangle is obtuse with side of length k opposite the obtuse angle.

4. The circle whose equation is $x^2+y^2-8x+12y=39.3936$ is tangent to the line $y=k$ where $k>0$. Find the value of k. Express your answer as a **decimal**.

5. The following ordered triples represent the lengths of the sides of triangles: $(3,4,5)$. $(5,6,7)$. $(6,8,10)$, $(12,35,37)$, $(1,\sqrt{3},2)$, $(45,49,64)$. If one of these ordered triples is selected at random, find the probability that the ordered triple selected can represent the lengths of the sides of a right triangle. Express your answer as a common fraction reduced to lowest terms.

6. A right triangle has vertices $(-3,1)$, $(2,4)$, and $(5,-1)$. Find the exact length of the hypotenuse.

7. Find the length of the chord connecting the points of intersection of the two circles whose equations are $(x-5)^2+(y-5)^2=25$ and $(x+3)^2+(y-5)^2=25$.

8. Triangle ΔABC has area 2800 square units. The midpoints of the three sides are joined to form ΔDEF. The midpoints of the three sides of ΔDEF are then joined to form ΔXYZ. Find the area of ΔXYZ.

9. The minute and hour hands of Hahn Deeman's watch overlap (one hand is on top of the other) exactly every 68 minutes and 20 seconds. According to Hahn's watch, he began work at 8:00 A. M. and finished at 4:00 P. M. that same day. Find the number of **minutes**, according to an accurate clock, that Hahn worked that day. Express your answer as an improper fraction reduced to lowest terms.

10. Find the length of the inradius of a right triangle whose legs have length 9 and 40.

11. A right cylindrical tube has an outside circumference of 14, and has a height of 13. Six turns of a wire are helically wound so that the ends of the wire coincide with the ends of the same cylindrical element. Find the length of the wire.

12. The point $(k,-9)$ is one trisection point of the segment joining $(2,-1)$ to $(8,-13)$. Find the value of k.

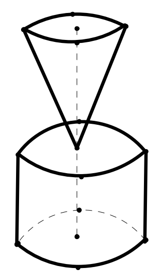
13. Find the degree measure of the central angle that intercepts an arc of length 18π in a circle with area 625π. Express your answer as a decimal.

14. From the point $(2,4)$, the two lines that are tangent to the circle $x^2+y^2+4x-4y=2$ are drawn. The equations of one of the two tangent lines can be expressed as $kx-y+w=0$. The equation of the other of the two tangent lines can be expressed as $x+py+f=0$. If k, w, p, and f are integers, find the value of $(k+w+p+f)$.

15. A convex polygon has consecutive vertices $(3,9)$, $(2,10)$, $(4,12)$, $(7,13)$, $(8,12)$, and $(6,9)$. Find the area of this polygon. Write your answer as a reduced improper fraction.

16. A rectangle with sides of length 39 and 252 is inscribed in a circle. Find the exact circumference of the circle.

17. A right circular cone with its apex pointing downward is resting with its apex on the center of the circular base of a right circular cylinder with the cone and cylinder sharing the same vertical axis. That is, the base of the cone is parallel to the bases of the cylinder and the apex of the cone is at the center of the top base of the cylinder. The height of the cone is 8, and the radius of the circular base of the cone is 2. The radius of the circular base of the cylinder has a length of 3.247. At the start, the cone is full of water and the cylinder is empty. Water drips down through a small hole in the apex of the cone into the cylinder forming new circular water “bases” in both the cone and cylinder. (Assume the hole is small enough so that its radius can be disregarded for calculation purposes.) Compute k so that the radius of the new circular base corresponding to the water left in the cone is k and the height of the water in the cylinder (height is measured from the bottom circular base of the cylinder to the new circular base of the upper level of the water in the cylinder) is k. Express your answer as a decimal rounded to the nearest thousandth.



18. “Picture in a picture” places a smaller rectangle inside a larger, similar rectangle. The rectangles share a common vertex and two consecutive sides of the smaller rectangle overlap (are contained in) corresponding sides of the larger rectangle. The inside rectangle has side lengths half the corresponding length of the outer rectangle and the length of the inside rectangle is 4 inches more than the width of the inside rectangle. If the difference in areas of the two rectangles is 135 square inches, find the perimeter of the outer rectangle.

19. A regular polyhedron has 6 distinct vertices and 12 distinct edges. If the length of one of these edges is 8, find the total surface area of this regular polyhedron.

20. Given triangle ΔOAB with $O(0,0)$, $A(10,0)$ and $B(6,6)$. Let P be the point where the angle bisector of $\angle OBA$ intersects $\overline{OA}$. Let Q be the centroid of ΔOAB. Find the length of $\overline{PQ}$. Express your answer as a decimal rounded to 4 significant digits.

2011 SA
Geometry

Name ________**ANSWERS**______

School ______________________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $(-10,-24)$ (Must be this ordered pair.)

2. 94.25 (Must be this decimal.)

3. 30

4. 3.56 (Must be this decimal.)

5. $\frac{2}{3}$ (Must be this reduced common fraction.)

6. $2\sqrt{17}$ (Must be this exact answer.)

7. 6

8. 175 (Square units optional.)

9. $\frac{4510}{9}$ (Must be this reduced improper fraction.)

10. 4

11. 85

12. 6

13. $129.6°$ (Must be this decimal, degrees optional.)

14. -10

15. $\frac{29}{2}$ (Must be this reduced improper fraction.)

16. 255π (Must be this exact answer.)

17. 0.915 OR $.915$ (Must be this Decimal.)

18. 56 (Inches optional.)

19. $128\sqrt{3}$ (Must be this exact answer.)

20. 2.001 (Must be this decimal.)

1. Find the y-coordinate **only** of the point in which the vertical line passing through $(2,5)$ intersects the parabola whose equation is $y = x^2 - 2x + 3$.

2. Let b and x represent real numbers and let k be an element of the set $\{2,3,4,5,6,7,8\}$. Find the **sum** of all distinct values of k such that $\left(b^x\right)\left(b^k\right) = \left(b^{(x+k)}\right)$.

3. The Wooden Carpentry Company will send a team of 3 people to work on a project. The Wooden Carpentry Company has 6 carpenters and 8 trainees. If a team consists of 1 carpenter and 2 trainees, how many different teams are possible?

4. Find the degree of the following polynomial: $x^3 + 5x^4 - 13x^2 + 15x - 78$.

5. Find the **ordered triple** that represents one of the trisection point of the segment joining $(-14,6,29)$ to $(-32,207,35)$ if the trisection point is closer to $(-14,6,29)$.

6. The first term of an arithmetic sequence is 21, and the last term of this arithmetic sequence is 1. The sum of all the terms, except for the first and last in this arithmetic sequence, is 132. Find the second term of this arithmetic sequence. Express your answer as an improper fraction reduced to lowest terms.

7. Let $i = \sqrt{-1}$. $\left|\dfrac{i}{1-7i}\right| = \dfrac{\sqrt{k}}{w}$ where k and w represent positive integers. Find the smallest possible value of $(k + w)$.

8. Let a, b, c, and d represent positive integers and let the arithmetic mean of a, b, c, d and e be e. Let k be the smallest possible positive integer such that e **must** be a positive integer if $a+b+c+d$ is an integral multiple of k. Find the value of k.

9. Let x and y be positive integers with $x<y$ such that x is the value of a free throw and y is the value of a field goal in KATIE basketball. The largest score that is impossible to obtain in KATIE basketball is 119, and the values of x and y are such that $x+y$ is a minimum. If it is only possible to score via a free throw or a field goal, find the score obtained by making 3 field goals and 2 free throws in KATIE basketball.

10. Let $i=\sqrt{-1}$. Find the complex conjugate of $3-12i^3$.

11. Assume k and w are elements of the natural domain for the inequality $\frac{3}{x+2} \geq \frac{2}{x-4}$. Let k be the largest **integral** negative value that is **not** a member of the solution set for the given inequality. Let w be the largest **integral** positive value that is **not** a member of the solution set for the given inequality. Find the value of $(k+w)$.

12. An ellipse has its center at $(1,2)$, has its major axis parallel to the y-axis, and passes through the points $(3,2)$ and $(1,6)$. The equation of this ellipse can be expressed in the form $\frac{(y-k)^2}{a}+\frac{(x-h)^2}{b}=1$. Find the value of $(a+2b+3h+4k)$.

13. A bag contains only red, white, blue, and green marbles. The bag contains 17 red marbles, 24 white marbles, x blue marbles, and fewer than 13 green marbles. If the probability that a marble drawn at random from the bag is blue is exactly 25%, find the <u>*sum*</u> of all possible distinct values of x.

14. A group of $2n$ mathematicians consisting of exactly n men and of exactly n women are having a dinner meeting. One of the mathematicians, Cindy, notes that there are k distinct arrangements possible if the group were to sit at a round table with no distinguishing marks and if men and women must alternate. Another of the mathematicians, Tom, notes that there would be $126k$ distinct arrangements possible if the group were to sit at a round table with no distinguishing marks and if men and women did not necessarily alternate. Find the value of k.

15. The equation for the figure representing the set of all points in the plane that are equidistant from the point $(1,3)$ and the line whose equation is $y=-3$ can be written in the form $(x-h)^2 = wy$. Find the value of $(h+w)$.

16. If $12!-11!=(10!)x$, find the value of x.

17. Let n be a positive integer such that $1+3+6+\cdots+\frac{n(n+1)}{2}=7(1+2+3+\cdots+n)$. Find the value of n.

18. Let a, b, c, and d be single digit positive integers such that $f(x)=ax^3+bx^2+cx+d$. If $f(7)=974$ and $f(10)=2561$, find the value of $(a+2b+3c+4d)$.

19. If $x \neq 0$ and $\dfrac{x^k}{x^{(3(k+2))}} = x^2$, find the value of k.

20. Let p and w be integers such that 2, p, and w are the roots for x of $x^3+ax^2+bx-2268=0$. Let 3, $w+2$, and $p+3$ be the roots for y of $y^3+cy^2+dy-4095=0$. Find the value of $(b+d)$.

2011 SA

Algebra II

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 3
2. 35
3. 168
4. 4
5. $(-20, 73, 31)$ (Must be this ordered triple.)
6. $\frac{253}{13}$ (Must be this reduced improper fraction.)
7. 12
8. 4
9. 61
10. $3-12i$ OR $-12i+3$
11. 12
12. 35
13. 62
14. 2880
15. 13
16. 121
17. 19
18. 34
19. –4
20. 2919

1. Let $i=\sqrt{-1}$. If k and w are real numbers such that $\dfrac{k+wi}{19-7i}=2$, find the value of $(k+w)$.

2. Let k represent a positive integer such that $3<k<1023$. Find the largest possible value of k such that $\sin^2(k°)+\cos^2(k°)-1=0$.

3. **(Always, Sometimes, or Never true)** For your answer, write *the whole word* **Always, Sometimes,** or **Never**—whichever is correct.

   If a function is continuous at $x=c$, then the limit of the functions value does **not** exist as $x\to c$.

4. For the equation $x^2+x+k=0$, the square of the difference between the roots for x is 40 more than the sum of the squares of these roots for x. Find the value of k for which this is true.

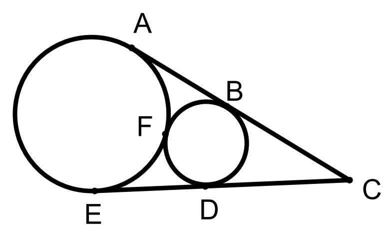
5. Find the value of $\displaystyle\sum_{k=1}^{\infty}\left(\left(\frac{1}{2}\right)^k\right)$.

6. Let $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$ represent vectors such that $\vec{a}=(3,2)$, $\vec{b}=(-4,3)$, and $\vec{c}=(17,6)$. Find the **ordered pair** representing $\vec{d}$ if $\vec{a}-\vec{b}=\vec{c}+\vec{d}$.

7. If $(a+b)^{18}$ is expanded and completely simplified, find the sum of the squares of the numerical coefficients.

8. Find the value of the indicated sum: $\sum_{x=3}^{4}(3x+4^{x})$.

9. In the diagram, $\overleftrightarrow{AC}$ and $\overleftrightarrow{EC}$ are common external tangents of the two circles with points of tangency at A, B, D, and E. The circles are tangent at F and have radii of lengths 3 and 8. Then $\sin\angle BCD = \frac{k\sqrt{w}}{f}$ where k, w, and f, are positive integers. Find the smallest possible value of $(k+w+f)$.



10. Let x represent the degree measure of an angle such that $\tan(x)=\sqrt{3}$. If $180<x<270$, find the value of x.

11. Let x represent an integer such that $0\le x\le 360$. Find the sum of all distinct values of x such that $2(\sin(x°)+\cos(x°))<\sqrt{1+\sin(x°)\cos(x°)}$.

12. A parabola has its line of symmetry parallel to the x-axis and has its vertex at $(7,2)$. The point $(3,-8)$ lies on the parabola. Find the ***x*-coordinate only** of the focus of this parabola. Express your answer as a **decimal**.

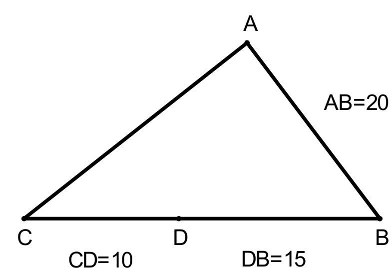
13. In taking a ten problem multiple choice test with 5 choices for each problem, a student randomly guesses on all ten questions. Find the probability that the student guessed at least three correct answers out of the ten. Express your answer as a **decimal** rounded to the nearest ten-thousandth.

14. Let $f(x)=x^7+x^6+x^5+x^4+x^3+x^2+x+1$. If k is a positive integer such that $53<k<115$, find the sum of all distinct k such that the numerical remainder when the polynomial $f(x^k)$ is divided by the polynomial $f(x)$ **must** be 8.

15. Let k represent the degree measure of an angle such that $\sin(33°) = \cos(k°)$. If $270° < k < 360°$, find the value of k ..

16. The sum of the last two terms of a eight term geometric progression of real terms is $\frac{2}{9}$. The sum of the third and fourth terms of this geometric progression is 18. Find the sum of all eight terms of this geometric progression. Express your answer as an improper fraction reduced to lowest terms.

17. In ΔABC, $\angle ABC = 70°$ and $AB = 20$. D lies on $\overline{BC}$ such that $CD = 10$ and $DB = 15$. Let E be a point on $\overline{AB}$ such that $CE + ED$ is as small as possible. Find that value of $CE + ED$. Express your answer as a decimal rounded to the nearest hundredth.



18. Let $C(n,k) = \dfrac{n!}{k!(n-k)!}$ where n and k represent positive integers. Find the value of n such that $C(n,7) = 330$.

19. When $(2x+3y)^5$ is expanded and completely simplified, the coefficient of one of the terms is 240. Find the **exponent** of x for that term.

20. If x is a real number, find the number of distinct values of x that satisfy the equation $x = 50\cos(x)$.

2011 SA
Pre-Calculus

Name ANSWERS

School ________________

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 24
2. 1022
3. Never (Must be the whole word.)
4. -20
5. 1
6. $(-10,-7)$ (Must be this ordered pair.)
7. 9,075,135,300 (Commas optional.)
8. 341
9. 165
10. 240 (Degrees optional.)
11. 47475 (Degrees optional.)
12. 0.75 OR .75 (Must be this decimal.)
13. 0.3222 OR .3222 (Must be this decimal.)
14. 672
15. 303 (Degrees optional.)
16. $\frac{1640}{9}$ (Must be this reduced improper fraction.)
17. 37.74 (Must be this decimal.)
18. 11
19. 4
20. 31

NO CALCULATORS

1. If $1.2x = 30$, find the value of x.

2. The expression $2\sqrt{27} + 5\sqrt{\frac{4}{3}}$ can be simplified to a single term $\frac{k\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the least possible value of $(k + w + p)$.

3. For all values of x, $\frac{2x+5}{4} + \frac{3x-4}{5}$ can be simplified to a single fraction $\frac{kx+w}{p}$. Find the least positive value of $(k + w + p)$.

4. A triangle with sides of lengths 42, 144, and 150 is inscribed in a circle whose circumference is $k\pi$. Find the value of k.

5. b is a y-intercept of a relation if the point $(0, b)$ lies on the graph of the relation. Find the sum of all possible distinct y-intercept(s) of $(y-5)^2 = x + 9$.

6. In a circle whose equation is $x^2 + y^2 + 10x - 14y - 26 = 0$, $(-11, 15)$ is one endpoint of a diameter of the circle. Find the coordinates of the other endpoint of this diameter.

7. **(Always, Sometimes, or Never)** For your answer, write *the whole word* ***Always*****, Sometimes,** or **Never**—whichever is correct.

   If a sphere is inscribed in a cube, then the ratio of the surface area of the sphere to the surface area of the cube is the same as the ratio of the length of a diameter of the sphere to the length of an edge of the cube.

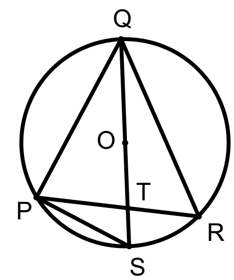
8. When expressed in base fifty-six, $N!$ terminates in a block of exactly 11 zeroes. Compute the largest positive integer N with this property. Express your answer for N in base ten notation.

NO CALCULATORS

NO CALCULATORS

9. In simplest radical form, $\frac{2}{\sqrt[3]{243}} = \frac{k\sqrt[3]{w}}{p}$, where k, w, and p are positive integers. Find the least possible value of $(k+w+p)$.

10. Find the value of $(168542)^2 - (168539)^2$.

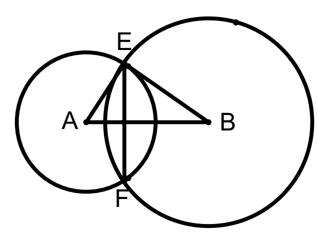


11. Given: P, Q, R, and S lie on a circle with a center at O. Q, O, T, and S lie on a diameter. $\overset{\frown}{PQ} = 112°$, $\angle RPS = 27°$, points P, T, and R are collinear. Find the degree measure of $\angle PQR$.

12. A woman working alone and at a constant rate can do a job in w hours. A man working alone and at a constant rate can do the same job in m hours. The woman works alone at her constant rate for 2 hours; the man then joins her and each works at his/her constant rate until the job is finished. Assume no loss of efficiency and assume that the rates remain constant. Find the length of time in hours after the woman started until the job is finished. Express your answer as a simplified single fraction and express your answer in terms of w and m.

13. Let $5(x+y) = xy$. If x and y are positive integers, find the sum of all possible distinct values of x.

14. In the diagram, $\odot A$ and $\odot B$ intersect in points E and F. $\odot A$ has a radius whose length is 5, and $\odot B$ has a radius whose length is 9. $\overline{AB}$, the segment connecting the centers of the circle, has a length of 12. $EF = \frac{k\sqrt{w}}{p}$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.



NO CALCULATORS

15. Let x and y be positive integers with $x > y$. Find the **ordered pair** (x, y) such that the quotient of x and y and the positive difference of x and y are equal.

16. Let x, y, k and $\sqrt{141+k}$ be positive integers and $k < 321$. Let $S = x + y + k$. Find the sum of all possible distinct values of S such that $x^2 = 141 + k - y^2$.

17. Let a and b be non-zero real numbers. $\left(\frac{1}{4}a^2b^{-3}\right)^{-1}\left(\frac{1}{2}a^{-3}b^2\right)^{-2}\left(2a^3b^{-3}\right)^{-4} = pa^kb^w$ where p is positive and k and w are integers. Find the least value of $(k + w + p)$.

18. Let $A = \{3, 4, 5, 6\}$. From A, three distinct elements are selected at random and used as the lengths of the three sides of a triangle. Find the probability that this triangle is obtuse. Express your answer as a common fraction reduced to lowest terms.

19. The 10-digit number $abc35b62ca$, where $a > b > c$ represent non-zero digits, is divisible by 792. Find the ordered triple (a, b, c)

20. Let $x \in \{111, 114, 118, 123, 129, 136, 144, 153, 163, 174\}$. Find the sum of all distinct x such that $\frac{x^3 + 9x^2 + 17x + 6}{6}$ is an integer.

NO CALCULATORS

2011 SA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 25
2. 34
3. 51
4. 150
5. 10
6. $(1,-1)$ (Must be this ordered pair.)
7. Never (Must be the whole word.)
8. 76 OR 76_{10} OR 76_{ten}
9. 14
10. 1011243
11. 61 (Degrees optional.)
12. $\frac{(mw+2w)}{m+w}$ OR $\frac{(m+2)w}{m+w}$ (Must be single fraction, equivalent commutations acceptable.)
13. 46
14. 33
15. $(4,2)$ (Must be this ordered pair.)
16. 608
17. 4
18. $\frac{1}{2}$ (Must be this reduced common fraction.)
19. $(8,7,4)$ (Must be this ordered triple.)
20. 686

NO CALCULATORS

1. **(Always, Sometimes, or Never true)** For your answer, write <u>*the whole word*</u> **Always, Sometimes,** or **Never**—whichever is correct for the following statement:

   "The converse of a false statement is a true statement."

2. Find the **sum** of all distinct integers in the domain of the real-valued function $f(x)=\sqrt{8x-x^2}$.

3. When $6x^3+4x^2+kx+3$ is divided by $(x-2)$, the remainder is 7. Find the value of k .

4. Let $\|(k,w)\|$ represent the **norm** of the vector represented by (k,w). Find the exact value of $\|(7,9)\|$.

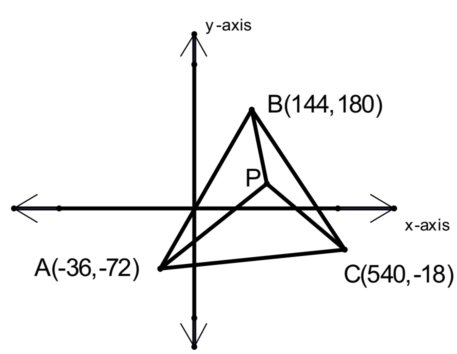
5. An ellipse has an equation of $\frac{16(x-5)^2}{289}+\frac{(y+3)^2}{64}=1$. The area of this ellipse can be expressed in the form $k\pi$. Find the value of k .

6. Bob and Judy are playing a game with four dice whose faces are numbered as follows:

Orange Die	4,	4,	4,	4,	4,	4	
Blue Die	8,	8,	2,	2,	2,	2,	2
Scarlet Die	7,	7,	7,	1,	1,	1	
Gray Die	6,	6,	6,	6,	0,	0	

   Bob must select a die first. After Bob selects his die, and after he rolls that die, Judy must select her die. Judy then rolls once. The winner is the one who rolled the higher number. Bob selects the scarlet die and rolls. If Judy applies the poorest possible strategy, find the probability that Judy will win the game. Express your answer as a common fraction reduced to lowest terms.

7. In the diagram with ΔABC and coordinates as shown, the ratio of the area of ΔPAB to the area of ΔPBC to the area of ΔPAC is $2:3:4$. Find the **ordered pair** that represents point P .



NO CALCULATORS

8. Find the reciprocal of the sum of the reciprocals of 2 and 4. Express your answer as an improper fraction reduced to lowest terms.

9. Find the **sum** of all distinct values of k for which the relation S will **not** be a function if $S=\{(|k+2|+3,19),(8,7),(15,1)\}$.

10. If $2^x-7=0$, then $x=\dfrac{\log_4 k}{\log_{64} w}$ where k and w are positive integers. Find the smallest possible value of $(2k+3w)$.

11. Given the four letters: A, B, C, D. $ABCAD$ and $ABABD$ are two examples of a five letter movement. In a five letter movement, one must start with the letter A and end with the letter D. No two consecutive spots in a five letter movement can consist of the same letter. For example, ABBCD would **not** be a five letter movement. Choosing only from the four given letters, find the number of distinct five letter movements.

12. Let b and c be real numbers such that $|b|\le 3$ and $|c|\le 3$, and such that at least one of those two variables (b and c) must represent an integer. For how many different values of x can the two solutions for x of the equation $x^2+bx+c=0$ be equal?

13. $\begin{bmatrix} 2 & -3 \\ a & c \end{bmatrix}\begin{bmatrix} a & 4 \\ c & 2 \end{bmatrix}=\begin{bmatrix} 16 & 2 \\ 20 & 0 \end{bmatrix}$. Find the value of $(a+c)$.

14. The arithmetic mean of four numbers, none of which is a prime, is 21. A prime number between 26 and 30 is added to the list to make five numbers. Let k and w be two **new** prime numbers between 16 and 40 that are added to the list to make a total of seven numbers. The arithmetic mean of these seven numbers is 23. Find the least possible value of $|k-w|$.

NO CALCULATORS

15. Line L passes through the point represented by $(7,-9)$ and is perpendicular to the line passing through points represented by $(21,10)$ and $(7,8)$. Line L can be represented as $\{(7+t,-9+kt)\}$. Find the value of k.

16. **(Always, Sometimes, or Never true)** For your answer, write <u>*the whole word*</u> **Always, Sometimes,** or **Never**—whichever is correct for the following statement:

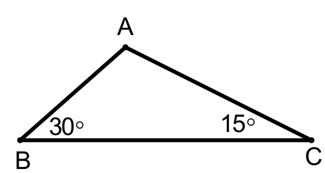
    "If x is a real number, then $|x|<7$ is equivalent to $-7<x<7$."

17. Cindy tosses a fairly weighted penny six times. Find the probability that at least four consecutive tosses were heads. Express your answer as a common fraction reduced to lowest terms.

18. Let $i=\sqrt{-1}$, and let k and w represent integers. If the product of the roots for x for the quadratic equation $x^2+5ix=kx+wi$ is $-35i$, find the value of w.

19. One of the angle bisectors of the angles formed by the graphs of $15x-20y+17=0$ and $20x+15y-12=0$ is represented by the equation $kx-y+w=0$ where k and w are integers. Find the value of $(k+w)$.

20. In ΔABC, $BC=12$, $\angle ABC=30°$, and $\angle ACB=15°$. The area of ΔABC can be expressed as $k\sqrt{w}-p$ where k, w, and p are positive integers. Find the smallest possible value of $(k+w+p)$.



2011 SA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. Sometimes (Must be the whole word.)
2. 36
3. -30
4. $\sqrt{130}$ (Must be this exact answer.)
5. 34
6. $\frac{1}{3}$ (Must be this reduced common fraction.)
7. $(172, 52)$ (Must be this ordered pair.)
8. $\frac{4}{3}$ (Must be this reduced improper fraction.)
9. -8
10. 38
11. 20
12. 9
13. -2
14. 14
15. -7
16. Always (Must be the whole word.)
17. $\frac{1}{8}$ (Must be this reduced common fraction.)
18. 35
19. 8
20. 39

Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. If $2x+4.3=38.78$, find the value of $4x-16.23$.

2. Find the value of $\log_5 184$.

3. It takes 58.49 seconds for Carol's computer to "boot up." Last week Carol decided to "boot up" her computer 123 times. How many total **minutes** did Carol use last week to "boot up" her computer?

4. If $3.008(x-2.456y)+4.113(2.142x-6.518y)=6.002(x-4.567)$ is simplified and expressed in the form $y=mx+b$, find the value of $(m+b)$.

5. Find the value of $\sum_{k=1}^{k=13}\left(7.6k-\left(\sqrt{626}+\frac{\sqrt[3]{27.3k}}{\sqrt{25.2k}}\right)\right)$.

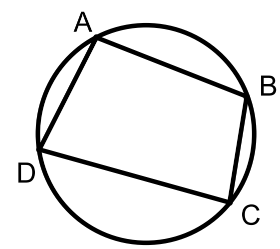
6. A parabola has an equation of $x^2+36.848y=0$. The directrix of this parabola is represented by the equation $y=k$. Find the value of k.

7. A sum of $10,000 is invested at an interest rate of 12% annual percentage rate. If compounded quarterly, in how many years will that sum be worth $50,000?

8. The smaller of two consecutive positive integers is an integral multiple of 37, and larger of these two integers is an integral multiple of 43. Find the smaller of these two positive integers if that positive integer contains at least 5 digits, and these 5 digits can only be 7's and/or 0's. For example, 70007 would be a possible 5-digit number to consider for the smaller integer; in checking, 70007 won't work because it is **not** an integral multiple of 37. Write your answer as an integer. Do *not* use scientific notation.

9. Consider the graph of $G = \begin{cases} x = 8\cos t \\ y = 5\sin t \end{cases}$, $t \in [0, 2\pi]$. Find the largest value for t so that the distance from the origin $(0,0)$ to $P(x,y) = 7$.

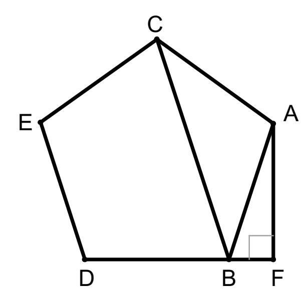
10. In the diagram, quadrilateral $ABCD$ is inscribed in the circle. $AB = 10$, $BC = 14$, $CD = 26$, and the area of $ABCD$ is $\frac{15\sqrt{41055}}{16}$. Find AD if $AD < 15$. Express your answer as a decimal.



11. Find the sum of the two distinct values of x such that $(\log(x))^2 + 3.114\log(x) + 2.023 = 0$.

12. The total surface area of a sphere is $k\pi$ square units where k is a 4-digit integer. The volume of this same sphere is $w\pi$ cubic units where w is a 6-digit integer. Find the sum of all distinct possibilities for the number of units in the length of a radius of the sphere.

13. $ABDEC$ is a regular pentagon with $\overline{DB}$ extended through B to F. $AF = 1$, and $\angle AFB = 90°$. Find the area of ΔABC.

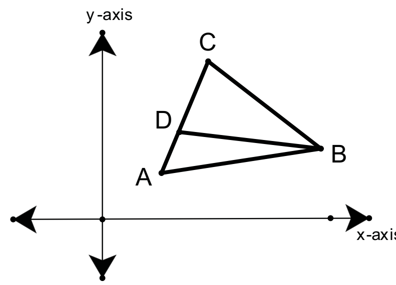


14. Find the 20th term of the sequence when $a_1 = 2$ and $a_n = 3\sqrt{2(1 + a_{n-1})}$ for $n \ge 2$.

15. In poker, four of a kind is a five-card hand in which four of the cards are of the same denomination, such as $10-10-10-10-3$ or $Jack - Jack - 5 - Jack - Jack$. From a standard 52 card deck, five cards are drawn at random without replacement. Find the probability of getting four of a kind. Express your answer as a common fraction reduced to lowest terms.

16. The volume of a right circular cylinder is 108.0π , and the height is 2.986. Find the length of a radius of one of the bases.

17. In the diagram, ΔABC lies entirely in Quadrant I, and $D(3,20)$ lies on $\overline{AC}$. The equation of $\overleftrightarrow{AC}$ is $y=6x+2$, and the equation of $\overleftrightarrow{AB}$ is $3y-x=23$. If $DC=7.642$ and $AB=11.12$, find the y-coordinate of the midpoint of $\overline{BD}$.



18. Find the radian measure of an angle whose degree measure is $86\frac{1}{3}°$.

19. A right circular cone has a height whose length is 8. The pointy end is cut off parallel to the base to form a frustum of the original cone. The volume of the frustum is 20% of the volume of the original right circular cone. The angle formed by the height drawn from the peak of the cone to the center of the base of the cone and a lateral edge of the cone is 12.58°. Find the length of a radius of the smaller base of the frustum.

20. Given the set of consecutive positive integers $\{1,2,3,4,5,\cdots,x,\cdots\}$. The numbers of this set are arranged in ascending order. A purple mark is placed under each number of the set. Of those numbers with one purple mark, beginning with the second (that is #2), a second purple mark is placed under every third number in the original ascending order (#2, #5, #8, etc.) Of those with two purple marks, beginning with the third (that is #8), a third purple mark is placed under every third number in the ascending set of numbers with two purple marks. Of those with three purple marks, beginning with the fourth (that is #35), a fourth purple mark is placed under every third number in the ascending set of numbers with three purple marks. This pattern of marking continues. If x is the smallest number with 8 purple marks under it, find the value of x. Write your answer as an integer without scientific notation.

2011 SA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 52.73 OR 5.273×10 OR 5.273×10^{1}
2. 3.240 OR 3.240×10^{0} (Must have the trailing zero.)
3. 119.9 OR 1.199×10^{2} (Minutes optional.)
4. 0.9717 OR 9.717×10^{-1}
5. 360.5 OR 3.605×10^{2}
6. 9.212 OR 9.212×10^{0}
7. 13.75 OR 1.375×10 OR 1.375×10^{1} (Years optional.)
8. 77700 (Must be this integer.)
9. 5.614 OR 5.614×10^{0}
10. 12.5 OR 12.50 (Must be this decimal.)
11. 0.1257 OR 1.257×10^{-1}
12. 93 (Units optional.)
13. 0.5257 OR 5.257×10^{-1}
14. 18.95 OR 1.895×10 OR 1.895×10^{1}
15. $\frac{1}{4165}$ (Must be this reduced common fraction.)
16. 6.014 OR 6.014×10^{0}
17. 15.76 OR 1.576×10 OR 1.576×10^{1}
18. 1.507 OR 1.507×10^{0} (Radians optional.)
19. 1.657 OR 1.657×10^{0}
20. 7109 (Must be this integer.)

1. Let $(\sqrt{k})(5\sqrt{32}) = w$. If k represents a positive integer and w represents a positive integer, find the smallest possible value of k.

2. If $8.12(x-3)=16.24$, let w be the value of $(3x-9)$. Let k be the length of a diagonal of a square with a perimeter of $24\sqrt{2}$. Find the value of $(k+w)$.

3. Let k and w be positive integers where $k > w$. Find the number of distinct ordered pairs (k,w) that exist such that $\sqrt{k} + \sqrt{w} = 19$.

4. In a convex hexagon, 3 congruent angles total 384 degrees in measure. Of the remaining three angles, the second has measure of 52 degrees more than the first and the third is 24 degrees less then twice the first. Find the largest degree measure of an angle of the hexagon.

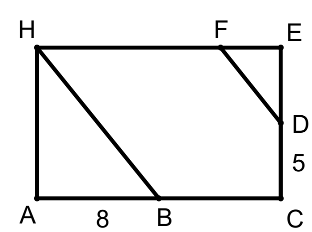
5. For all real numbers, $a \otimes b = (a+3)(2b-3)$. Let k be the longest length of the base of an isosceles triangle with perimeter 21 where all three sides have integral length. Find the value of $k \otimes (12 \otimes 5)$.

6. $(2x-9)$ is a factor of $14x^2 - kx - 45$. The system $\begin{cases} 5x + 9y = k \\ 36y + w = -20x \end{cases}$ has real solutions for (x,y). Find the value of w.

7. How many non-congruent right triangles have sides that are positive integers if the length of the hypotenuse is an integer less than 50?

8. Let k be the sum of all **positive** integers such that $(x-3)(x^2-17x+60) < 0$. Let w be the length of a radius of a circle whose circumference contains the same number of units as its area contains square units. Find the value of $(k+w)$.

9. $ACEH$ is a rectangle, $\angle AHB \cong \angle FDE$, B and D are midpoints of $\overline{AC}$ and $\overline{EC}$ respectively. $AB = 8$ and $DC = 5$. Find the perimeter of $HBCDF$. Write your answer as a decimal rounded to the nearest hundredth.



10. The perimeters of three faces of a rectangular solid are respectively 192, 240, and 320. Find the volume of the rectangular solid.

2011 SA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. 2	
2. 18	
3. 9	
4. 130	
5. 2484	
6. –212	
7. 18	
8. 56	
9. 44.21 (Must be this decimal.)	
10. 175168	

TOTAL SCORE: ____

(*enter in box above)

Extra Questions:

11. ANS
12. ANS
13. ANS
14. ANS
15. ANS

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. The multiplicative inverse of $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$ is equal to $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find $\left(|a| + |b| + |c| + |d|\right)$.

2. Let r be the numerical remainder when dividing $x^4 - 3x^3 + 5x^2 + 84x - 18$ by $(x-3)$. For all real numbers x and y, $f(x+y) = f(x) + f(y)$. If $f(1) = 3$, let $f(10) = k$. Find the value of $(r+k)$.

3. Let k be the value in base ten of the base three numeral 12110_{three}. Let S be the sum of the reciprocals of the three distinct roots of $x^3 - \frac{3}{4}x^2 + \frac{13}{72}x - \frac{1}{72} = 0$. Find the value of $(k+S)$.

4. If x is an integer such that $7 \le x \le 21$, find the sum of all distinct values for x such that 441_x is the square of an integer.

5. Start by placing the integer 1 in one of two sets. Continue by placing each succeeding positive integer into one of the two sets such that neither set contains any subset of three distinct integers that are in arithmetic progression. Find the largest possible sum of the distinct integers in one of the two sets.

6. Let k be the value of the constant term of $\left(x^2 + \frac{1}{x^2}\right)^4$. Let wx^7y^p be a term in the expansion of $\left(x + \frac{1}{2}y\right)^{10}$. Find the sum $(k+w+p)$.

7. $AcisB°$ represents the polar form notation for complex numbers as vectors in standard position in the complex plane, sometimes written as $A\left(\cos B°, i\sin B°\right)$, and where $i = \sqrt{-1}$. Find the distance between the tips of the vectors represented by $6\sqrt{2}cis45°$ and $8\sqrt{3}cis120°$. Write your answer as a decimal rounded to four significant digits.

8. Find the value of $\sum_{k=1}^{\infty}\left(45(\frac{1}{3})^k\right)$.

9. Find the distance from the point $(2,3,8)$ to the plane whose equation is $5x + 12y + 84z + 47 = 0$.

10. Let $f(x) = 3x^2 + 2x$ and $g(x) = x + 2$. Find the value of $f\left(g(5)\right) + g\left(f(5)\right)$.

2011 SA

Jr/Sr 2 Person Team

School ANSWERS

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. 1	
2. 309	
3. 160	
4. 210	
5. 22	
6. 24	
7. 14.25 (Must be this decimal.)	
8. 22.5 OR $22\frac{1}{2}$ OR $\frac{45}{2}$	
9. 9	
10. 248	

TOTAL SCORE: ______ **(*enter in box above)**

Extra Questions:

11. ANS

12. ANS

13. ANS

14. ANS

15. ANS

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

FRESHMAN-SOPHOMORE RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 1

1. Evaluate $\frac{b^3 - 21}{5b + 29.897036}$ when $b = 3.19$. Write your answer as a reduced common fraction.

2. Let ANS be the slope of the line passing through $(22.64, k + 0.1)$ and $(1.84, -4.1)$. Find the value of k.

3. $\overline{EF}$ is a diameter of a circle. If E has coordinates $(ANS, 4)$ and F has coordinates $(3, 2)$, find the exact value of the length of the radius of this circle.

4. Let ANS be the length of each edge of a regular hexagon. Find the exact area of this regular hexagon.

ANSWERS

1. $\frac{1}{4}$ (Must be this reduced common fraction.)

2. 1

3. $\sqrt{2}$ (Must be this exact radical.)

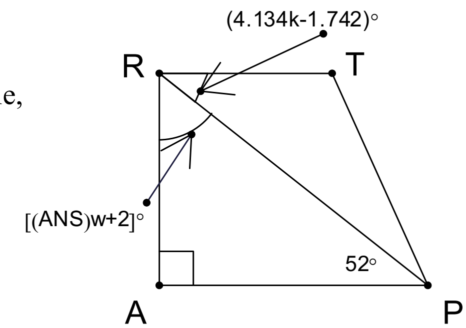
4. $3\sqrt{3}$ (Must be this exact radical.)

FRESHMAN-SOPHOMORE RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 2

1. Solve for x when $5x-3(x-4)=18$.

2. Find the sum of the two solutions for x when $2|x-ANS|+1=5$.

3. $TRAP$ is a trapezoid with $\overline{TR} \parallel \overline{AP}$. $\angle A$ is a right angle, $m\angle APR = 52°$, $m\angle ARP = [(ANS)w+2]°$, and $m\angle PRT = (4.134k-1.742)°$. Find the sum $(k+w)$.



4. A clock with hour and minute hands is set to exactly 1:00 PM. Find the measure of the acute angle formed by the hour and minute hands after ANS minutes elapse. Write your answer as a decimal.

ANSWERS

1. 3

2. 6

3. 19

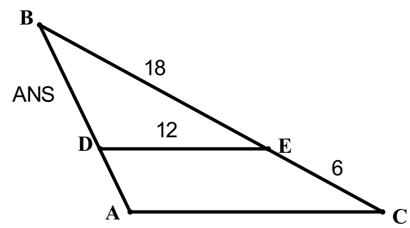
4. 74.5 (Must be this exact decimal, degrees optional.)

FRESHMAN-SOPHOMORE RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 3

1. The ordered pair (k, w) is the solution to the system where $y = \frac{1}{2}x$ and $2x + 3y = -7$. Find the sum $(k + w)$.

2. $\left(\frac{3}{4d}\right)\left(\frac{2d^4}{c^3}\right)\left(8c^6d^{-3}\right)^{-2}\left(5c^5\right)^3\left(d^{-2}\right)^{ANS} = Nc^kd^w$. Find the value of $(k + w)$.

3. In the diagram, $\frac{BD}{DA} = \frac{BE}{EC}$. $BE = 18$, $EC = 6$, $DE = 12$, and $BD = ANS$. Find the perimeter of Quadrilateral $ACED$.



4. Looking directly from the side at a car with 16-inch wheels, the 16-inch wheel appears to be a solid silver circular disk with diameter of 16 inches and the axle is the center. The black rubber tire then extends 5 inches all around the wheel with the top of the tread (tire) forming a concentric circle. Find the total distance a point on the top of the tread travels as it rotates around the axle when the tire makes ANS revolutions? Write your answer as an integer rounded to the nearest **foot**.

ANSWERS

1. -3

2. 15

3. 39 (Units optional.)

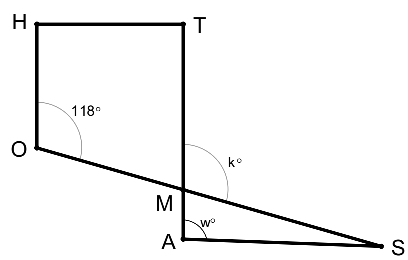
4. 265 (Must be this integer, feet optional.)

FRESHMAN-SOPHOMORE RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 4

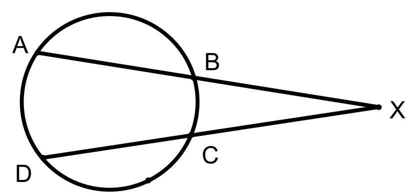
1. Find the sum of the solutions for the equation $x^2 - 26 = 11x$.

2. Set $A = \{ANS, 9, 10, 11, 12, 15, 16\}$. Let $C = \{2, 9, 7\}$. The arithmetic mean of C is 6 since the average of 2, 9, and 7 is 6. The median of C is 7 because 7 is the middle data point if the data are arranged *in order*. Set A has median k and arithmetic mean w. Find the sum $(k + w)$.

3. In the diagram, $\overline{HO} \parallel \overline{TA}$. $m\angle HOM = 118°$, $m\angle TMS = k°$, $m\angle MAS = w°$, and $m\angle MSA = (ANS)°$. Find the sum $(k + w)$.



4. Given the circle with two secant segments meeting at point X, $\overline{AB} \cong \overline{CD}$ and $m\widehat{ADC} = (ANS)°$. Find the degree measure of $\angle AXD$.



ANSWERS

1. 11

2. 23

3. 213 (Degrees optional.)

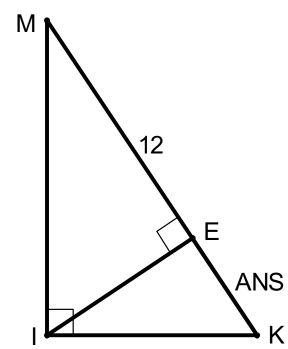
4. 33 (Degrees optional.)

FRESHMAN-SOPHOMORE RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 5

1. Let x be a member of $\{1,2,3,4,5,6,7,8,9,10\}$. For how many distinct values of x will the expression $\left[x\left(1+\frac{1}{2}\right)+0.5\right]$ be a whole number?

2. Let $k=2(ANS)$. Solve for x when $\frac{2x}{x-1}+\frac{2}{3}=\frac{k}{x-1}$.

3. ΔMIK is a right triangle with $\overline{MI}\perp\overline{IK}$. $\overline{IE}\perp\overline{MK}$ at point E. $ME=12$ and $KE=ANS$. Find the exact area of ΔMIK.



4. Let ANS represent the numerical area of Rhombus $RHOM$. One Angle of $RHOM$ measures $45°$. Find the numerical length of the shorter diagonal of $RHOM$. Write your answer as a decimal rounded to the nearest tenth of a unit.

ANSWERS

1. 5

2. 4

3. $32\sqrt{3}$ (Must be this exact answer.)

4. 6.8 (Must be this decimal, units optional.)

JUNIOR-SENIOR RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 1

1. Alavertz rents cars for \$48 a day and \$0.24 per mile driven. Tom rented a car for 3 days at a total cost of \$210. How many miles did Tom drive?

2. Find the remainder when $P(x) = 2x^3 - 21x^2 + 19x - (ANS)$ is divided by $(x - 11)$.

3. If x represents a positive integer, find the smallest possible value for x such that $\log(x + ANS) > 2.06$.

4. The $2\frac{3}{4}$ National Bank chartered buses with maximum capacity of 120 riders to take customers to a dinner and show in Milwaukee. ANS customers signed up and the price was set at \$150 per ticket. The bank agreed to reduce the price for *every* customer by \$1 per ticket for each additional ticket sold. How many *total* tickets should be sold to maximize the revenue for this trip?

ANSWERS

1. 275 (Miles optional.)

2. 55

3. 60

4. 105 (Tickets optional.)

JUNIOR-SENIOR RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 2

1. Find the value of $\dfrac{\frac{2}{5}+\frac{1}{2}}{\frac{3}{15}-\frac{1}{10}}$.

2. Pizza Party Palace offers two options for its customers. On the regular plan, a customer pays a $5 fee and $0.50 per game played. Or the customer can pay $(\$ANS)$ as a fee and only $0.25 per game played. What is the least number of games a customer must play to make the second plan less expensive than the regular plan?

3. Find the perimeter of a regular nonagon inscribed in a circle with radius of length ANS. Round your answer to the nearest hundredth of a unit.

4. ANS should be a decimal in the form $k.w$ where k represents the integer part of the decimal and w is the integer formed by the digits after the decimal point. $P(x)=x^2+bx+c$ is a quadratic polynomial with integer coefficients whose zeroes sum to w and have product k. $Q(x)=x^2+Bx+C$ is also a quadratic polynomial with integer coefficients whose zeroes are each double the zeroes of $P(x)$. Find the sum $(B+C)$.

ANSWERS

1. 9

2. 17 (Games optional.)

3. 104.66 (Must be this decimal, units optional.)

4. 284

JUNIOR-SENIOR RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 3

1. Solve for x if $\frac{1}{x+3}+\frac{5}{x^2-9}=\frac{2}{x-3}$.

2. The harmonic mean of some terms is found by dividing the number of terms by the sum of the reciprocals of the terms. For example, the harmonic mean of 2, 3 and $\frac{1}{12}$ is $\frac{18}{77}$ because $\frac{3}{\frac{1}{2}+\frac{1}{3}+12}=\frac{18}{77}$. For your problem, find the harmonic mean of $\frac{44}{3}$ and ANS.

3. Let $ANS = k$. Points R, S, and P have coordinates $R(-1,6)$, $S(-4,2)$, and $P(x,y)$. In vector notation, $\overline{RP} = k\overline{RS}$. Find the value of x, the x-coordinate of point P.

4. Let $ANS = y^2 - 6x + 2y + 45$. The graph of this equation is a parabola with focus (k,w). Find the sum $(k+w)$.

ANSWERS

1. -4

2. -11

3. 32

4. 2.5 or $\frac{5}{2}$ or $2\frac{1}{2}$

JUNIOR-SENIOR RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 4

1. A father is 4 times as old as his daughter. In six years, he will be 3 times as old as she is then. How old will the daughter be in six years? (Assume all ages are whole numbers of years.)

2. How many integral solutions exist such that $3|x-7|+6 \le ANS$?

3. $ANS = x^2 + 2xy + y^2$. Find the exact distance between the two branches of the graph of this relation.

4. (Notation: $f(x) = Arc\tan x$ represents the inverse tangent function.) Let $k = ANS$. Find the value of $\cos(2Arc\tan k)$. Write your answer as a reduced common fraction.

ANSWERS

1. 18 (Years optional)

2. 9

3. $3\sqrt{2}$ (Must be this exact radical.)

4. $-\frac{17}{19}$ or $\frac{-17}{19}$ or $\frac{17}{-19}$ (Must be a reduced common fraction.)

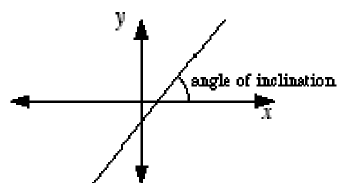
JUNIOR-SENIOR RELAY COMPETITION
ICTM 2011 DIVISION A STATE FINALS ROUND 5

1. In this problem, $i=\sqrt{-1}$. $\dfrac{7+3i}{4i}$ can be simplified to a complex number of the form $k+wi$. Find the sum $(k+w)$.

2. For all real numbers x, $f(x+1)=(ANS)f(x)+1$ and $f(0)=6$. Find the value of $f(5)$.

3. Let $k=ANS$. $f(x)=3\sin x+k\cos x$ with x measured in degrees. Find the least positive angle such that $f(x)=0$. Round your answer to the nearest degree.

4. The angle of inclination for a line is the angle α, $0°\le\alpha<180°$, measured counterclockwise from the part of the x-axis in the positive direction from the line. Let ANS be the angle of inclination, in degrees, for a line that passes through the point $(5,4)$. This line crosses the $x-axis$ at $(k,0)$. Find the value of k as a decimal rounded to the nearest hundredth of a unit.



ANSWERS

1. -1

2. -5

3. 59 (Degrees optional.)

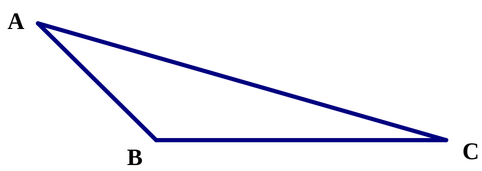
4. 2.60 (Must be this decimal, last “0” is significant, no ordered pair.)

ICTM State 2011
Division A Oral

Symmetry and Patterns

1. Give an example of a strip pattern of type *pma2* made using the symbol 😉.

2. a. Show how/why triangle ABC, given below, can tile the plane.



   b. Is it possible to create a semi-regular tiling using only regular quadrilaterals and regular hexagons (including at least one of each)? If it is possible, show how; if it is not possible, explain why.

   c. What is the measure of an interior angle of a regular n-gon? For how many odd integral values of n will this n-gon tile the plane?

3. The golden ratio, ϕ, is equal to the expression $\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}}}}$.
What is the value of a?

ICTM State 2011
Division A Oral

Extemporaneous Questions

1. One of the two symbols below is a cyclic rosette (with notation *cn*) and the other is a dihedral rosette (with notation *dn*). Explain which symbol is of which type and for what value of *n*.

2. Give two integers that have a geometric mean of 6. Is your answer unique?

3. Which, if any, or the letters A, C, E, I, L, S have a shape that is preserved by reflection and/or rotation? Consider the letters in the font as printed here.

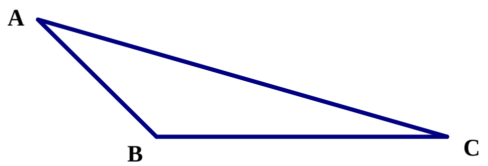
ICTM State 2011
Division A Oral

Symmetry and Patterns

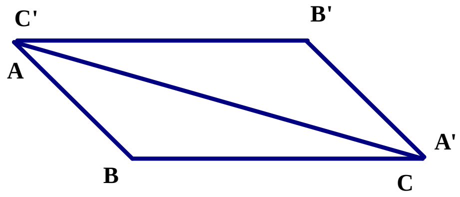
1. Give an example of a strip pattern of type *pma2* made using the symbol ☺.
   Here is a sample:

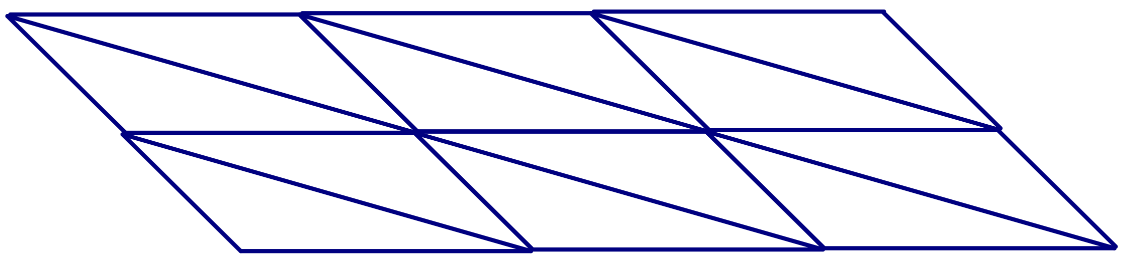
The strip pattern must have a vertical reflection line and a half-turn, but no horizontal reflection line.

2. a. Show how/why triangle ABC, given below, can tile the plane.



If triangle ABC and a copy are fit together to form a parallelogram, strips can be created and duplicated to tile the plane. One possibility is shown below:





Students may also note that in triangle ABC the measures of the three angles is 180^{o} a pattern that has two of each of the angles at the corner point and matching sides will tile the plane, since that will give the pattern 360^{o} at each corner.

b. Is it possible to create a semi-regular tiling using only regular quadrilaterals and regular hexagons (including at least one of each)? If it is possible, show how; if it is not possible, explain why.

No. The sum of the angles meeting at each corner point must be 360^o. Regular quadrilaterals (squares) have an interior angle of 90^o and regular hexagons have an interior angle of 120^o. In order to tile with these figures there must be integer values *x* and *y* so that 90*x* + 120*y* = 360 where both *x* and *y* are greater than or equal to 1. No such values are possible.

c. What is the measure of an interior angle of a regular *n*-gon? For how many odd integral values of *n* will this *n*-gon tile the plane?

Each interior angle of a regular *n*-gon measures $\left(180 - \frac{360}{n}\right)^{o}$

(or an equivalent expression)

Only one odd value will allow it to tile the plane: n=3 (in a triangle).

3. The golden ratio, ϕ, is equal to the expression $\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}}}}$. What is the value of *a*?

One possible solution is given below:

$$\phi = \sqrt{a+\sqrt{a+\sqrt{a+\ldots}}}$$
$$\phi^2 = a+\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}}$$
$$\phi^2 = a+\phi$$
$$\phi^2 - \phi = a$$
$$\phi^2 - \phi = 1$$
$$a = 1$$

Students may also use variations on this solution, and may work out $\phi^2 - \phi$ with a calculator. They should note that $\phi = \frac{1+\sqrt{5}}{2}$.

ICTM State 2011
Division A Oral

Extemporaneous Questions

1. One of the two symbols below is a cyclic rosette (with notation *cn*) and the other is a dihedral rosette (with notation *dn*). Explain which symbol is of which type and for what value of *n*.


The right symbol ("recycle" symbol) is the cyclic rosette because it only has rotation symmetry, not reflection symmetry; the left symbol ("biohazard" symbol) is the dihedral rosette because it also has reflection symmetry. Both symbols coincide with themselves under rotations of 120°, so for both, *n* = 3.

2. Give two integers that have a geometric mean of 6. Is your answer unique?

Students should note that any two integers with a product of 36 will have a geometric mean of 6. Answers should include at least one of 1 and 36, 2 and 18, 3 and 12, 4 and 9, or 6 and 6.

3. Which, if any, or the letters A, C, E, I, L, S have a shape that is preserved by reflection and/or rotation? Consider the letters in the font as printed here.

A, C, E, and I are preserved under reflection (students might note the line of reflection).
I and S are preserved under rotation (180^{o}).