1. A fair, standard cubical die is rolled. Find the probability that the number showing on the upper face is an integral multiple of 3. Express your answer as a common fraction reduced to lowest terms.

2. If $f(3)=43$ when $f(x)=x^3+2x^2+kx+4$, find the value of k.

3. The list price of a television set was \$480. If the set sold at a 20% discount, find the number of dollars in the price for which the set was sold.

4. The arithmetic mean (average) of 8 different positive integers is 10. Find the largest possible value for any one of those 8 positive integers.

5. Express the repeating decimal $2.1\overline{5}$ (where only the "5" repeats) as an improper fraction reduced to lowest terms.

6. If x is a real number, find the sum of all distinct values of x such that $\dfrac{\frac{x+2}{5}-5}{8+\frac{4}{x}}$ and $\dfrac{x^2-23x}{40x+20}$ are **not** equivalent real numbers. Write your answer as an exact **decimal**.

7. Abatoop, Bullseye, and Clingading are lined up in a straight line at random. Find the probability that Abatoop is directly next to Clingading. Express your answer as a common fraction reduced to lowest terms.

8. Tom mixes some chocolate candy at \$4 per pound with 20 pounds of coconut at \$3.20 per pound so that a fair price for the mixture is \$3.40 per pound. Find the number of pounds of chocolate candy that Tom used. Express your answer as an improper fraction reduced to lowest terms.

9. Cindy invests part of \$8,000 at 6% annual interest and invests the rest of the \$8,000 at 5% annual interest. The annual interest income is \$452. Find the number of dollars invested at 6%.

10. Let x and y be positive integers. One year at the ICTM math contest, it was discovered that $\frac{1}{x}$ of the number of Grade 9 attendees had received a D as their semester math grade, that $\frac{1}{y}$ (where $\frac{1}{y} > \frac{1}{x}$) of Grade 9 attendees had received a C as their semester math grade, and that the rest of the Grade 9 attendees had received either an A or a B for their semester math grade. If the total number of the Grade 9 attendees having either a C or a D was $\frac{1}{93}$ of the total number of Grade 9 attendees, find the smallest number possible for Grade 9 attendees who had received a C as their semester math grade.

11. Let $x > 0$, $y > 0$, and $z > 0$. If $xy = 3$, $yz = 40$, and $xz = 30$, find the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$. Express your answer as an improper fraction reduced to lowest terms.

12. Lydia and Gerald are painting the interior of a home and purchased 5 gallons of paint. On the first day, Lydia used $\frac{1}{2}$ gallon of paint and Gerald used $1\frac{1}{4}$ gallons of paint. The next day Lydia used twice as much paint as did Gerald, and together the two finished using up the 5 gallons of paint. Find the number of gallons of paint that Lydia used altogether on the two days. Express your answer as an improper fraction reduced to lowest terms.

13. Let k be a positive integer such that $k < 20$. The roots for x of $x^2 + kx + \frac{1}{2}w = 0$ are real. The roots for y of $y^2 + wy + k = 0$ are also real, and the sum of these real roots for y is -4. Find the sum of all possible distinct values of k.

14. When the system $\begin{cases} 2x - y = 1 \\ x^2 - xy + y^2 = 91 \end{cases}$ is solved, there are two ordered pairs of the form (x, y) in the solution. For one of those ordered pairs, $x > 0$ and $y > 0$. Find that **ordered pair** (x, y).

15. As an example, if $J = 1$, $U = 2$, $D = 3$, and $Y = 4$, then the four-digit number $JUDY$ is 1234, the two-digit number $UD = 23$, and the single-digit number $Y = 4$. If $JUDY$ represents a four-digit number, UD represents a two-digit number, and J and Y are single digit numbers, find the sum of all distinct four-digit numbers $JUDY$ such that $\sqrt{JUDY} = J + UD + Y$.

16. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

There are four cardboard squares—a $2" \times 2"$, a $4" \times 4"$, a $6" \times 6"$, and an $8" \times 8"$. From each of these squares, a $1" \times 1"$ square is removed at one corner. Then a $1" \times 1"$ square is also removed at the corner diagonally opposite the first corner. For how many of the four altered squares can the remainder of the cardboard be completely covered by $2" \times 1"$ rectangular strips without overlapping or without going outside the cardboard?

A) None B) Only the $2" \times 2"$ C) Only the $4" \times 4"$
D) Only the $6" \times 6"$ E) Only the $8" \times 8"$ F) All can be covered this way.

Note: Be sure to write the correct capital letter as your answer.

17. In the system $\begin{cases} 3x + 5y = 235 \\ 14x - ky = 94 \end{cases}$, the letters x, y and k all represent **positive** integers. Find the sum of all possible distinct values of $(x + y)$.

18. The product of two consecutive positive integers is 16 more than the smaller of the two integers. Find the sum of the two consecutive positive integers.

19. The smaller of two consecutive positive integers is an integral multiple of 23, and the larger of the two consecutive positive integers is an integral multiple of 29. An example of a number that consists of 4 consecutive digits is 2534. Find the smallest possibility for the smaller integer if the smaller integer consists of 4 consecutive digits.

20. Find the sum of all distinct positive integral values of x for which $x^2 + 3x + 147$ is the square of an integer.

2012 SA
Algebra I

Name ANSWERS

School

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. =

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{3}$ (Must be this reduced common fraction.)

2. -2

3. 384 ($ optional.)

4. 52

5. $\frac{97}{45}$ (Must be this reduced improper fraction.)

6. -0.5 or $-.5$ (Must be this decimal.)

7. $\frac{2}{3}$ (Must be this reduced common fraction.)

8. $\frac{20}{3}$ (Must be this reduced improper fraction, pounds optional.)

9. 5200 ($ optional.)

10. 3

11. $\frac{73}{60}$ (Must be this reduced improper fraction.)

12. $\frac{8}{3}$ (Must be this reduced improper fraction, gallons optional.)

13. 7

14. $(6,11)$ (Must be this ordered pair.)

15. 8020

16. A (Must be this capital letter.)

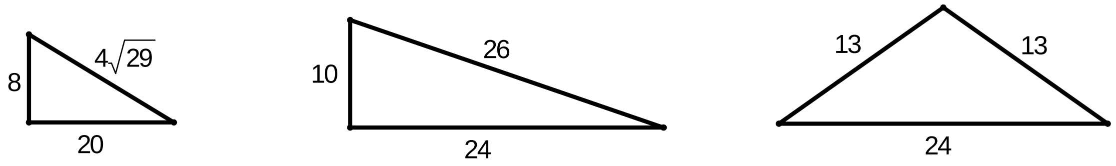
17. 207

18. 9

19. 6785

20. 189

1. If one of the triangles pictured below, with lengths as shown, is selected at random, find the probability that the area of the triangle can be written in the form 30k where k is a whole number. Express your answer as a common fraction reduced to lowest terms.



2. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

A rectangle is drawn in a coordinate plane. Which of the following transformations of the rectangle will shift it 7 units to the left?

A) Multiply each *x*-coordinate by 7.
B) Multiply each *y*-coordinate by 7.
C) Add 7 to each *x*-coordinate.
D) Subtract 7 from each *x*-coordinate.
E) Add 7 to each *y*-coordinate.
F) Subtract 7 from each *y*-coordinate.

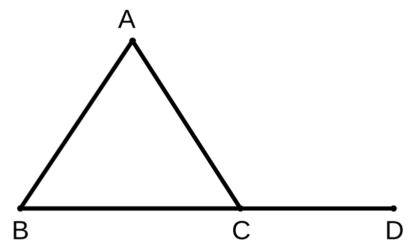
Note: Be certain to write the correct capital letter as your answer.

3. Find the slope of a line that is perpendicular to the line whose equation is $3x+2y=60$. Express your answer as a common fraction reduced to lowest terms.

4. **(Always, Sometimes, or Never True)** For your answer, write the <u>*whole word*</u> **Always, Sometimes,** or **Never**—whichever is correct.

If two decagons are regular, then the two decagons are similar.

5. In the diagram (not necessarily drawn to scale), points B, C, and D are collinear, and $\overline{AB} \cong \overline{BC}$. If $\angle BAC = (3.65x+24.8)^\circ$ and $\angle ACD = (8.65x+57.52)^\circ$, find the degree measure of $\angle ABC$, rounded to the nearest degree.



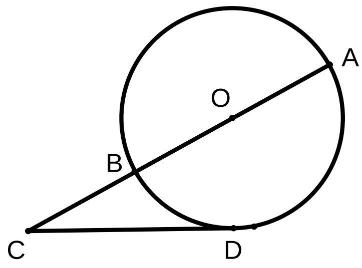
6. The lengths of the three sides of a triangle are 6, 25, and 29. Find the length of the altitude drawn to the shortest side.

7. From a regular hexagon, a regular decagon, and a regular dodecagon, one of the 28 possible interior angles is selected at random. Find the probability that the angle selected has a degree measure that is an integral multiple of 30. Express your answer as a common fraction reduced to lowest terms.

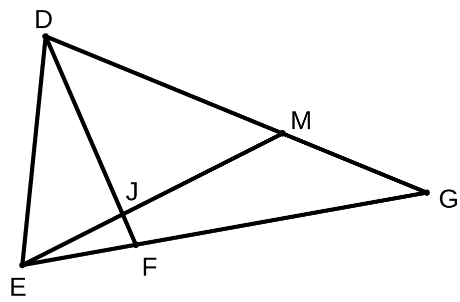
8. **(Always, Sometimes, or Never True)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

The median to one of the shorter sides of an obtuse triangle divides the obtuse triangle into two triangles of equal area.

9. In the diagram, A, B, and D lie on the circle with center O, $\overline{CD}$ is tangent to the circle at D, and C, B, O, and A are collinear. If $\overline{AB}$ is a diameter, $CD = 12$, and $CB = 4$, find the area of ΔCOD.

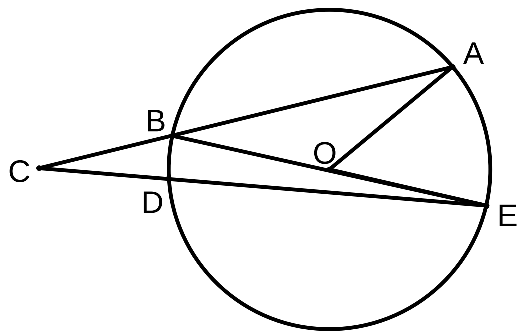


10. In the diagram, E, F, and G are collinear; E, J, and M are collinear; and D, M, and G are collinear. The area of ΔDEG is 1197. If $DM : MG = 7:4$ and $EF : FG = 2:5$, find the area of ΔEFJ.



11. Rounded to the nearest tenth, find the length of an apothem of a regular octagon if the length of a side of the regular octagon is 12. Express your answer as a **decimal**.

12. Points A, B, D, and E lie on the circle with center at O. Point O lies on diameter $\overline{BE}$. Point B lies on $\overline{AC}$, and point D lies on $\overline{CE}$. If $\angle AOB = 102°$ and $\angle BEC = 32°$, find the degree measure of $\angle ACE$.

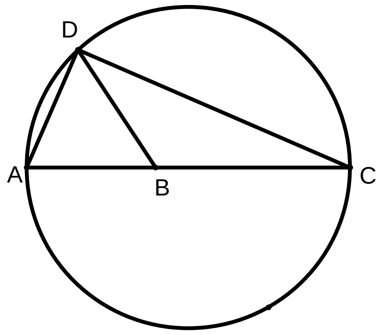


13. A sector with a $60°$ central angle is cut out of a circle with radius of 6, and the remaining $300°$ sector is folded to form a right circular cone. Find the volume of this right circular cone. Express your answer as a **decimal** rounded to the nearest hundredth.

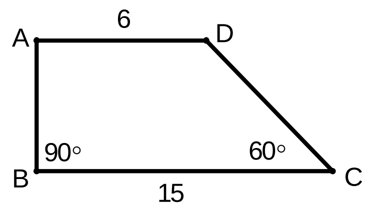
14. An original circle has a radius whose length is 10. A new circle has a radius whose length is 96% of the length of a radius of the original circle. What % of the area of a semi-circle of the original circle is the area of a semi-circle of the new circle? Give an exact **decimal percentage** answer. Do **not** attach the % sign.

15. Find the number of minutes that will elapse between the first time after 7:00 that the minute hand and the hour hand will form a $96°$ angle and the second time after 7:00 that the minute hand and the hour hand will form a $96°$ angle. Express your answer as an improper fraction reduced to lowest terms.

16. In the diagram, points A, D, and C lie on the circle whose radius has a length of 26.5. Point B lies on diameter $\overline{AC}$. $AD = 28$, and the area of $\Delta DBC = 342$. Find the area of ΔABD.

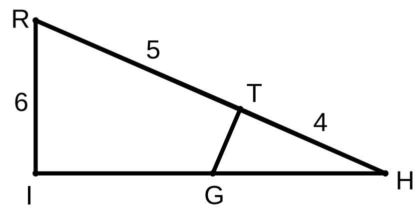


17. In the diagram, $\overline{AD} \parallel \overline{BC}$, $AD = 6$, $BC = 15$, $\angle ABC = 90°$, and $\angle DCB = 60°$. The trapezoid is revolved one complete revolution around $\overline{AB}$. The volume of the solid formed by the revolution can be expressed as $k\pi\sqrt{3}$. Find the value of k.



18. Find the total surface area of a cube whose edge has a length of 3.2. Express your answer as a **decimal**.

19. In the diagram, G lies on $\overline{HI}$, T lies on $\overline{RH}$, $\overline{GT} \perp \overline{RH}$, and $\overline{RI} \perp \overline{HI}$. If $RI = 6$, $RT = 5$, and $HT = 4$, then the area of quadrilateral $RTGI$ can be expressed , in simplest radical form as $\frac{k\sqrt{w}}{f}$, where k, w, and f are positive integers. Find the value of $(k + w + f)$.



20. A triangle has vertices at $(0,0)$, $(12,0)$, and $(6,14)$. Find the sum of the distances of the circumcenter from the 3 sides of this triangle. Express your answer as a **decimal** rounded to 4 significant digits.

2012 SA
Geometry

Name ANSWERS

School ______

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{2}{3}$ (Must be this reduced commn fraction.)
2. D (Must be this capital letter.)
3. $\frac{2}{3}$ (Must be this reduced common fraction.)
4. Always (Must be this whole word.)
5. 72 (Degrees optional.)
6. 20
7. $\frac{9}{14}$ (Must be this reduced common fraction.)
8. Always (Must be this whole word.)
9. 96
10. 48
11. 14.5 (Must be this decimal.)
12. 7 (Degrees optional.)
13. 86.83 (Must be this decimal.)
14. 92.16 (Must be this decimal, no % attached.)
15. $\frac{384}{11}$ (Must be this reduced improper fraction, minutes optional.)
16. 288
17. 1053
18. 61.44 (Must be this decimal.)
19. 39
20. 12.24 (Must be this decimal.)

1. One member of the 4 members of the set $\left\{169,\ 224,\ \sqrt{3^2+4^2},\ 81+19\right\}$ is selected at random. Find the probability that the number selected is the square of an integer. Express your answer as a common fraction reduced to lowest terms.

2. Two hikers, each walking at a constant rate, start walking at constant rates and in the same direction from the city line on the same route. The second hiker whose speed is 5 mph. starts 2 hours after the first hiker whose speed is 4 mph. Find the number of hours for which the second hiker walks in order to catch up to the first hiker.

3. Bud invested k dollars for one year at an annual percentage rate of 5% compounded annually. If the amount of **interest** earned on this one year investment was $400, find the value of k.

4. Find the value of $\log_{27}\left(9\left(\frac{1}{27}\right)^{(-2)}\right)$. Express your answer as an improper fraction reduced to lowest terms.

5. If x is an integer, find the sum of all distinct values of x such that $\frac{x-4}{x-9}-3\geq 0$.

6. If $x^5-9x^4-34x^3-27x^2+30x+72$ is factored with respect to the integers, then the third degree factor is kx^3+wx^2+px+f. Find the value of $\left(k+w+p+f\right)$.

7. If three fair, standard cubical dice are thrown, find the probability that the sum of the numbers showing on the three uppermost faces is 8. Express your answer as a common fraction reduced to lowest terms.

8. A recent survey revealed that 76% of adult residents of Illinois did not know of the existence of the ICTM State Math Contest and that 45% of adult residents of Illinois had math phobia. Let x be the percentage of adult residents of Illinois who did not know of the existence of the ICTM State Math Contest and also who had math phobia. Then the most accurate limiting statement about x that we can make with certainty based only on the given information is $k\% \le x \le w\%$. Find the value of $(k+w)$. **Note**: $(k+w)$ will be a whole number.

9. Let $i=\sqrt{-1}$. Then $-2i^2+\left(\sqrt{-4}\right)\left(\sqrt{4}\right)-\left(\sqrt{-3}\right)\left(\sqrt{-3}\right)-2i^5=a+bi$ where a and b are real numbers. Find the value of $(2a+3b)$.

10. When 1, 2, 3, 4, and 5 are substituted for x in a polynomial expression for x, the results are respectively 1, 24, 61, 112, and 177. If $P(x)$ is the polynomial expression of lowest degree with integral coefficients satisfying the given, find $P(51)$.

11. A line whose equation is $40y=9x+k$ is parallel to a line whose equation is $y=\frac{9}{40}x+17$. If $k>1000$ and if the distance between the two lines is 40, find the value of k.

12. Find the sum of all distinct values of x such that $\left(\log_k(x^2)\right)\left(\log_{12}k\right)=2$.

13. Given the points $A(1,9)$, $B(-7,21)$, and $C(12,3)$. The absolute value of the distance from the centroid of ΔABC to the line represented by $y=5x+2$ can be expressed in simplest radical form as $\frac{\sqrt{k}}{w}$ where k and w are positive integers. Find the value of $(k+w)$.

14. Owen plans to invest \$5000 in a savings account that pays an annual percentage rate of 7% and is compounded quarterly. After the interest is credited at the end of one year, find the value of Owen's investment. Round your answer to the **nearest dollar, and express your answer as that whole number**.

15. The three vertices of a triangle are at $(4,0)$, $(1,3)$, and $(1,6)$. Find the smallest possible value of the sum of the squares of the distances of a point to the three vertices of the triangle.

16. Find the eighth term of an arithmetic progression whose first term is 3 and whose 31st term is 73. Express your answer as an improper fraction reduced to lowest terms.

17. A bag contains exactly 6 marbles—3 red, 2 white, and 1 blue. A boy draws a marble at random, replaces the marble, and continues to draw in this fashion. Find the probability that after 5 draws he has drawn 3 marbles of one color and 2 marbles of a second color. Express your answer as a common fraction reduced to lowest terms.

18. Find the length of the major axis of the ellipse whose equation is $\frac{x^2}{49}+\frac{y^2}{25}=1$.

19. A rectangular park with dimensions 125 feet by 230 feet is surrounded by a sidewalk of uniform width of x feet along the sides of the park. If the area of the sidewalk is between 2900 square feet and 7900 square feet and if x is an integer, find the sum of all distinct possible values for x. Assume all edges of the sidewalk are composed of straight line segments.

20. In Triangle ABC, $AB=16$, $BC=5$, and $AC=19$. $\overrightarrow{AD}$ bisects $\angle CAB$, and $\overrightarrow{CD}$ bisects $\angle ACB$. Expressed in simplest radical form, $AD=k\sqrt{w}$ where k and w are positive integers. Find the value of $(k+w)$.

2012 SA
Algebra II

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{2}$ (Must be this reduced common fraction.)

2. 8 (Hours optional.)

3. 8000 ($ optional.)

4. $\frac{8}{3}$ (Must be this reduced improper fraction.)

5. 21

6. –1

7. $\frac{7}{72}$ (Must be this reduced common fraction.)

8. 66

9. 16

10. 18301

11. 2320

12. 0 or zero

13. 52

14. 5359 ($ optional.)

15. 24

16. $\frac{58}{3}$ (Must be this reduced improper fraction.)

17. $\frac{95}{324}$ (Must be this reduced common fraction.)

18. 14

19. 49 (Feet optional.)

20. 59

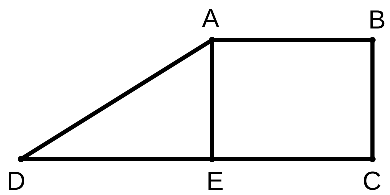
1. The seven letters from the word *HEXAGON* are placed in a bag. A letter is drawn at random from the bag. Find the probability that the letter drawn was a letter that came after the letter *J* in the alphabet. Express your answer as a **common fraction** reduced to lowest terms.

2. $\dfrac{8!}{3!k!} = 56$. Find the value of k.

3. How many non-congruent triangles ABC are possible if $\angle B$ is a given acute angle, $BC = 12$, and $AC = 15$?

4. It is known that y varies inversely as the cosine of x. If the constant of variation is 18, find the value of y when x is $60°$.

5. In Quadrilateral $ABCD$, point E lies on $\overline{DC}$, $\angle ADC = 32°$, $\overline{AB} \parallel \overline{DC}$, $\overline{AE} \perp \overline{DC}$, and $\overline{BC} \perp \overline{DC}$. If $AB = \sqrt{27.23}$, and $BC = 12.47$, find DC. Express your answer as a **decimal** rounded to the nearest hundredth.



6. When $(kx + wy)^9$ is expanded and completely simplified, one of the terms is $84000000x^6y^3$. If k and w are positive integers and if $1 < k < w$, find the value of (kw).

7. Find the value of $\sum_{n=1}^{4}\left(n\left(\sum_{k=1}^{3}(k+1)^2\right)\right)$.

8. A bag contains exactly 3 marbles—1 red, 1 white, and 1 blue. A girl draws a marble at random, replaces the marble, and continues to draw in this fashion. Find the probability that after 6 draws she has drawn at least 1 marble of each color. Express your answer as a **common fraction** reduced to lowest terms.

9. The sum of the last two terms of a seven term geometric progression of real terms is -0.625. The sum of the first two terms of this geometric progression is 20. Find the sum of all seven terms of this geometric progression. Express your answer as an **exact decimal**.

10. If $\csc(\theta)$ is $\frac{13}{5}$ and $\tan(\theta) < 0$, find the value of $\cos(\theta)$. Express your answer as a common fraction reduced to lowest terms.

11. Given the following four sets: $\{1,2,3\}$, $\{1,3,5\}$, $\{1,4,7\}$, $\{1,5,6\}$. If one of the sets is selected at random, find the probability that the total population standard deviation (σ_X) is greater than $\sqrt{5}$. Express your answer as a **common fraction** reduced to lowest terms.

12. The polar graph of $r = 4\cos(k\theta)$ can be described as a rose with 12 leaves. If $k > 0$, find the value of k.

13. In the $xy-plane$, line p is perpendicular to the vector $(-4,7)$ and line p passes through the point $(2,4)$. Find the absolute value of the distance of the point $(101,76)$ from line p. Express your answer as a **decimal** rounded to 4 significant digits.

14. From the set $\{8,11,12,13,19,23,27,29\}$, one number is selected at random. Find the probability that the number selected is within one (population) standard deviation from the arithmetic mean of the set. Express your answer as a **common fraction** reduced to lowest terms.

15. Find the sum of all distinct values of x for which the graph of $f(x)=\dfrac{x^3-120+79x-16x^2}{x^4+193x^2-24x^3-570x+400}$ has vertical asymptotes.

16. Sam borrows \$3375 from a friend. He arranges to pay back \$40 the first month, \$45 the second month, \$50 the third month, and so on by paying \$5 more each month than he did the preceding month. If no interest is charged, how many months will it take Sam to repay the loan? (Note: Sam makes the payments at the end of each month.)

17. Given: $ax^3-4bx^2+cx-10d=0$, $1\le a\le 9$, $2\le b\le 10$, and $0\le d\le 5$. Let $k\le\dfrac{50}{9}$. If the probability that the product of the roots for x is less than or equal to k is 0.4, find the value of k.

18. Find the cosecant of one of the interior angles of a regular polygon with 24 sides. Express your answer as a **decimal** rounded to the nearest thousandth.

19. Let $f(x)=x^3-13.5x^2+42x+39$. The set of values for k such that $f(x)-k$ will have three distinct real zeroes is $\{k : w<k<p\}$. Find the value of $(w+p)$. Express your answer as an **exact decimal**.

20. $\overline{AM}$, $\overline{BN}$, and $\overline{CP}$ are the respective medians of ΔABC. K is the centroid of ΔABC. $AM=37.02$, $BN=33.18$, and $CP=39.51$. Find the degree measure of $\angle MKC$. Express your answer rounded to the nearest hundredth of a degree.

2012 SA
Pre-Calculus

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{3}{7}$ (Must be this reduced common fraction.)

2. 5

3. 1 or one

4. 36

5. 25.17 (Must be this decimal.)

6. 50

7. 290

8. $\frac{20}{27}$ (Must be this reduced common fraction.)

9. 26.875 (Must be this decimal.)

10. $-\frac{12}{13}$ or $\frac{-12}{13}$ or $\frac{12}{-13}$ (Must be this reduced common fraction.)

11. $\frac{1}{4}$ (Must be this reduced common fraction.)

12. 6

13. 13.40 (Must be this decimal, trailing zero is necessary.)

14. $\frac{5}{8}$ (Must be this reduced common fraction.)

15. 11

16. 30 (Months optional.)

17. 4

18. 3.864 (Must be this decimal.)

19. 91.5 (Must be this decimal.)

20. 51.26 (Must be this decimal., degrees optional.)

NO CALCULATORS

1. There are 24 students in a fraternity with exactly two fraternity members named Lee and exactly two fraternity members named Jeffrey. If one member of the fraternity is selected at random, find the probability that the person selected is named Jeffrey. Express your answer as a common fraction reduced to lowest terms.

2. A bicycle, selling at a 40% discount of the regular price, sold for \$90. Find the number of dollars in the regular price of the bicycle.

3. **(Always, Sometimes, or Never True)** For your answer, write the *whole word* **Always, Sometimes,** or **Never**—whichever is correct.

   If two quadrilaterals are similar, then the ratio of their areas is equal to the square of the ratio of their perimeters.

4. A square is circumscribed about a circle, and then a circle is circumscribed about the square. If a point is selected at random in the interior of the bigger circle, find the probability that the point selected is in the interior of the smaller circle. Express your answer as a common fraction reduced to lowest terms.

5. A star is formed by extending the sides of a regular octagon until they reach their initial point of intersection (i.e., regular polygon $ABCDEFGH$ will have $\overline{AB}$ extended to meet $\overline{CD}$ extended, $\overline{BC}$ extended to meet $\overline{DE}$ extended, etc.). Find the sum of the degree measures of the angles at the 8 vertices of the star.

6. How many distinct **ordered pairs** of the form (x, y) exist such that $2x + y = 5$ if x and y are positive integers?

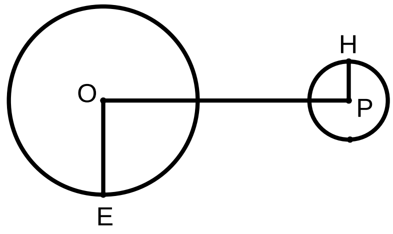
7. If $4x^4 + 4x^2y^2 + 25y^4$ is factored over the integers, the result is $(ax^2 + by^2 + cxy)(ax^2 + by^2 - cxy)$ where $a > 0$, $b > 0$, and $c > 0$. Find the value of $(a + b + c)$.

8. Given the system: $\begin{cases} y = 2k + 3 \\ y = w + 7 \end{cases}$ If both k and w are positive integers, find the smallest possible value of $(k + w)$.

NO CALCULATORS

NO CALCULATORS

9. In the diagram, E lies on the circle with center at O, and H lies on the circle with center at P. $EO = 8$, $HP = 3$, and $OP = 13$. Find the length of a common **external** tangent segment of the two circles.



10. The graphs of $2y + x + 7 = 0$ and $4y + kx + 11 = 0$ meet and form a right angle. Find the value of k.

11. Find the largest possible value of y such that $x + 2y = 5$, $|x| < 2$, and x is an integer.

12. The three vertices of Triangle ABC are $A(-6,19)$, $B(6,-15)$, and $C(27,-3)$. Find the orthocenter of Triangle ABC. Express your answer as an **ordered pair** in the form (x, y).

13. The degree measure of the supplement of an angle is six more than three times the degree measure of the complement of the angle. Find the degree measure of the angle.

14. One of the subsets of the set $\{a,b,c,d\}$ is selected at random. Find the probability that the subset selected has exactly three elements. Express your answer as a common fraction reduced to lowest terms.

15. The orthocenter of a triangle is located at $(7,6)$, and the circumcenter of the same triangle is located at $(19,15)$. Find the ordered pair (x, y) that represents the location of the centroid of this triangle. Express your answer as an **ordered pair** in the form (x, y).

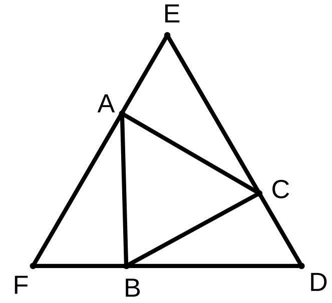
16. If $4^{43}(2^{83} - 1) + 2^{65} - 1$ were computed and the answer is written in base two, compute the number of 0's that would appear in the base two representation of the answer. Give your answer to this problem in base ten.

NO CALCULATORS

NO CALCULATORS

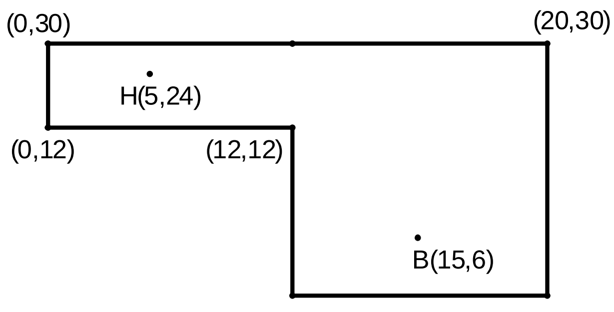
17. During the first week, a book is priced at a whole number of dollars. During the second week the price of the book is i**ncreased** by an amount that is $\frac{1}{2}$ the price of the first week. During the third week the price of the book is **decreased** by an amount that is $\frac{1}{2}$ the price of the second week. During the fourth week the price of the book is **increased** by an amount that is $\frac{1}{2}$ the price of the third week. The price of the book during the fourth week is k times the price of the book during the first week. Find the value of k.

18. In the diagram, ΔDEF is equilateral, points A, B, and C lie on $\overline{EF}$, $\overline{DF}$, and $\overline{DE}$ respectively. $\overline{EA} \cong \overline{FB} \cong \overline{DC}$. $EF = 1$, and $EA = \frac{2}{5}$. The area of ΔABC can be expressed in simplest radical form as $\frac{k\sqrt{w}}{f}$ where k, w, and f are positive integers. Find the value of $(k + w + f)$.



19. For all non-zero values for the variables, $\frac{288x^3y^5z^{-2}}{4x^{-2}y^9z^4} = 2^a3^b x^c y^d z^e$. Find the value of $(a - b + c - d + e)$.

20. On a miniature golf course as shown, the ball is at $B(15,6)$. A golfer wishes to strike the ball so that it hits the barrier running from $(0,30)$ to $(20,30)$ and then immediately rebounds into the hole at $H(5,24)$. To do so, the ball must strike the barrier at $(k,30)$. Find the value of k.



NO CALCULATORS

2012 SA

School ANSWERS

Fr/So 8 Person

(Use full school name – no abbreviations)

_______ Correct X 5 pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{12}$ (Must be this reduced common fraction.)

2. 150 ($ optional.)

3. Always (Must be this whole word.)

4. $\frac{1}{2}$ (Must be this reduced common fraction.)

5. 720 (Degrees optional.)

6. 2 or two

7. 11

8. 5

9. 12

10. -8

11. 3

12. $(10,-9)$ (Must be this ordered pair.)

13. 48 (Degrees optional.)

14. $\frac{1}{4}$ (Must be this reduced common fraction.)

15. $(15,12)$ (Must be this ordered pair.)

16. 21 or 21_{ten} or 21_{10}

17. $\frac{9}{8}$ or $1\frac{1}{8}$ or 1.125

18. 110

19. 4

20. 7

NO CALCULATORS

1. The vertices of a triangle are at $(0,0)$, $(4,0)$, and $(0,k)$. If k is selected at random from the set $\{1,2,3,4,5\}$, find the probability that the triangle is a right triangle.

2. $\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 6 \\ -2 & -1 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 4 \\ -3 & k & -9 \\ -2 & -1 & -5 \end{vmatrix}$. Find the value of k.

3. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

Which of the following has symmetry with respect to the *x*-axis, but **not** with respect to the *y*-axis?

A) $x^2 + y^2 = 5$ B) $5x - |y| = 2$ C) $x^2 + y = 7$
D) $xy + 3y^2 - 2x = 8$ E) $|x| - y^2 = 7$

Note: Be certain to write a capital letter as your answer.

4. The lengths of the legs of a right triangle are respectively $5x+3y$ and $2x$. The length of the hypotenuse of this right triangle is $11x-3y$. The ratio of x to y is $k:w$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

5. If $\left(2x^2 - x^{-1}\right)^7$ is expanded and completely simplified, find the coefficient of the term that contains x^5.

6. Given the system: $\begin{cases} y - 2z = 1 \\ x + y = 5 \\ x + 3z = 5 \end{cases}$. Find the value of $(x+2y+3z)$.

NO CALCULATORS

7. Find the average of all the distinct abscissas of the points on the graph of $8x^2 + 13y^2 = 936$ if the points lie in either Quadrant I or in Quadrant II.

8. When $y^2 + my + 8$ is divided by $(y-3)$, the quotient is $f(y)$ and the remainder is k. When $y^2 + my + 8$ is divided by $(y-5)$, the quotient is $g(y)$ and the remainder is w. If $k = w$, find the value of m.

9. The function whose rule is $f(x) = x(x+2)(x-1)(x-5)(3x+682)(2x-12)(7x-595)$ has a rational zero **between** two consecutive integers. Find the sum of those two consecutive integers.

10. Find the exact value of $\cos(324)° - \sin(162)°$.

11. Let vectors $\vec{c} = (1, -4, 3)$ and $\vec{d} = (4, 0, 1)$. Find the inner (dot) product $\vec{c} \bullet \vec{d}$.

12. In ΔABC, let $AB = c$, $AC = b$, and $BC = a$. $\angle BAC = 60°$ and $(a+b+c)(a+b-c) = \frac{27}{13}ab$. If the length of all sides of the triangle are (positive) integers, find the smallest possible value of c.

13. A set of six scores has a range of 85, a mode of 42, a median of 41, and an arithmetic mean of 39. If one of the scores is 13, find the highest score.

NO CALCULATORS

14. From a standard deck of 52 cards with four suits and 13 ranks per suit, two cards are selected at random without replacement. Find the probability that both cards selected are from the same suit. Express your answer as a **common fraction** reduced to lowest terms.

15. $C(13,1)+C(13,3)+C(13,5)+C(13,7)+C(13,9)+C(13,11)+C(13,13)=2^x$. Find the value of x.

16. Find the center of the ellipse whose equation is $4x^2+9y^2-48x+72y+144=0$. Express your answer as an **ordered pair of the form** (x, y).

17. Let $M=\{A,B,C,D,E,F,G,H,I\}$, consisting of the first nine letters of the alphabet. Without replacement, Judy draws four distinct members of Set M without looking at them. Katie looks at the four members drawn and says: "Judy, you have selected at least one vowel. What is the probability that you chose exactly one vowel?" Report Judy's correct response as a **common fraction** reduced to lowest terms.

18. If $\log_{12} 2+\log_{12} 3+\log_{12} 4+\log_{12} k = 2$, find the value of k.

19. A box contains 135 chips, each of which is either scarlet, gray, blue, or orange. There is at least one chip of each color. The number of gray chips is 4 times the number of scarlet chips. The number of blue chips is greater than two times the number of gray chips, and the number of orange chips is more than three times the number of gray chips. If the number of orange chips is more than the number of blue chips, find the smallest possible number of orange chips.

20. If $\sin(5x-7)^\circ-\cos(3x+9)^\circ=0$, find the sum of all distinct solutions for x such that $0^\circ < x < 360^\circ$.

2012 SA

Jr/Sr 8 Person

School ________ANSWERS__________

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 1

2. 2

3. B (Must be this capital letter.)

4. 47

5. –560

6. 11

7. 0 or zero

8. –8

9. –455

10. $\frac{1}{2}$ (Must be this reduced common fraction.)

11. 7

12. 15

13. 91

14. $\frac{4}{17}$ (Must be this reduced common fraction.)

15. 12

16. $(6,-4)$ (Must be this ordered pair.)

17. $\frac{20}{37}$ (Must be this reduced common fraction.)

18. 6

19. 58 (Chips optional.)

20. 1634 (Degrees optional.)

Unless otherwise specified in the problem, give all answers correct to four significant digits.

1. Find the value of $\sqrt[3]{7-\sqrt{13}} + \sqrt[6]{8047+\sqrt{342.1}}$

2. If $2.6x+12.1y=368.437$ and $y=7x^2-15x-2028.7532$, find the largest possible value for y.

3. The lengths of two of the sides of a right triangle are 65 and 97. The length of the third side is also an integer. Find that integer. Express your answer as an **exact integer**.

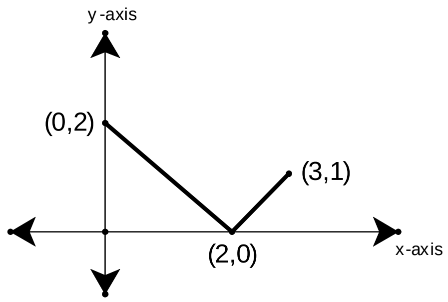
4. Find the value of $\dfrac{1.687}{\left(\dfrac{2.143}{\left(\dfrac{1.542+\dfrac{5.687}{7.862}}{1.534}\right)}\right)}$.

5. A circle with an area of 496.432 has a chord whose distance from the center of the circle is 11.78. Find the length of the chord.

6. Let N be the greatest common divisor of 291,600 and 45,000. Let M be the greatest common divisor of 43,659 and 68,607. Find the least common integral multiple of N and M. Express your answer as an **exact integer**.

7. If $i=\sqrt{-1}$, find the value of $(1.549-1.549i)^8$.

8. The graph of $y = f(x)$ is shown to the right and consists of the union of a line segment from $(0,2)$ to $(2,0)$ and a line segment from $(2,0)$ to $(3,1)$. The graph of $y = 2f(x-1)$ also consists of the union of two line segments with a common endpoint. The three endpoints of the segments on the graph of $y = 2f(x-1)$ lie on a parabola of the form $y = ax^2 + bx + c$. Find the value of $(a+b+c)$. Express your answer as an **exact integer**.



9. Find the positive real solution to $x^{x^{x^{x^{x^{x}}}}} = 58673$.

10. If $f(x) = 1.73x(1-x)$, find the value of $f^{\infty}(0.24)$ where f^{∞} means to iterate infinitely many times. Express your answer as a **decimal** rounded to **ten** decimal places. .

11. At a cost of \$1.83 per square **foot**, find the total cost of painting the total surface area of a cube with an edge of 1.032 **yards**? Express your answer in **dollars and cents** with the cents rounded to the nearest cent. Do **not** use scientific notation.

12. You draw a card at random from a standard 52 card deck with 4 suits and 13 ranks per suit. If the card drawn is a face card rank (K, Q, or J), you win. Otherwise, you replace the card, reshuffle the deck, and draw again at random. Find the probability that it will take more than 4 draws before a face card appears (and the game is over.)

13. Find the least possible area of a triangle with sides of length 62.48 and 89.72 where the angle opposite the side of length 62.48 is $33°16'$.

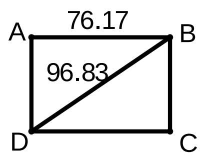
14. Let $f(x) = x^3 - 3.114x^2 - 23.68x + 53.27$. If $-13 < x < -1$, find the largest possible value of $f(x)$.

15. In Triangle ABC, $AB = 495$, and $\angle ABC = 60°$. If all sides of Triangle ABC have integral side-lengths, find the largest possible length of $\overline{AC}$ if $AC < 700$. Express your answer as an **exact integer**.

16. Find the value of the determinant: $\begin{vmatrix} \cos(59.58)° & \sec(2.386)° \\ \sin(24.23)° & \tan(58.53)° \end{vmatrix}$.

17. Shania has 3 sticks, one of length 8, a second with a length less than 11, and a third with a length less than 12. Find the probability the 3 sticks can form a triangle. Express your answer as a **common fraction** reduced to lowest terms.

18. In rectangle $ABCD$ as shown, $AB = 76.17$ and $BD = 96.83$. Find the perimeter of the rectangle.



19. If r represents the length of a radius of a sphere, find the sum of all distinct r such that $r < 2.5$ and such that the volume of the sphere is an integer. Round your answer for that sum to the nearest integer and express your answer as that **integer.**

20. Points A, B, C, D lie on a circle in that order in a clockwise direction. Chords $\overline{AC}$ and $\overline{BD}$ intersect at point E. $AE = 14$, $BE = 8$, $AB = 8$, and $DE = 10$. Rounded to the nearest degree, find the degree measure of $\angle ABC$. Express your answer as a **whole number**.

2012 SA Calculator Team

School **ANSWERS**

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 5.981 OR 5.981×10^0

2. 33.91 OR 3.391×10^1 OR 3.391×10

3. 72 (Must be this integer.)

4. 1.163 OR 1.163×10^0

5. 8.775 OR 8.775×10^0

6. 1,247,400 OR 1247400 (Must be this integer.)

7. 530.3 OR 5.303×10^2

8. 4 (Must be this integer.)

9. 1.784 OR 1.784×10^0

10. 0.4219653179 OR .4219653179 (Must be this decimal.)

11. \$105.25 (Must be in dollars and sense form.)

12. 0.3501 OR .3501 OR 3.501×10^{-1}

13. 898.8 OR 8.988×10^2 (Square units optional.)

14. 80.19 OR 8.019×10^1 OR 8.019×10

15. 693 (Must be this integer.)

16. 0.0003670 OR .0003670 OR 3.670×10^{-4}

17. $\frac{175}{264}$ (Must be this reduced common fraction.)

18. 271.9 OR 2.719×10^2

19. 123 (Must be this integer.)

20. 134 (Must be this integer, degrees optional.)

1. If $RS = 224$, and M is the midpoint of $\overline{RS}$, N is the midpoint of $\overline{MS}$, O is the midpoint of $\overline{NS}$, P is the midpoint of $\overline{OS}$, and Q is the midpoint of $\overline{PS}$, then find the length of $\overline{NQ}$.

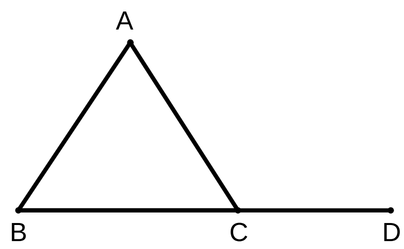
2. If the same value for y satisfies both systems, find the value of k.

$$\begin{cases} 15x + 12y = 171 \\ 5x - y = 17 \end{cases} \text{ and } \begin{cases} 6x + 15y = 66 \\ -3x + ky = 131 \end{cases}$$

3. Find the numerical average of the degree measures of one exterior angle of a regular pentagon and one interior angle of a regular 20-side polygon.

4. If a, b, and c are positive integers, how many distinct ordered triples of the form (a,b,c) exist such that $3a + b + 4c = 15$?

5. In the diagram, points B, C, and D are collinear, and $\overline{AB} \cong \overline{AC}$. Let $\angle ACD = (x + 70)°$, $\angle ABC = (2x - 16)°$, and $\angle ACB = k°$. Let w and f be the largest two positive prime integers less than 100 such that $w - f = 6$. Find the value of $(k + w + f)$.



6. In Feezball, teams can only score in 2 ways: a sug is worth 3 points, and a touchy is worth 7 points. In Feezball, Champaign beat Urbana by 44 to 37. For their respective scores, Champaign made the minimum number of sugs, and Urbana made the maximum number of sugs. Find the **total** number of sugs scored.

7. The length of the hypotenuse of a right triangle is 65, and the sum of the lengths of the two legs of this right triangle is 89. Find the length of the smaller leg.

8. Let k be the sum of the reciprocals of the two roots of $24x^2 - 5x - 1 = 0$. Let w be the distance from the point $(2,5)$ to the closest point on the circle whose equation is $(x-14)^2 + (y+30)^2 = 49$. Find the value of $(k + w)$.

9. The trinomial $2x^2 + 9x - 35$ factors into $(x+a)(2x+b)$. A square with sides of length $12\sqrt{2}$ is inscribed in a circle with a radius whose length is c. Find the value of $(3a + 2b + c)$.

10. Find the least positive integer greater than 4778 that leaves a remainder of 8 when divided by each of the following five integers: 9, 10, 11, 15, 24.

1. If $RS = 224$, and M is the midpoint of $\overline{RS}$, N is the midpoint of $\overline{MS}$, O is the midpoint of $\overline{NS}$, P is the midpoint of $\overline{OS}$, and Q is the midpoint of $\overline{PS}$, then find the length of $\overline{NQ}$.

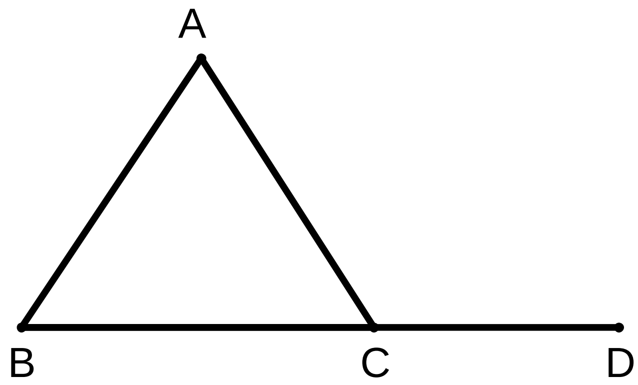
2. If the same value for y satisfies both systems, find the value of k.

$$\begin{cases} 15x+12y=171 \\ 5x-y=17 \end{cases}$$

$$\begin{cases} 6x+15y=66 \\ -3x+ky=131 \end{cases}$$

3. Find the numerical average of the degree measures of one exterior angle of a regular pentagon and one interior angle of a regular 20-side polygon.

4. If a, b, and c are positive integers, how many distinct ordered triples of the form (a, b, c) exist such that $3a + b + 4c = 15$?



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9. The trinomial $2x^2 + 9x - 35$ factors into $\left(x+a\right)\left(2x+b\right)$. A square with sides of length $12\sqrt{2}$ is inscribed in a circle with a radius whose length is c. Find the value of $\left(3a+2b+c\right)$.

10. Find the least positive integer greater than 4778 that leaves a remainder of 8 when divided by each of the following five integers:
9, 10, 11, 15, 24.

2012 SA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. 49	______
2. 13	______
3. 117 (Degrees optional.)	______
4. 5	______
5. 240	______
6. 13	______
7. 33	______
8. 25	______
9. 23	______
10. 7928	______

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. ANS
12. ANS
13. ANS
14. ANS
15. ANS

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let $k = \log_2(\frac{1}{8})$. Let $A\pi$ be the area of the geometrical figure whose equation is $4x^2 + 4y^2 = 1$. Find the value of $(k + A)$. Express your answer as a **decimal**.

2. Find the value of $\sum_{k=0}^{360}\left(\sin\left(k°\right)\right)$.

3. The first term of an arithmetic sequence with a common difference of d is 1, and the sum of the first six terms of this arithmetic sequence is 246. Let k be the absolute value of the distance from the point $(2,5)$ to the line whose equation is $4x + 3y + 67 = 0$. Find the value of $(d + k)$.

4. The equation of a hyperbola is $9x^2 - 16y^2 - 18x - 64y - 199 = 0$. One of the foci of this hyperbola is (x, y) where $x > y$. Find this focus. Express your answer as an **ordered pair** of the form (x, y).

5. If $\log_8(\sqrt{15^5}) = a$ and $\log_2(15) = b$, then $a = kb$. Find the value of k. Express your answer as a common fraction reduced to lowest terms.

6. Let a and b be integral divisors of 226 such that $1 < a < b < 226$. Let k be the positive geometric mean of a and b. $AB = 51$, $BC = 40$, and the area of ΔABC is 924. Let w be the largest possible degree measure of $\angle ABC$. Rounded to the nearest **integer**, find the numerical value of $(k + w)$.

7. Find the **distance** between the point of intersection of the lines with equations of $y = 4x + 2$ and $2y = 3x + 29$ **and** the vertex of the parabola whose equation is $y = 5x^2 - 30x + 1$.

8. Let $a \in \{1,2,3,4,5,6,7,8,9\}$ and let $b \in \{3,4,6,7,8,10,11,12,13\}$. How many distinct ordered pairs (a,b) exist such that the solution set for x of the equation $|2x - a| = |x - b|$ is $\{a, -a\}$?

9. Let x be a positive integer such that $x < 10$. Let k be the sum of all distinct values of x such that the expression $x^2 - x + 43$ is a prime number. Let w be the largest possible positive integer such that $\frac{75!}{2^w}$ is an integer.

10. Let $g(x) = x^5 - 4x^4 + x^3 - 13x^2 - 49$. The reciprocal of each solution for y of the cubic equation $y^3 + hy^2 + wy + 4 = 0$ is also a root for y. Find the value of $(g(3) + w)$.

1. Let $k = \log_2\left(\frac{1}{8}\right)$. Let $A\pi$ be the area of the geometrical figure whose equation is $4x^2 + 4y^2 = 1$. Find the value of $(k + A)$. Express your answer as a **decimal**.

2. Find the value of

$$\sum_{k=0}^{360}\left(\sin\left(k^\circ\right)\right).$$

3. The first term of an arithmetic sequence with a common difference of d is 1, and the sum of the first six terms of this arithmetic sequence is 246. Let k be the absolute value of the distance from the point $(2,5)$ to the line whose equation is $4x+3y+67=0$. Find the value of $(d+k)$.

4. The equation of a hyperbola is $9x^2-16y^2-18x-64y-199=0$. One of the foci of this hyperbola is (x, y) where $x > y$. Find this focus. Express your answer as an **ordered pair** of the form (x, y).

5. If $\log_8(\sqrt{15^5}) = a$ and $\log_2(15) = b$, then $a = kb$. Find the value of k. Express your answer as a common fraction reduced to lowest terms.

6. Let a and b be integral divisors of 226 such that $1 < a < b < 226$. Let k be the positive geometric mean of a and b. $AB = 51$, $BC = 40$, and the area of ΔABC is 924. Let w be the largest possible degree measure of $\angle ABC$. Rounded to the nearest **integer**, find the numerical value of $(k + w)$.

7. Find the **distance** between the point of intersection of the lines with equations of $y = 4x + 2$ and $2y = 3x + 29$ **and** the vertex of the parabola whose equation is $y = 5x^2 - 30x + 1$.

8. Let

$a \in \{1,2,3,4,5,6,7,8,9\}$

and let

$b \in \{3,4,6,7,8,10,11,12,13\}$

How many distinct ordered pairs (a,b) exist such that the solution set for x of the equation

$|2x-a|=|x-b|$ is $\{a,-a\}$?

9. Let x be a positive integer such that $x < 10$. Let k be the sum of all distinct values of x such that the expression $x^2 - x + 43$ is a prime number. Let w be the largest possible positive integer such that $\frac{75!}{2^w}$ is an integer.

10. Let $g(x) = x^5 - 4x^4 + x^3 - 13x^2 - 49$. The reciprocal of each solution for y of the cubic equation $y^3 + hy^2 + wy + 4 = 0$ is also a root for y. Find the value of $\left(g(3) + w\right)$.

2012 SA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. -2.75 (Must be this decimal.)	
2. 0 or zero	
3. 34	
4. $(6,-2)$ (Must be this ordered pair.)	
5. $\frac{5}{6}$ (Must be this reduced common fraction.)	
6. 130	
7. $2\sqrt{1090}$ (Must be this exact answer.)	
8. 5	
9. 78	
10. -202	

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. ANS
12. ANS
13. ANS
14. ANS
15. ANS

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

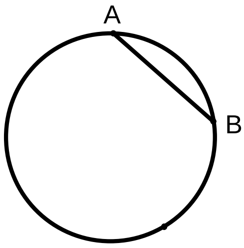
FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 1
ICTM 2012 DIVISION A STATE FINALS

1. Given the system: $\begin{cases} 3x+2y=10 \\ 2x+3y=20 \end{cases}$. Find the value of $(y-x)$.

2. Given the system: $\begin{cases} 3x+2y=ANS \\ 2x+3y=2(ANS) \end{cases}$. Find the value of $(x+y)$.

3. A regular polygon has ANS sides. Find the degree measure of one of the interior angles of the polygon.

4. In the diagram, A and B lie on the circle. $m\overset{\frown}{AB}=(ANS)^\circ$. The length of the diameter of the circle is 60. Find the distance from the midpoint of $\overline{AB}$ to the point on the circle that is closest to the midpoint of $\overline{AB}$.



ANSWERS

1. 10

2. 6

3. 120 (Degrees optional.)

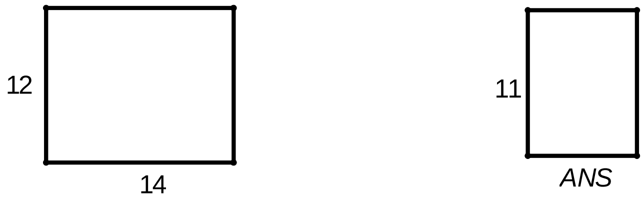
4. 15

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 2
ICTM 2012 DIVISION A STATE FINALS

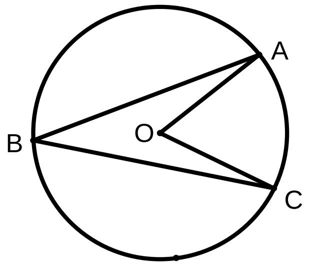
1. If $f(x) = x^2 - 5x$, find the value of $f(-2)$.

2. If $x > 0$, find the value of x such that $x^2 + (ANS)x - 32 = 0$.

3. The two rectangles shown have sides with lengths as shown. By how much does the length of the diagonal of the rectangle on the left exceed the length of the diagonal of the rectangle on the right? Express your answer as a **decimal** rounded to the nearest thousandth.



4. In the diagram, point O is the center of the circle, and points A, B, and C lie on the circle. $\angle ABC = 37.2°$, and $OA = ANS$. Find the area of the sector bounded by radii $\overline{OA}$ and $\overline{OC}$ and minor arc $\overset{\frown}{AC}$. Express your answer as a **decimal** rounded to the nearest thousandth.



ANSWERS

1. 14

2. 2

3. 7.259 (Must be this decimal.)

4. 34.212 (Must be this decimal.)

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 3
ICTM 2012 DIVISION A STATE FINALS

1. Find the value of x such that $\frac{x-2}{x+3}=6$.

2. Find the integral value of $(3^2)(4^2)(19^2)(12^{(-2)})(20^2)(19^{(-2)})(5^{(-2)})(7^2)(2^{(ANS)})$.

3. Find the area of the circle whose equation is $16x^2+32x+16y^2=ANS+199.064$. Express your answer as a **decimal** rounded to the nearest thousandth.

4. In ΔABC, the altitude and the median from vertex A are drawn to $\overline{BC}$. The angle determined by $\overline{AB}$ and the median is bisected by the altitude. If the median has a length of 61.234, and the altitude has a length of ANS, find the length of $\overline{AC}$. Express your answer as a **decimal** rounded to the nearest thousandth.

ANSWERS

1. -4

2. 49

3. 51.849 (Must be this decimal.)

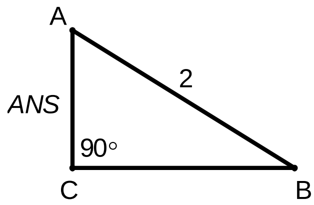
4. 110.634 (Must be this decimal.)

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND A
ICTM 2012 DIVISION A STATE FINALS

1. If 27 more than half of a number is the same as twice the number, find the number.

2. In the following system, solve for y **only:** $\begin{cases} 6x - 2y = ANS \\ 2x + 6y = 12 \end{cases}$. Express your answer as a common fraction reduced to lowest terms.

3. Find the area of the right triangle with measures as shown. Express your answer as a **decimal** rounded to the nearest thousandth.



4. A rectangular solid has lengths of its three dimensions as follows: 1.843, 2.879, ANS. Find the total surface area of the rectangular solid. Express your answer as a **decimal** rounded to the nearest thousandth.

ANSWERS

1. 18

2. $\frac{9}{10}$ (Must be this reduced common fraction.)

3. 0.804 OR $.804$ (Must be this decimal.)

4. 18.205 (Must be this decimal.)

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND B
ICTM 2012 DIVISION A STATE FINALS

1. Evaluate: $\left(5-\sqrt{80.5+\frac{1}{2}}\right)^3$.

2. For all real numbers x and y, $x \otimes y = x^y - y$. Find the value of $ANS \otimes 2$.

3. The volume of a cube is ANS cubic units. Find the number of units in the length of a diagonal **of one of the faces** of the cube. Express your answer as a decimal rounded to the nearest thousandth.

4. Two point-sized bugs leave a point on a flat horizontal plane at the same time. One bug travels straight north at 3 feet per minute, and the other bug travels straight east at 4 feet per minute. Find the number of feet between the bugs after they have been traveling for ANS minutes. Express your answer as a decimal rounded to the nearest hundredth.

ANSWERS

1. -64

2. 4094

3. 22.624 (Must be this decimal, units optional.)

4. 113.12 (Must be this decimal, feet optional.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 1
ICTM 2012 DIVISION A STATE FINALS

1. The arithmetic mean (average) of the five members of the set $\{85, 2, 16, x, 63\}$ is 40. Find the value of x.

2. Let S equal the sum of the terms of the arithmetic sequence $1, 2, 3, 4, \cdots, ANS$. Find the value of $S + 561$.

3. Each side of an equilateral triangle is the same length as each side of a square whose area is ANS. The area of the equilateral triangle is $k\sqrt{3}$. Find the value of k.

4. A class has exactly $\sqrt{ANS}$ students, and exactly 5 are girls. If a team of four students is selected at random from the members of that class, find the probability that there is at least one girl on the team. Express your answer as a common fraction reduced to lowest terms.

ANSWERS

1. 34

2. 1156

3. 289

4. $\frac{377}{476}$ (Must be this reduced common fraction.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 2
ICTM 2012 DIVISION A STATE FINALS

1. If $x = 3.854$ and $y = 20.146$, find the value of $\frac{x+y-|x-y|}{12} + \frac{x+y+|x-y|}{12}$.

2. How many of the following four statements are correct?

(A) If $4.1x + 8.3 = 74.72$, then $x = 16.2$.
(B) $1+2+3+4+5+\cdots+22 = 242$.
(C) If $f(x) = 703x + 14.67$, then $f(ANS) = 2826.67$.
(D) $\sum_{n=1}^{n=ANS} 2^n = 30$.

3. Find the value of the determinant: $\begin{vmatrix} 526 & 83 & ANS \\ 0 & -5 & 0 \\ 2 & -39 & 0 \end{vmatrix}$

4. If $(\log_3(x))(\log_x(2x))(\log_{(2x)}(y)) = (\log_2(ANS+2))$, find the value of y.

ANSWERS

1. 4

2. 3

3. 30

4. 243

JUNIOR-SENIOR RELAY COMPETITION ROUND 3
ICTM 2012 DIVISION A STATE FINALS

1. A single digit is selected at random from the number 39,752. Find the probability that the selected digit is prime. Express your answer as a common fraction reduced to lowest terms.

2. Find the sum of the series: $\sum_{n=0}^{\infty} 2(ANS)^n$.

3. $f(x) = x+4$, $g(x) = x-3$, $h(x) = x+7$, and $r(x) = x^2+2$. Find the value of $r\Big(h\Big(g\Big(f(ANS)\Big)\Big)\Big)$.

4. Find the sum of all distinct prime integers that are greater than ANS and less than 344.

ANSWERS

1. $\frac{4}{5}$ (Must be this reduced common fraction.)

2. 10

3. 326

4. 668

JUNIOR-SENIOR RELAY COMPETITION ROUND A
ICTM 2012 DIVISION A STATE FINALS

1. Find the sum of the two distinct roots of the equation $2x^2 + 5x + 6 = 0$. Express your answer as an improper fraction reduced to lowest terms.

2. Given the two infinite geometric series: $1 + \frac{1}{4} + \frac{1}{16} + \cdots$ and $-5 + ANS + \cdots$. Find the sum of these two infinite geometric series. Express your answer as an improper fraction reduced to lowest terms.

3. Stan and Bob are standing at the same spot on a perfectly level road intersection near Champaign. Assume that Stan walks due East at the rate of $\frac{-3(ANS) - 6}{5}$ mph. and that Bob walks due North at the rate of $\frac{-3(ANS) - 11}{5}$ mph. Find the number of miles between Stan and Bob 5 hours after they started to walk assuming both started to walk at the same time. Assume that the roads on which they walk remain perfectly level.

4. If Ginny collects only dimes and quarters and has a box with ANS coins having a total value of $4.00, find the number of dimes that Ginny has.

ANSWERS

1. $-\frac{5}{2}$ OR $\frac{-5}{2}$ OR $\frac{5}{-2}$ (Must be this reduced improper fraction.)

2. $-\frac{26}{3}$ OR $\frac{-26}{3}$ OR $\frac{26}{-3}$ (Must be this reduced improper fraction.)

3. 25 (Miles optional.)

4. 15

1. If $f(x) = 17x + 8$, by how much does $3(f(4))$ exceed $2(f(5))$?

2, Find the value of $\begin{vmatrix} 13 & 1 \\ 2 & 4 \end{vmatrix} - \begin{vmatrix} ANS & 4 \\ 3 & 1 \end{vmatrix}$.

3. Rounded to the nearest whole number, find the length of a radius of a circle whose equation is $x^2 + (ANS)x + y^2 + 16y = 2$.

4. A parabola whose equation is $y = x^2 - 6x + 4$ is shifted ANS units to the right and seven units upward. The resulting vertex of the shifted parabola is located at (h, k). Find the **ordered pair** (h, k).

ANSWERS

1. 42
2. 20
3. 13
4. $(16, 2)$ (Must be this ordered pair.)

ICTM Mathematics Contest
State 2012 - Division A Oral

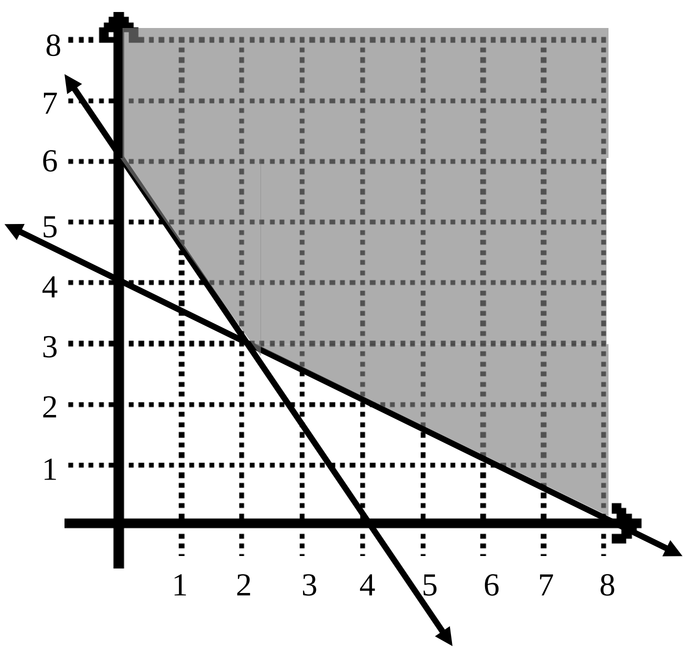
1. a. A tour bus company operates two routes. *x* represents the number of buses that run on Route 1 and *y* represents the number of buses that run on Route 2 in a given week. Route 1 yields a profit of $2000 per bus and Route 2 yields a profit of $1000 per bus. Resource constraints for the company are $x + y \le 20$ and $3x + y \le 30$. How many buses should run on each route in the given week to maximize profit?

   b. Describe what the resource constraint $x + y \le 20$ means in terms of the resources of the tour bus company.

   c. The tour bus company needs to send some of the buses in for maintenance and finds that they only have 12 buses available for Route 2. What additional resource constraint should be used to describe this situation?

2. Given the feasible region graph and resource constraints as shown, with cost function $C = Ax + By$ where $A \ne 0$ and $B \ne 0$. If there are multiple points that will minimize the cost function, what is the relationship between A and B?

$x + 2y \ge 8$
$3x + 2y \ge 12$
$x \ge 0,\ y \ge 0$



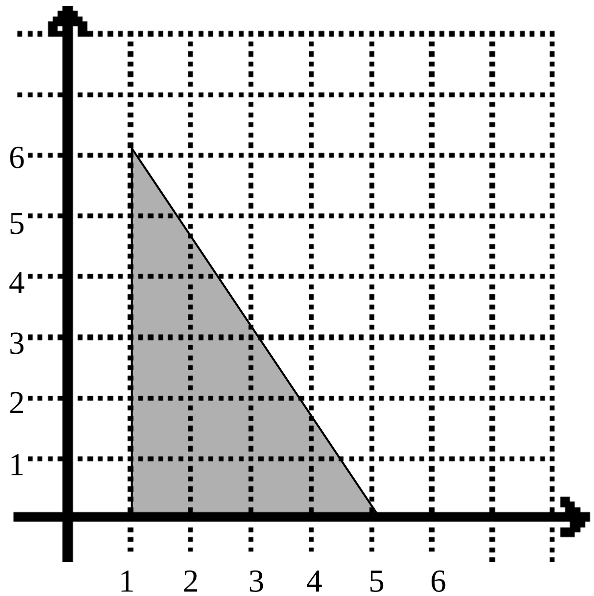
3. A given feasible region has vertices at $(5,6)$, $(7,4)$ and $(4,5)$ and profit function $P = 4x + ky$. What value(s) of *k* will give a maximum profit of 40?

ICTM Mathematics Contest
State 2012 - Division A Oral Extemporaneous questions

Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. *x* represents the number of chairs and *y* represents the number of benches manufactured by a company. In order to meet the resource constraints of the company, the following feasible region is determined.



a. Give 4 possibilities for the number of chairs and benches the company can produce.

b. Give 1 possibility for the number of chairs and benches the company would NOT be able to produce.

2. What are the characteristics required of a feasible region for a linear programming mixture problem so that the corner point principle will work? Describe/draw a graph that would NOT qualify as a feasible region.

3. A patient taking vitamin pills must have at least 16 units of Vitamin A, at least 5 units of Vitamin B and at least 20 units of Vitamin C. She can choose between a pill that contains 8 units of A, 1 unit of B and 2 units of C, and a second pill that contains 2 units of A, 1 unit of B and 7 units of C. What are the resource constraints that can be used to determine how many of each pill she should take? You do not have to graph the resource constraints.

1. a. A tour bus company operates two routes. x represents the number of buses that run on Route 1 and y represents the number of buses that run on Route 2 in a given week. Route 1 yields a profit of \$2000 per bus and Route 2 yields a profit of \$1000 per bus. Resource constraints for the company are $x + y \le 20$ and $3x + y \le 30$. How many buses should run on each route in the given week to maximize profit?

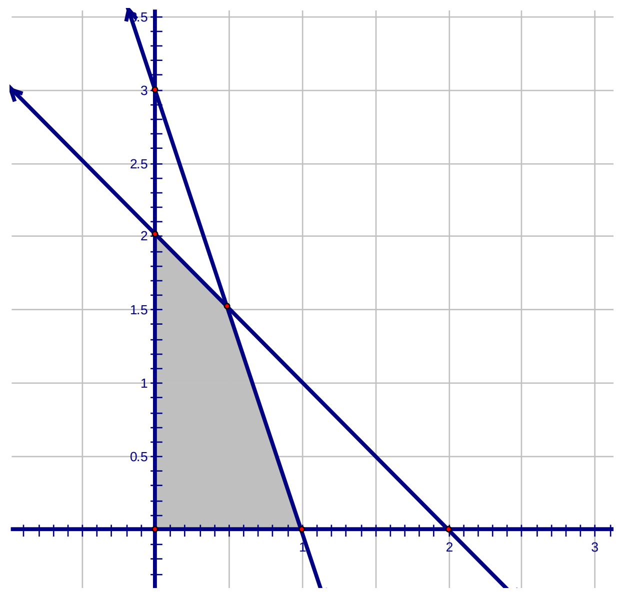
Profit function is $\boldsymbol{P} = 2000\boldsymbol{x} + 1000\boldsymbol{y}$

Corner points are at (0,0), (10,0), (5,15) and (0,20)

Profits at the corner points are:

(0,0) 2000(0)+1000(0)=0
(10,0) 2000(10)+1000(0)=20000
(5,15) 2000(5)+1000(15)=25000
(0,20) 2000(0)+1000(20)=20000

Company should run 5 buses on Route 1 and 15 buses on Route 2



Note: Graph scale is 5 units per grid line.

b. Describe what the resource constraint $x + y \le 20$ means in terms of the resources of the tour bus company.

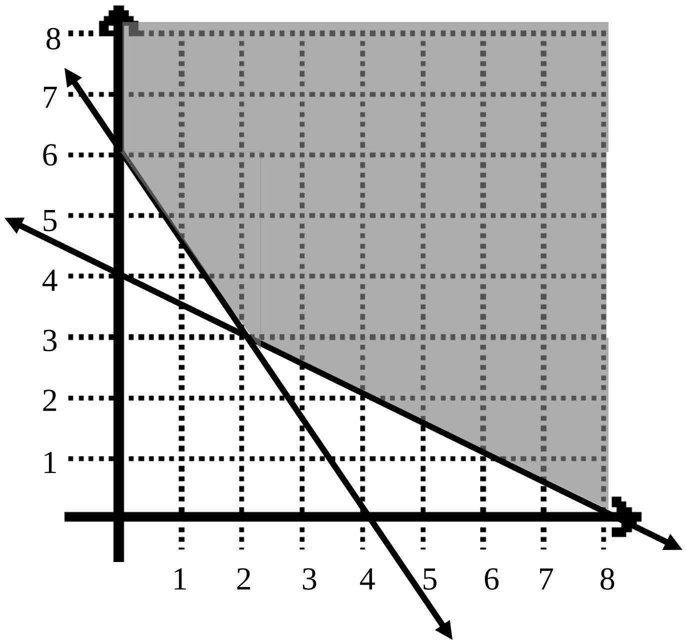
The total number of buses that the company has available is 20.

c. The tour bus company needs to send some of the buses in for maintenance and finds that they only have 12 buses available for Route 2. What additional resource constraint should be used to describe this situation?

Add the resource constraint $y \le 12$

2. Given the feasible region graph and resource constraints as shown, with cost function $C = Ax + By$ where $A \neq 0$ and $B \neq 0$. If there are multiple points that will minimize the cost function, what is the relationship between A and B?

$x + 2y \geq 8$
$3x + 2y \geq 12$
$x \geq 0, \ y \geq 0$



To have multiple solutions, the profit line must lie on a segment that is a boundary of the feasible region. Thus A and B must have the ratio A:B = 1:2 or A:B = 3:2 to correspond to the equations of the lines that are the boundaries of the regions. Students do not need to give both ratios; either should be given credit.

3. A given feasible region has vertices at $(5,6)$, $(7,4)$ and $(4,5)$ and profit function $P = 4x + ky$. What value(s) of k will give a maximum profit of 40?

Substitute each vertex with the maximum profit:

$$4(5) + \boldsymbol{k}(6) = 40 \rightarrow \boldsymbol{k} = \frac{10}{3} \ \textbf{\textit{or}} \ 3.\overline{3}$$

$$4(7) + \boldsymbol{k}(4) = 40 \rightarrow \boldsymbol{k} = 3$$

$$4(4) + \boldsymbol{k}(5) = 40 \rightarrow \boldsymbol{k} = \frac{24}{5} \ \textbf{\textit{or}} \ 4.8$$

Test each value of k for all vertices:

If $\boldsymbol{k} = \frac{10}{3}$: $4\boldsymbol{x} + \frac{10}{3}\boldsymbol{y}$

$4(5) + \frac{10}{3}(6) = 40$

$4(7) + \frac{10}{3}(4) = \frac{124}{3}$ ***or*** $41.\overline{3}$ ** ***Max***

$4(4) + \frac{10}{3}(5) = \frac{98}{3}$ ***or*** $32.\overline{6}$

If $\boldsymbol{k} = 3$: $4\boldsymbol{x} + 3\boldsymbol{y}$

$4(5) + 3(6) = 38$

$4(7) + 3(4) = 40$ *** ***Max***

$4(4) + 3(5) = 31$

If $\boldsymbol{k} = \frac{24}{5}$: $4\boldsymbol{x} + \frac{24}{5}\boldsymbol{y}$

$4(5) + \frac{24}{5}(6) = \frac{244}{5}$ ***or*** 48.8 ** ***Max***

$4(7) + \frac{24}{5}(4) = \frac{236}{5}$ ***or*** 47.2

$4(4) + \frac{24}{5}(5) = 40$

Only a value of k=3 gives the maximum value of 40 when all vertices are checked.

ICTM Mathematics Contest
State 2012 - Division A Oral

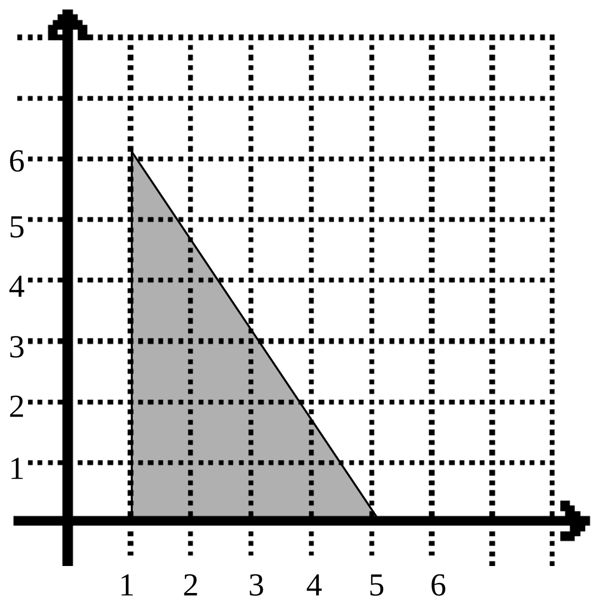
Extemporaneous questions

JUDGE'S SOLUTIONS

Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. x represents the number of chairs and y represents the number of benches manufactured by a company. In order to meet the resource constraints of the company, the following feasible region is determined.



a. Give 4 possibilities for the number of chairs and benches the company can produce.

Students should give examples corresponding to the coordinates of any points that lie on the borders or within the feasible region.

b. Give 1 possibility for the number of chairs and benches the company would NOT be able to produce.

Students should give an example corresponding to the coordinates of a point that lies outside the feasible region

2. What are the characteristics required of a feasible region for a linear programming mixture problem so that the corner point principle will work? Describe/draw a graph that would NOT qualify as a feasible region.

The feasible region must be a convex polygon in the first quadrant (it does not have "dents" or "holes")

As an example, students may show/describe a non-convex polygon, a polygon with a hole in the graph, or a polygon that is not in the first quadrant.

3. A patient taking vitamin pills must have at least 16 units of Vitamin A, at least 5 units of Vitamin B and at least 20 units of Vitamin C. She can choose between a pill that contains 8 units of A, 1 unit of B and 2 units of C, and a second pill that contains 2 units of A, 1 unit of B and 7 units of C. What are the resource constraints that can be used to determine how many of each pill she should take? You do not have to graph the resource constraints.

Resource Constraints are:

$8x + 2y \geq 16$ **Vitamin A**

$x + y \geq 5$ **Vitamin B**

$2x + 7y \geq 20$ **Vitamin C**

Students may also include $x \geq 0$ ***and*** $y \geq 0$ ***but points should not be deducted if these are not given.***