1. A certain number is five more than 90% of 90. Find the value of that certain number.

2. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   There are 7 sections of seats in an auditorium. Each section contains more than 150 seats and less than 201 seats. Which of the following could be the number of seats in this auditorium?

   A) 158 B) 1050 C) 1056 D) 1368 E) 1407 F) 1804

   Note: Be certain to write the correct capital letter as your answer.

3. The slope of the line that contains $P(5,k)$ and $Q(8,w)$ is 3. Find the value of $(w-k)$.

4. In the town of Johnston Village, there are 2329 females and 2055 males. The fractional part of the total of these females and males that are male can be expressed as $\frac{k}{w}$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

5. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

   You have just purchased an item and paid \$39.06 which included a 5% sales tax. The cost of the item before the sales tax was added was:

   A) \$37.20 B) \$37.14 C) \$37.11 D) \$37.10 E) \$37.09

   Note: Be certain to write the correct capital letter as your answer.

6. For all real numbers x, let $\oplus x = x+2$ if x is even and let $\oplus x = 2x-1$ if x is odd. Let $\odot x = x-1$. Based on these definitions, $(\oplus 7)(\odot 4) = \odot k$. Find the value of k.

7. Two cartons have respective weights of $6x-31$ and $3x+10$ pounds. If the average weight of the two cartons is 66 pounds, the heavier carton weighs how many more pounds than the lighter carton?

8. For two quadratic polynomials, $P(x)$ and $Q(x)$, $P(x)+Q(x)=7x^2+3x+1$ and $P(x)-Q(x)=3x^2+7x-5$. The polynomial $Q(x)=ax^2+bx+c$. Find the ordered triple (a,b,c)

9. The x-intercepts of the graph of $y=x^2-2x+k$ are 6 units apart. Find the value of k.

10. If $ax+ac+dx+dc=12$ and $a+d=4$, find the value of $(x+c)$.

11. Find the value of x such that $\frac{4}{x-3}+\frac{1}{x+3}=\frac{8}{x^2-9}$. Express your answer as a **decimal**.

12. Find the value of base x if $\left(\frac{24_x}{30_x}\right)=\left(\frac{3_x}{4_x}\right)$. Express your answer for x in base ten.

13. Given $4x^2+kx-36$ with one factor of $2x-4$. Find the value of k.

14. Assume that stocks are only priced (or traded) in tenths with no rounding; for example, 23.7, 23.8, 23.9, 24, 24.1, etc. Between April 1, 2008 and April 1, 2009, Stock X traded in the range of greater than 62 and less than 71. On April 2, 2008, Stock X traded at $\frac{3}{4}$ the price of Stock W. On March 31, 2009, the price of Stock W was 0.1 lower than it had been on April 2, 2008, and the price of Stock X on March 31, 2009, was $\frac{2}{3}$ the price of Stock W at that time. Find the price of Stock X on March 31, 2009. Express your answer as an exact **decimal** using tenths.

15. The sum of the digits of a three-digit number is 16. The hundreds digit is $\frac{1}{3}$ of the tens digit, and the units digit is four times the hundreds digit. Find the number.

16. One of the roots for x of the quadratic equation $x^2 - 2x + k = 0$ is $1 - \sqrt{10}$. Find the value of k.

17. Find the minimum value of the expression $x + y + z$ given that x, y, and z are positive integers and $8x + y + 12z = 62$.

18. A bag has 1 red and 3 blue marbles. You take out two marbles at the same time. Find the probability that they are the same color. Write your answer as a reduced common fraction.

19. A line contains points A, B and C. If the equation of the line is $2x + y = k$, and the coordinates of points A, B and C are $(2, a)$, $(b, -2)$ and $(1, 4)$ respectively, find the sum $(a + b)$.

20. Find the largest integer that divides 300, 417, and 764 with remainders R_1, R_2, and R_3, respectively, such that $R_2 = R_1 + 3$ and $R_3 = R_2 + 5$.

2013 SA

Algebra I

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 86

2. D (Must be this capital letter.)

3. 9

4. 47

5. A (Must be this capital letter.)

6. 40

7. 10 (Pounds optional.)

8. $(2,-2,3)$ (Must be this ordered triple.)

9. -8

10. 3

11. -0.2 OR $-.2$ (Must be this decimal.)

12. 16 OR 16_{10} OR 16_{ten}

13. 10

14. 62.6 (Must be this decimal.)

15. 268

16. -9

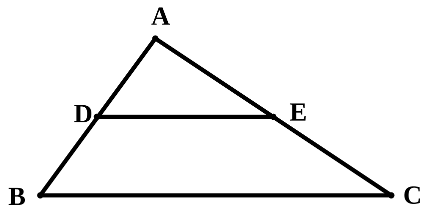
17. 8

18. $\frac{1}{2}$ (Must be this reduced common fraction.)

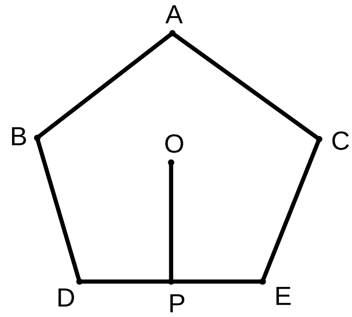
19. 6

20. 114

1. In the diagram, not necessarily drawn to scale, points, A, D, and B are collinear, and points A, E, and C are collinear. $\overline{DE} \parallel \overline{BC}$. If $AD = 18a$, $BD = 45a$, $AE = 70x$, and $EC = kx$, find the value of k.

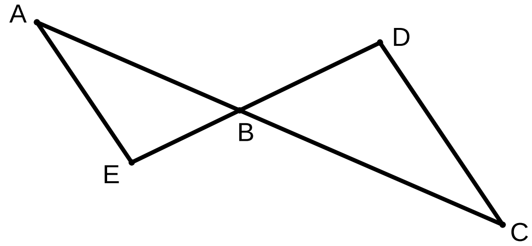


2. Let $ABDEC$ be a regular pentagon with center at point O. Point P lies on $\overline{DE}$ such that $\overline{OP} \perp \overline{DE}$. If $\overline{OD}$ were drawn, find the degree measure of $\angle DOP$.



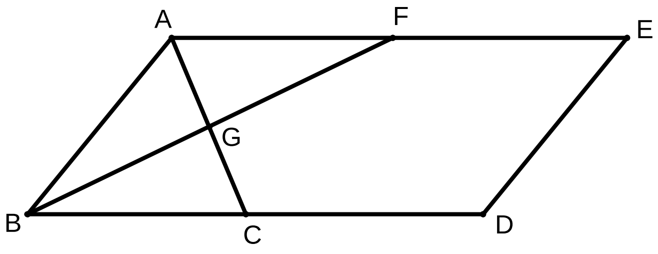
3. Given: $A(1,2)$, $B(3,9)$, and $C(15,6)$. In ΔABC, find the **ordered pair** that represents the point at which the median from B intersects $\overline{AC}$.

4. In the diagram, not necessarily drawn to scale, $\overleftrightarrow{AE} \parallel \overleftrightarrow{DC}$, $BE = 8$, $AB = 12$, $DC = 25$, and $BC = 16$. Find the length AE.



5. The two longer sides of a rectangle have a total length of 96, and one of the diagonals of this rectangle has a length of 52. Find the length of one of the shorter sides of the rectangle.

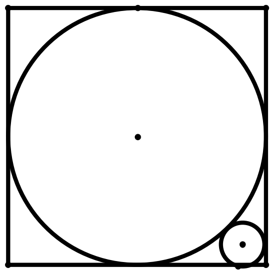
6. In the diagram, $ABDE$ is a parallelogram with F on $\overline{AE}$ and C on $\overline{BD}$. $\overline{AC}$ and $\overline{BF}$ intersect at G. If $AG = 8$, $GF = 12$, and $BG = 2.88$, find the length GC. Express your answer as an **exact decimal**.



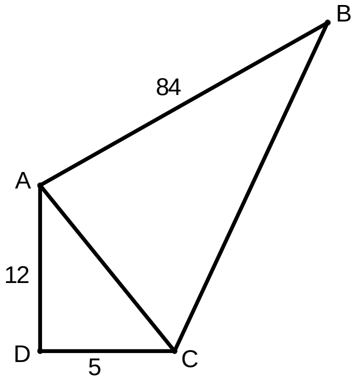
7. A trapezoid has sides with respective lengths 2, 41, 20, 41. Find the length of an altitude of this trapezoid.

8. A triangle whose sides have lengths of $(19+k)$, $(38+k)$, and $(46+k)$, where k is a positive integer, has an area that is also an integer. Find the smallest possible value of k.

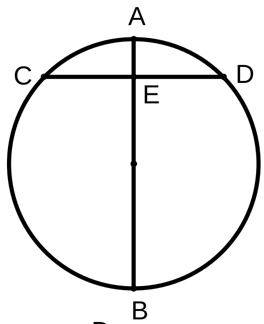
9. In the diagram, a circle is inscribed in a square with a perimeter of 32. A smaller circle is then inscribed in one corner of the square and tangent to the circle as shown in the diagram. The length of the radius of the small circle can be expressed in simplest radical form as $k-w\sqrt{f}$ where k, w, and f are positive integers. Find the smallest possible value of $(k+w+f)$.



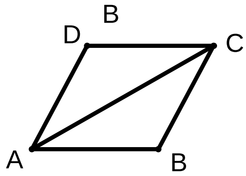
10. In the diagram, $\overline{AD} \perp \overline{CD}$ and $\overline{AB} \perp \overline{AC}$. AB = 84, AD = 12, and CD = 5. Find the area of quadrilateral $ABCD$.



11. In the diagram not necessarily drawn to scale, points A, D, B, and C lie on the circle, $\overline{AB}$ is a diameter, $AE < BE$, and $\overline{CD} \perp \overline{AB}$ at point E. The radius of the circle has a length of 19.5, and $CD : AB = 12 : 13$. The ratio $AE : CE = k : w$, where k and w are positive integers. Find the smallest possible value of $(2k+w)$.



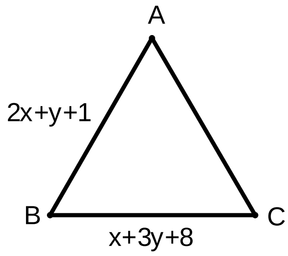
12. In the diagram, $ABCD$ is a rhombus. If the degree measure of $\angle ADC$ is 7 times the degree measure of $\angle BAC$, find the degree measure of $\angle BCD$.



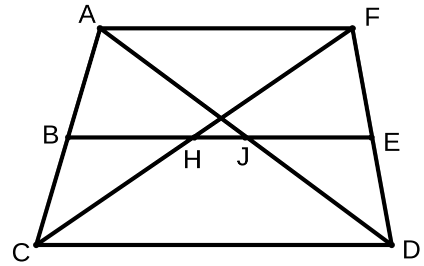
13. Two cubes have edges whose respective lengths are 4 and 5. The ratio of the volume of the smaller cube to the volume of the larger cube is $k : w$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

14. In ΔABC, $AB = 2013$, $BC = 2014$, and $AC = 2015$. Let D be the point on $\overline{BC}$ where $\overrightarrow{AD}$ intersects $\overline{BC}$. Let $\overline{EF}$ (interior to ΔABC) be the common external tangent of the inscribed circles of ΔABD and ΔACD respectively. Let point G be the intersection of $\overline{AD}$ and $\overline{EF}$. Let H and J be the points of tangency with the inscribed circle of ΔABD on $\overline{AB}$ and $\overline{BD}$ respectively, and let I and K be the points of tangency with the inscribed circle of ΔACD on $\overline{AC}$ and $\overline{CD}$ respectively. Let L and M be the points of tangency with $\overline{AD}$ and the inscribed circles of ΔABD and ΔACD respectively. Find the length of $\overline{AG}$.

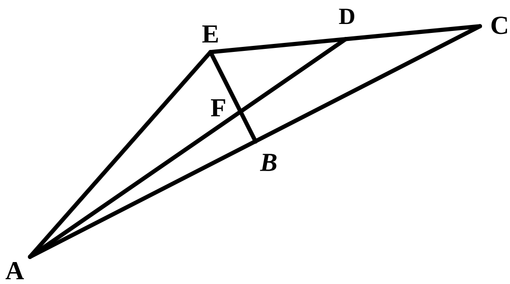
15. In the diagram, ABC is an equilateral triangle with side-lengths as shown. If x and y represent positive integers, and if the perimeter is more than 107, find the smallest possible area of ΔABC.



16. In the diagram, $ACDF$ is a trapezoid with $\overline{AF} \parallel \overline{CD}$. B is the midpoint of $\overline{AC}$. H is the midpoint of $\overline{CF}$. E lies on $\overline{FD}$, points B, H, J, and E are collinear. J lies on $\overline{AD}$. If $AF = 20$ and $CD = 32$, find the length of $\overline{BE}$.



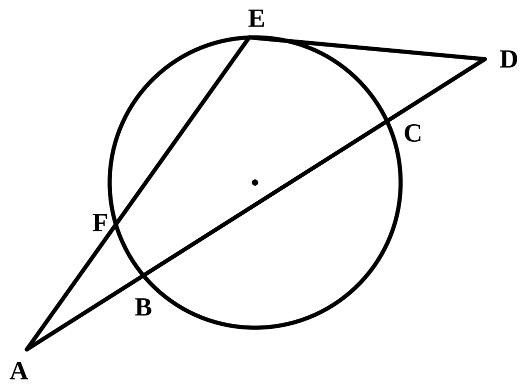
17. In the given diagram, $\overline{AB} \cong \overline{BC}$, $\overline{ED} \cong \overline{DC}$, $EF = FB + 3$ and $AF = 3(FD) - 5$. How many units longer is $\overline{AF}$ than $\overline{FB}$?



18. Find the **sum** of the number of degrees in the interior angles of a convex polygon with 25 sides.

19. A quadrilateral with side-lengths of 4, 5, 7, and k is inscribed in a circle. The area of this quadrilateral is non-integral but can be expressed, in simplest radical form, as $w\sqrt{p}$. If k, w, and p are positive integers, and if $(w+p)$ is a prime integer, find the sum of all distinct possibilities for k.

20. In the given diagram, $\overline{DE}$ is tangent to the circle at point E, $AF = 4$, $EF = 6$, $AB = 5$ and $DE = 3\sqrt{2}$. What is the length of $\overline{AD}$? (Note: the diagram is not drawn to scale.)



2013 SA

Geometry

Name ANSWERS

School

(Use full school name – no abbreviations)

_______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 175

2. 36 Degrees optional.)

3. $(8,4)$ (Must be this ordered pair.)

4. 18.75 OR $18\frac{3}{4}$ OR $\frac{75}{4}$

5. 20

6. 1.92 (Must be this exact decimal.)

7. 40

8. 49

9. 22

10. 576

11. 7

12. 40 (Degrees optional.)

13. 189

14. 1007

15. $400\sqrt{3}$ (Must be this exact area.)

16. 26

17. 7

18. 4140 (Degrees optional.)

19. 26

20. 11

1. The graph of the equation $y = x^2 - 8x + 13$ is reflected over the horizontal line $y = -3$. Find the y-intercept (the y-coordinate of the ordered pair when graphed) of this reflected graph.

2. If $f(x) = x^3 + 2x^2 - 3$, find the value of $f(-7)$.

3. Find the exact distance between the centers of the two circles whose respective equations are $x^2 + 4x + y^2 + 2y = -4$ and $(x+5)^2 + (y-1)^2 = 81$.

4. Let $i = \sqrt{-1}$ and let k represent a real number. If $k - 5i + x = -7 + 2i$ is solved for x, then $x = -10 + 7i$. Find the value of k.

5. Let n represent a positive integer such that $0 < n < 91$. If $n!$ is an integral multiple of 11, find the **sum** of all possible distinct values of n.

6. Let $A = \{1, 2, 3, 4, 5\}$. Let k, w, p, and f represent numbers from A. It is known that exactly two of the four variables represent the same number. How many distinct possibilities exist for the sum of all the distinct numbers represented by the four variables?

7. Find the value of $(\log_8 625)(\log_{25} 20)(\log_{20} 8)(\log_5 25)$.

8. Given the arithmetic sequence: $-10,-9,-8,\cdots,10$. Find the **sum** of all distinct members k of that arithmetic sequence for which $0.5^k < 0.4$.

9. Points A, B, C, and D are the vertices of a rectangle. Point E is in the interior of this rectangle. $AE = 17$, $EC = 31$. The lengths of $\overline{DE}$ and $\overline{BE}$ are integers. If $DE < BE$ and if $BC = 4\left(DE\right)$, find the area of rectangle $ABCD$. Express your answer as a decimal rounded to 4 significant digits.

10. **(Always, Sometimes, or Never)** For your answer, write the <u>whole word</u> **Always, Sometimes,** or **Never**—whichever is correct.

    In a rhombus whose diagonals are not equal in length, the length of the shorter diagonal is equal to the length of one of the sides of the rhombus.

11. An ellipse has vertices of its major axis at $\left(4,0\right)$ and $\left(-4,0\right)$, and its latus rectum has a length of 1. Find the length of the minor axis of this ellipse.

12. The Fibonacci sequence is defined as follows: $F_0 = 0$, $F_1 = 1$, and $F_{(n+1)} = F_n + F_{(n-1)}$ for all integral $n \geq 1$. Let k be an integer such that $3 \leq k \leq 14$. Find the sum of all distinct values of k such that $F_{(k+1)}$ is relatively prime to F_k.

13. ΔABC with $A(\sqrt{104},\sqrt{1850})$ and $B(\sqrt{3744},\sqrt{16650})$ is a right triangle with $\overline{AB}$ as the hypotenuse and with all side-lengths as integers. If the length of the longer leg of ΔABC is k, find the sum of all possible values of k.

14. In a regular pentagon, each diagonal has a length of 1. The perimeter of the pentagon can be written in reduced simplest radical form $\dfrac{k\sqrt{w}-p}{f}$ where k, w, p, and f represent positive integers. Find the smallest possible value of $\left(k+w+p+f\right)$.

15. In a finite geometric sequence, the last term is 1458, the common ratio is -3, and the sum of the terms is 1094. Find the second term of this geometric sequence.

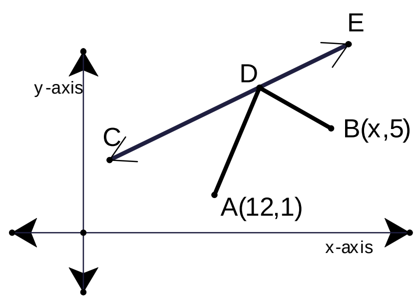
16. In how many distinct points do the real graphs of $x^2 + y^2 = 9103$ and $x^2 = 9102y$ intersect?

17. In a bridge game with a standard 52 card deck (four suits with 13 ranks each), each of 4 persons is dealt 13 cards at random. If Judy is one of these 4 persons, find the probability that Judy was dealt exactly 4 spades, exactly 4 hearts, exactly 3 diamonds, and exactly 2 clubs. Express your answer as a decimal rounded to 4 significant digits.

18. If $x - 7 = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots + \frac{n}{2^n} + \cdots$, find the value of x.

19. In the set of single digit integers $\{3, 6, k, w\}$, the geometric mean is 6. Find the sum $(k + w)$.

20. In the diagram, not necessarily drawn to scale, the line containing points C, D, and E has the equation of $y = 0.5x + 2$. $\angle ADC = 42.56°$, and $\angle BDE = 72.02°$. Find the distance from $A(12,1)$ to $B(x,5)$. Express your answer as a decimal rounded to 4 significant digits.



2013 SA Algebra II

Name ANSWERS

School ______________________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. –19

2. –248

3. $\sqrt{13}$ (Must be this exact answer.)

4. 3

5. 4040

Both answers accepted on appeal

6. 7 OR 11 (Sums optional.)

7. 4

8. 54

9. 689.6 (Must be this decimal.)

10. Sometimes (Must be this whole word.)

11. $2\sqrt{2}$ (Must be this exact answer.)

12. 102

13. 176

14. 17

15. –6

16. 2

17. 0.01796 OR .01796 (Must be this decimal.)

18. 9

19. 17

20. 8.832 (Must be this decimal.)

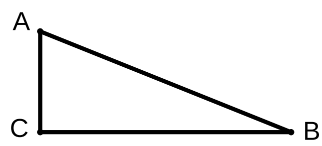
1. The first term of a geometric sequence is $\frac{1}{2}$, and the second term is $\frac{1}{4}$. Find the **sum** of the fourth and fifth term of this geometric sequence. Express your answer as a common fraction reduced to lowest terms.

2. Given that f is an even function and that $f(2)=4$, find the value of $f(-2)$.

3. Let $i=\sqrt{-1}$. If $|21+ki|=75$ and $k<0$, find the value of k.

4. Let $A=\{8,4,2,k\}$ If k is equally likely to be any one of the first 100 positive integers, find the probability that A will consist of 4 distinct numbers that can be arranged in some order to form a geometric sequence. Express your answer as a common fraction reduced to lowest terms.

5. In ΔABC with right angle at C, $\frac{AC}{BA}=\frac{9}{41}$. Find $\sin\angle BAC$. Express your answer as a common fraction reduced to lowest terms.



6. Let $a>0$. Dealing only with real numbers, $\log_a(2x+5)=3$ and $\log_a(3x+40)=6$. Find the value of x. Express your answer as a **decimal**.

7. When 2^{2013} is written as an integer, how many digits are there in the integer?

8. The three dimensional vector $(1,2,3)$ is perpendicular to the three dimensional vector $(5,-16,k)$. Find the value of k.

9. Consider the sequence $\{x_n\}$ defined as $x_{(n+1)} = \frac{6x_n}{7} + \frac{5491}{7x_n}$ for integral n such that $n \geq 1$ and for which $x_1 = 11$. As n increases without bound, the limiting value of x_n is k. Find the value of k.

10. If $\frac{(x+k)!}{x!} - x = 1$ for all possible values of x where x and k represent positive integers, find the value of k.

11. For all values of x, $\sin(3x) = k\sin(x) - w\sin^3(x)$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

12. Let T be the transformation $(x,y) \to (5x+7, ky+9)$ with $k > 0$. If the distance between the images of $(1,3)$ and $(-4,6)$ for transformation T is 20 times the distance between $(1,3)$ and $(-4,6)$, then the value of k can be expressed as $\frac{w\sqrt{f}}{p}$ in simplest radical form where w, f, and p are positive integers. Find the smallest possible value of $(w+f+p)$.

13. Three monkeys randomly toss 24 indistinguishable ping pong balls into three containers labeled A, B, C. If each ball goes in one of the containers and is equally likely to end up in any one of the containers, find the number of distinct outcomes that are possible in which each container has at least one ball. The order in which the balls are placed in the containers does not matter.

14. Let x and n represent two digit positive integers for which the tens digit is non-zero and for which $n > x$. Find the sum of all possible distinct values of n if the sum of the consecutive positive integers from 1 through $(x-1)$, inclusive, is 118 more than the sum of the consecutive positive integers from $(x+1)$ through n, inclusive, where $(x+1) \neq n$.

15. Let $i = \sqrt{-1}$. If z represents a complex number, then $\bar{z}$ represents its complex conjugate. If $z = 5 - 4i$, then $4\bar{z} = k + wi$ where k and w represent real numbers. Find the value of $(k + w)$.

16. Let k and w be positive integers with $k > w$. A coin exists such that the ratio of the probability of tossing a head to the ratio of the probability of tossing a tail is $k : w$. If the coin is tossed six times, the probability of getting 4 heads and 2 tails is $\frac{768}{3125}$. If the coin is tossed five times, the probability of getting 3 heads and 2 tails is $\frac{128}{625}$. Find the smallest possible value of $(2k + 3w)$.

17. In ΔABC, $\sin(\angle BAC) + \cos(\angle BAC) = \frac{167}{145}$. If $\tan(\angle BAC) < 1$, find $\tan(\angle BAC)$ Express your answer as a common fraction reduced to lowest terms.

18. The polynomial equation $x^3 + kx^2 + wx + p = 0$ with k, w, and p representing integers has 5 and $3 + i$ as two of its roots for x. If $i = \sqrt{-1}$, find the value of w.

19. Find the value of the limit $\lim_{x \to 3}\left(\frac{x^2 - 10x + 21}{x - 3}\right)$.

20. Let the brackets $[\ ]$ represent the greatest integer function. If $12\left[\frac{x}{11}\right]^4 - 40\left[\frac{x}{11}\right]^3 + 9\left[\frac{x}{11}\right]^2 + 10\left[\frac{x}{11}\right] = 3$, then the set of all possible values for x can be denoted by $\{x : k \le x < w\}$ for some integers k and w. Find the value of $(k + w)$.

2013 SA
Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{3}{32}$ (Must be this reduced common fraction.)

2. 4

3. –72

4. $\frac{1}{50}$ (Must be this reduced common fraction.)

5. $\frac{40}{41}$ (Must be this reduced commn fraction.)

6. 0.75 OR .75 (Must be this decimal.)

7. 606

8. 9

9. $17\sqrt{19}$ (Must be this exact answer.)

10. 1

11. 7

12. 527

13. 253

14. 180

15. 36

16. 11

17. $\frac{24}{143}$ (Must be this reduced common fraction.)

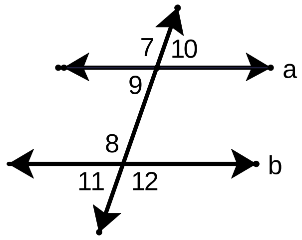
18. 40

19. –4

20. 77

NO CALCULATORS

1. In the diagram, parallel lines a and b are cut by a transversal. Of the 5 labeled angles 7, 8, 9, 11, and 12, how many of these angles **must** be congruent to $\angle 10$?



2. Let x and k represent positive integers such that $\left(x^2\right)\left(x^3\right) = k$ with $k > 1$. Find the smallest possible value of k.

3. Jack and Jill are standing on the outer rim of a circular track. The radius of the track (to the outer rim) is 80 feet. Jack and Jill are $60°$ apart. If Jack were to walk directly around the outer rim to Jill, the smallest number of feet Jack could walk would be $\frac{k\pi}{w}$ feet where k and w are positive integers. Find the smallest possible value of $(k+w)$.

4. Find the ordered pair of the form (x, y) representing a point that lies in Quadrant III if the ordered pair is a member of the solution set for the system $\begin{cases} x^2 + y^2 = 16 \\ 2x^2 - y^2 = -4 \end{cases}$

5. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never—**whichever is correct.

   If two distinct lines are perpendicular to the same plane, then those two distinct lines are parallel.

   Note: Be sure to write the whole word for your answer.

6. Ann, Bob and Cathy together have $51. Ann, Cathy and Don together have $40. Bob and Don together have $25. How many more dollars does Bob have than Don?

7. Let ΔJOE be an isosceles right triangle with hypotenuse $\overline{EO}$. Let ray $\overrightarrow{EL}$ bisect $\angle JEO$ and let ray $\overrightarrow{OA}$ bisect $\angle JOE$. If $\overrightarrow{EL}$ and $\overrightarrow{OA}$ intersect at S, find the degree measure of $\angle ESO$.

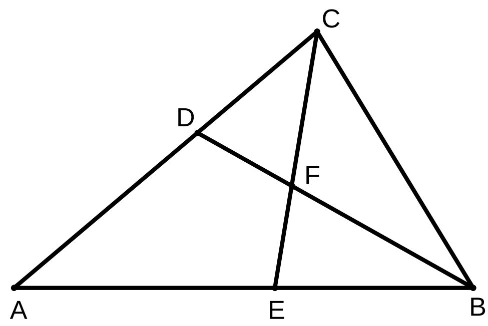
8. Solve for x: $\frac{\left(\sqrt{8}\right)^x (4)^{\frac{x}{2}}}{64^{(x-3)}} = \sqrt{32}$. Express your answer as a reduced common or improper fraction.

NO CALCULATORS

NO CALCULATORS

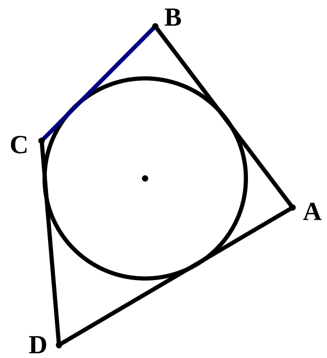
9. By how many square units does the area of a square whose diagonal has a length of $\sqrt{72}$ units exceed the number of square units in the area of a square whose perimeter is 8 units?

10. In the diagram, $\overrightarrow{BD}$ bisects $\angle CBA$, $\overrightarrow{CE}$ bisects $\angle ACB$, and $\overrightarrow{BD}$ and $\overrightarrow{CE}$ intersect at F. Let $\angle CAB = x°$, and let $\angle DFE = y°$. If $x + y = 123$, find the value of x.



11. In the equation $7+\left(\left(8\Psi 6\right)\Xi 3\right)=52$, Ψ and Ξ each represent a different arithmetic operation $\left(+,-,\times,\div\right)$. Assuming the definitions of Ψ and Ξ remain the same as in the given equation, find the value of $9+\left(\left(10\Psi 6\right)\Xi 3\right)$.

12. The circle shown is inscribed in quadrilateral $ABCD$. If $\mathrm{AB}=15$, $\mathrm{BC}=12$ and $\mathrm{CD}=14$, what is the length of $\overline{\mathrm{AD}}$?



13. In a circle, a chord of length 8 units is parallel to a longer chord of a fixed but unknown length. One possible location for the longer chord is one unit from the shorter chord. A second possible location for the longer chord is eleven units from the shorter chord. Find the number of units in the length of the longer chord.

14. Given $\begin{bmatrix} y^2-5y & 6 \\ 9 & 4x^2-100 \end{bmatrix} = \begin{bmatrix} x & y+7 \\ 2x-3 & 44 \end{bmatrix}$, find the ordered pair $\left(x, y\right)$.

NO CALCULATORS

NO CALCULATORS

15. **(Multiple Choice)** For your answer write the **capital letter(s)** which corresponds to the correct choice(s).

A right circular cone has a circular base with a radius of length $9x$ and a height of length $12x$. Which of the following figures with lengths as given has a volume that is equal to this given right circular cone?

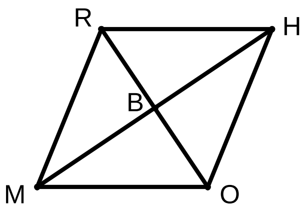
A) A cube with edge of $8\pi x$.
B) A rectangular solid with edges of 2π, $6x^2$, and $27x$.
C) A right circular cylinder with base radius of $6x$ and height of $12x$.
D) A right circular cylinder with base radius of $3x$ and height of $24x$.
E) A right circular cone with base radius of $12x$ and height of $9x$.

Note: Be certain to write the correct capital letter(s) as your answer.

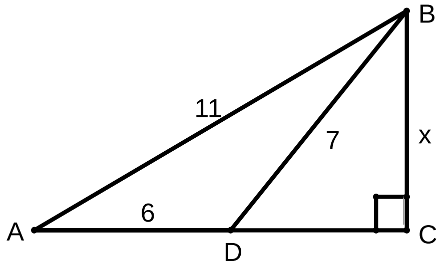
16. Solve for all x so that $\left|8x+2|x+4|\right|=12$.

17. Assuming non-zero denominators, when $\dfrac{1-\frac{4}{x^2}}{\frac{1}{x}-\frac{2}{x^2}}$ is expressed in simplest terms, the answer is $kx+w$ where k and w are integers. Find the value of $(k+w)$.

18. Let $RHOM$ be a rhombus with diagonals intersecting at B. If the perimeter of $RHOM$ is 144 and $RO=36$, find the length of $\overline{MH}$.



19. Given right triangle ΔABC with right angle at vertex C as shown. $AB=11$, $BD=7$, $AD=6$ and $BC=x$. Find the value of x.



20. Find the area of the triangle enclosed by the inequalities $y\le -x+1$, $y\le x+1$ and $y\ge -2$.

NO CALCULATORS

2013 SA

Fr/So 8 Person

School **ANSWERS**

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 2 (Angles optional.)

2. 32

3. 83

4. $\left(-2, -2\sqrt{3}\right)$ (Must be this ordered pair.)

5. Always (Must be this Whole word.)

6. 11 ($ optional.)

7. 135 (Degrees optional.)

8. $\frac{31}{7}$ (Must be this reduced improper fraction.)

9. 32 (Square units optional.)

10. 22

11. 66

12. 17

13. $6\sqrt{3}$ (Must be this exact answer, units optional.)

14. $(6, -1)$ (Must be this ordered pair.)

15. B (Must be this capital letter.)

16. $\frac{2}{5}, -2$ OR $0.4, -2$ OR $.4, -2$ (Must have both answers in either order.)

17. 3

18. $36\sqrt{3}$ (Must be this exact answer.)

19. $2\sqrt{10}$ (Must be this exact answer.)

20. 9

NO CALCULATORS

1. Find the multiplicity of the zero 3 for the polynomial $f(x) = 12x + 2x^3 - 11x^2 + 9$.

2. Let $i = \sqrt{-1}$. If $\sqrt{-144} + \sqrt{-225} + \sqrt{324} = k + wi$ where k and w represent real numbers, find the value of $(k + w)$.

3. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never—**whichever is correct.

   If $\sqrt{x}$ represents an irrational number, then $4.5 + \sqrt{x}$ represents a rational number.

4. Let $C(n,k) = \dfrac{n!}{k!(n-k)!}$ where k and n represent positive integers. Find the value of n such that $C(n,3) = 1771$.

5. If 375 is the 30th term of an arithmetic sequence whose 5th term is 500, find the common difference.

6. Given the system: $\begin{cases} \log_3(A) + \log_5(B) = 4 \\ \log_A(3) + \log_B(5) = 1 \end{cases}$. Find the **ordered pair** (A, B).

7. Find the absolute value of the difference between the two distinct values of x such that the three terms $x-2$, $x+1$, and $4x-8$, taken in that order, will form a geometric sequence.

NO CALCULATORS

8. Let $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$ represent vectors such that $\vec{a} = (-2,5)$, $\vec{b} = (-4,7)$, and $\vec{d} = (12,6)$. If $\vec{a} - \vec{b} = 3\vec{c} + \vec{d}$, then find the ordered pair representing $\vec{c}$. Express the elements of the ordered pair as reduced common or improper fractions.

9. The diagonals of a convex quadrilateral have lengths of 8 and 12, and the acute angle formed by the diagonals has a degree measure of 30. Find the area of the quadrilateral.

10. Let $B = \{1,\ x+3, 9, 11, 12, 2x\}$. The median of these six numbers is 10. Let k be the largest possible number and w the smallest possible number such that $k \le x \le w$. Find the value of $(k+w)$.

11. Let k be an integer such that the roots for x of $x^4 - x^3 + kx^2 - 4x - 48 = 0$ are 2 integers and 2 pure imaginary numbers. Find the value of k.

12. Sammy has a batting average of 0.250 (on the average, 1 hit out of every 4 official at bats). On this basis, find the probability that on his next 5 official at bats, Sammy will get a hit on his first official at bat, at least one hit out of his next 3 official at bats, and a hit on this fifth official at bat. Express your answer as a common fraction reduced to lowest terms.

13. Find the cross product of vector $(1,3,5)$ and vector $(-2,1,3)$. Express your answer as an ordered triple (x, y, z) with x positive.

NO CALCULATORS

14. The graph of f consists of the union of two line segments. The first line segment connects $(0,1)$ to $(2,2)$, and the second line segment connects $(2,2)$ to $(5,3)$. The graph of $y=-3f(x+3)-7$ also consists of the union of two line segments. One line segment connects $(-3,-10)$ to $(-1,-13)$, and the other line segment connects $(-1,-13)$ to (x,y). Find the ordered pair (x,y).

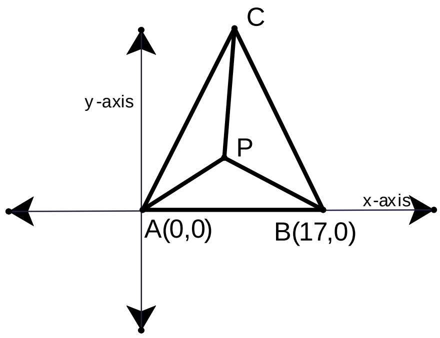
15. Find the value of $\lim_{x\to 5}\left(\frac{2x^2-7x-15}{x-5}\right)$.

16. Find the value of the indicated sum: $\sum_{2}^{5}(k^3+3)$.

17. One of the asymptotes of the conic whose equation is $\frac{(x-3)^2}{9}-\frac{y^2}{25}=1$ has the equation $y=kx+w$ where $k<0$. Find the value of $(60k+50w)$.

18. Let $f(x)=2x+18$ and $g(x)=3x-7$. If $f(x)=g(x)-7$, find the value of x.

19. In the diagram with coordinates as shown and P in the interior of ΔABC, $AC=25$, and $BC=26$. The ratio of the area of ΔPAB to the area of ΔPAC to the area of ΔPBC is $2:3:5$. Find the **ordered pair** that represents point P. Express your answer as an **ordered pair** with **each member** of the ordered pair expressed as an improper fraction reduced to lowest terms.



20. How many distinct integers k such that $1000<k<2000$ with four **different** digits are there such that the absolute value of the difference between at least two digits is 3?

NO CALCULATORS

2013 SA

Jr/Sr 8 Person

School ___**ANSWERS**___

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 2

2. 45

3. Never (Must be this whole word.)

4. 23

5. -5

6. $(9,25)$ (Must be this ordered pair.)

7. 4

8. $\left(-\frac{10}{3},-\frac{8}{3}\right)$ OR $\left(\frac{-10}{3},\frac{-8}{3}\right)$ (Must be this ordered pair)

9. 24

10. 11.5 OR $11\frac{1}{2}$ OR $\frac{23}{2}$

11. -8

12. $\frac{37}{1024}$ (Must be this reduced common fraction.)

13. $(4,-13,7)$ (Must be this ordered pair.)

14. $(2,-16)$ (Must be this ordered pair.)

15. 13

16. 236

17. 150

18. 32

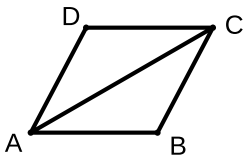
19. $\left(\frac{13}{2},\frac{24}{5}\right)$ (Must be this ordered Pair with reduced improper fractions.)

20. 330

Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. Find the number of degrees equivalent to 11.94 radians.

2. In the diagram, $ABCD$ is a rhombus. If the degree measure of $\angle ADC$ is 7.483 times the degree measure of $\angle BAC$, find the degree measure of $\angle BCD$.

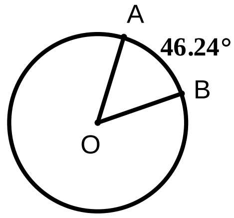


3. Find the real value of x such that $\sqrt{x+6.123}+x=14.98$.

4. Find the value of $\log\left(\sqrt{0.6789}\right)$.

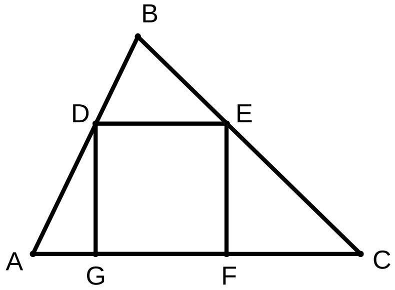
5. Three positive numbers are consecutive members of an arithmetic sequence. The **sum** of these three positive numbers is 935.7. The **product** of these three positive numbers is 17853655.04. Find the smallest of these three positive numbers.

6. In the diagram, point O is the center of the circle, points A and B lie on the circle, and the degree measure of arc $\overset{\frown}{AB}$ is shown. If $AB = 78.11$, find the area of the sector bounded by radii $\overline{AO}$, $\overline{BO}$, and $\overset{\frown}{AB}$.



7. Find the area of ΔABC given that $BC = 42.00$, $AB = 27.00$, and $\angle ABC = 115.0°$.

8. In the diagram (not necessarily drawn to scale), square $DEFG$ is inscribed in ΔABC with D on $\overline{AB}$, E on $\overline{BC}$, and $\overline{GF}$ on $\overline{AC}$. If $AC = 42.08$, $GF = 18.68$, and $\angle BAC = 41.46°$, find the area of ΔABC.



9. If $f(x) = (x-2.146)^3 + 4(x^2+3.17)^2 + 2kx - 130.8$, find the value of k such that $f(1.518) = -36.43$.

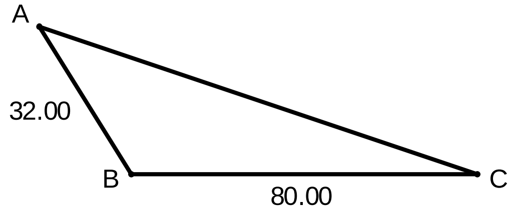
10. Sam is standing on his outside balcony in a residential building that is 679.7 feet tall. He throws a ball straight downward with the release of the ball coming at 348.8 feet from the horizontally flat ground. If the ball hits the ground after 2.000 seconds, find the absolute value of the number of feet per second in the initial velocity.

11. Find the smallest possible integer that is larger than the largest root of $x^3 - 45.8x^2 + 175.45x + 579.684 = 0$. Express your answer as an **integer**.

12. The apothem of a regular octagon has a length of 142.0. Find the perimeter of the regular octagon.

13. The 4 distinct roots for x of the quartic equation $x^4 + kx^3 + wx^2 + px + f = 0$ are 1.222, 2.334, 13.445, and 24.556. Find the value of f.

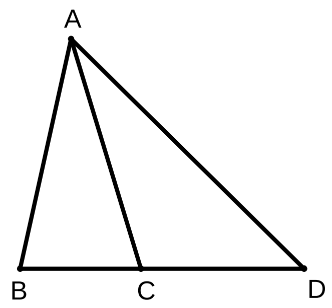
14. In ΔABC with side-lengths as shown, $\angle ABC = 123.4°$. Find the length of $\overline{AC}$.



15. Find the value of $2000^{2001} + 2008^{2009}$. Express your answer in **scientific notation**.

16. Find the length of the radius of a circle whose equation is $x^2 + 16.48x + y^2 + 8.236y = 23.74$.

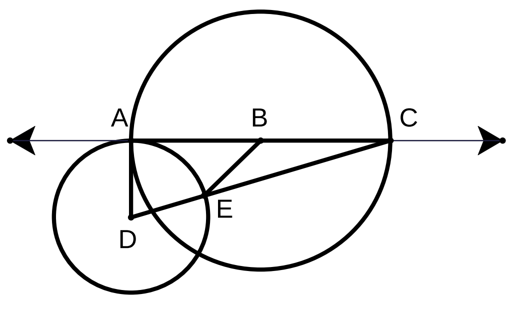
17. In the diagram (not necessarily drawn to scale), points B, C, and D are collinear. $\angle BAC \cong \angle CAD$, $AB = 5.111$, $BC = 2.847$, and $CD = 4.015$. Find the length AC.



18. In a recent girls track team 400 meter race, Alicia finished first with a time of 1 minute, 9**.**300 seconds, and Jenelle placed fourth with a time of 1 minute, 11**.**00 seconds. On the average, Alicia covered how many more meters per **ten seconds** than Jenelle?

19. A coin is so unbalanced that the probability of getting a head on at least three out of four throws is equal to the probability of getting a tail on a single throw. Find the probability of getting a head on a single throw.

20. Point A lies on both circles with centers at D and B. Point E lies on $\odot D$ and on $\overline{DC}$. The segment containing A, B, and C is a diameter, and $\overleftrightarrow{AC}$ is tangent to $\odot D$. The area of $\odot B$ is three times the area of $\odot D$. If $EC = 7$, find the area of the region bounded by $\overline{AB}$, $\overline{BE}$, and the minor arc $\overset{\frown}{AE}$.



2013 SA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 684.1 OR 6.841×10^2
2. 37.96 OR 3.796×10^1 OR 3.796×10
3. 10.86 OR 1.086×10 OR 1.086×10^1
4. -0.08410 OR $-.08410$ OR -8.410×10^{-2} (Trailing zero necessary.)
5. 111.8 OR 1.118×10^2
6. 3992 OR 3.992×10^3
7. 513.9 OR 5.139×10^2
8. 706.8 OR 7.068×10^2
9. -8.319 OR -8.319×10^0
10. 142.4 OR 1.424×10^2
11. 42 (Must be this integer.)
12. 941.1 OR 9.411×10^2
13. 941.7 OR 9.417×10^2
14. 101.2 OR 1.012×10^2
15. 1.788×10^{6635} (Must be in scientific notation.)
16. 10.42 OR 1.042×10 OR 1.042×10^1
17. 5.041 OR 5.041×10^0
18. 1.382 OR 1.382×10^0
19. 0.5722 OR $.5722$ OR 5.722×10^{-1}
20. 3.330 OR 3.330×10^0 (Trailing zero necessary.)

1. Find the area of a trapezoid with the shorter base of length 14, legs of length 13 and 20, and altitude of length 12.

2. If $15 \le x \le 55$, $5 \le y \le 15$, and $45 \le z \le 95$, let k be the greatest possible value of $\frac{x+z}{y}$. If $16a^2 - 9c^2 = 20$ and $4a + 3c = 10$, let $w = 4a - 3c$. Report as your answer the value $(k + w)$.

3. Let k be the smallest integer such that kx **must** be greater than $x + 2$, if $0.6 < x < 1$. All the sides of a right triangle have lengths that are integers. If 10 is the length of the smallest side, let w be the smallest possible length of the largest side. Find the value of $(k + w)$.

4. Players stand in a circle. Player 1 stays in. Player 2 is knocked out. Player 3 stays in. Player 4 is knocked out. This process continues, knocking every other Player out, until only one Player remains. Find the largest possible number of players from $\{20, 21, 22, 23, 24\}$ such that Player 13 is the last player remaining.

5. If x and y are integers such that $19 < x < 23$ and $-8 < y < -4$, let k be the largest possible value of the product xy. Let the "Nelson" of a number be defined as 36 less than 3 times the number. The number that is equal to its "Nelson" is w. Report as your answer the value $(k + w)$.

6. Let $k\pi$ be the area of the circumscribed circle of a triangle whose side-lengths are 12, 16, and 20. The three points $(4,5)$, $(-7,17)$, and $(-95, w)$ are collinear. Find the value of $(k + w)$.

7. Let k be the number of gallons of a 50% silver nitrate solution that are added to 15 gallons of a 30% silver nitrate solution to produce a 35% silver nitrate solution. Let w be the number of sides of a regular polygon whose degree measure of one of the exterior angles is 8. Find the value of $(k + w)$.

8. Let (x, y) be the coordinates of the point that is 0.25 of the way from $(-4,16)$ to $(32,56)$. Let $2^k = 8^{(w-2)}$ and $9^w = 3^{(k+3)}$. Find the value of $(k + w + x + y)$.

9. Let n be the number of distinct integer values for x such that $-20 \le 3x \le 17$. A cube has an edge of length e and a volume of v. If each edge of the cube is increased in length by 25%, the volume of the new cube is kv. Find the value of $(n + k)$. Express your answer as an improper fraction reduced to lowest terms.

10. Let k be the sum of the distinct positive prime factors of 6840. A hole that is 60 units in diameter is cut in a sheet of cardboard assumed to have no thickness. A sphere 68 units in diameter is set in the hole. Let w be the number of units the sphere extends below the surface of the cardboard. Find the value of $(k + w)$.

1. Find the area of a trapezoid with the shorter base of length 14, legs of length 13 and 20, and altitude of length 12.

2. If $15 \le x \le 55$, $5 \le y \le 15$, and $45 \le z \le 95$, let k be the greatest possible value of $\dfrac{x+z}{y}$.

If $16a^2 - 9c^2 = 20$ and $4a + 3c = 10$, let $w = 4a - 3c$.

Report as your answer the value $(k+w)$.

3. Let k be the smallest integer such that kx **must** be greater than $x+2$, if $0.6<x<1$. All the sides of a right triangle have lengths that are integers. If 10 is the length of the smallest side, let w be the smallest possible length of the largest side. Find the value of $\left(k+w\right)$.

4. Players stand in a circle. Player 1 stays in. Player 2 is knocked out. Player 3 stays in. Player 4 is knocked out. This process continues, knocking every other Player out, until only one Player remains. Find the largest possible number of players from $\{20, 21, 22, 23, 24\}$ such that Player 13 is the last player remaining.

5. If x and y are integers such that $19 < x < 23$ and $-8 < y < -4$, let k be the largest possible value of the product xy. Let the "Nelson" of a number be defined as 36 less than 3 times the number. The number that is equal to its "Nelson" is w. Report as your answer the value $(k + w)$.

6. Let $k\pi$ be the area of the circumscribed circle of a triangle whose side-lengths are 12, 16, and 20.
The three points $(4,5)$, $(-7,17)$, and $(-95,w)$ are collinear.
Find the value of $(k+w)$.

7. Let k be the number of gallons of a 50% silver nitrate solution that are added to 15 gallons of a 30% silver nitrate solution to produce a 35% silver nitrate solution. Let w be the number of sides of a regular polygon whose degree measure of one of the exterior angles is 8. Find the value of $(k+w)$.

8. Let (x, y) be the coordinates of the point that is 0.25 of the way from $(-4,16)$ to $(32,56)$.

Let $2^k = 8^{(w-2)}$ and $9^w = 3^{(k+3)}$.

Find the value of $(k + w + x + y)$.

9. Let n be the number of distinct integer values for x such that $-20 \le 3x \le 17$. A cube has an edge of length e and a volume of v. If each edge of the cube is increased in length by 25%, the volume of the new cube is kv. Find the value of $\left(n+k\right)$. Express your answer as an improper fraction reduced to lowest terms.

10. Let k be the sum of the distinct positive prime factors of 6840. A hole that is 60 units in diameter is cut in a sheet of cardboard assumed to have no thickness. A sphere 68 units in diameter is set in the hole. Let w be the number of units the sphere extends below the surface of the cardboard. Find the value of $(k+w)$.

2013 SA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. 294	______
2. 32	______
3. 31	______
4. 22	______
5. –82	______
6. 213	______
7. 50	______
8. 37	______
9. $\frac{893}{64}$ (Must be this reduced improper fraction.)	______
10. 47	______

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. ANS

12. ANS

13. ANS

14. ANS

15. ANS

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let w be a positive integer less than 200 such that $\sqrt{w}=\sqrt{k}+\sqrt{k}+\sqrt{k}$ where k is a positive integer. Find the sum of all distinct possible values of w.

2. Let $i=\sqrt{-1}$. Find the value of $\log_8|8+8i|$. Express your answer as an improper fraction reduced to lowest terms.

3. It is given that $f(x)=\dfrac{1-2x}{3}$ and $g(x)=\dfrac{1-3x}{2}$. Find $f(g(2))+g(f(3))+f(g(4))+g(f(5))$.

4. Let x be a positive integer such that $1<x<20$. Find the sum of all possible distinct values of x such that the sum of the degree measures of the interior angles of some convex polygon is $(x!)$ degrees.

5. The bases of a trapezoid have lengths of 13 and 34, and the non-parallel sides have lengths of 10 and 17. Find the length of the altitude of the trapezoid.

6. The roots for x of $x^3+kx^2+935x-847=0$ are three distinct positive integers. Let a be a positive integer greater than one. Let w be the sum of all distinct possible values for a such that $\log_a(4096)$ is a positive integer. Find the value of $(k+w)$.

7. Let k be the length of the third side of a triangle if the angle included between the sides of lengths 11 and 35 is $60°$. Let the **focus** of the parabola whose equation is $4y=x^2+8x+8$ be (h,w). Find the value of $(k+h+w)$.

8. Let $S=\log(\frac{1}{2})+\log(\frac{2}{3})+\log(\frac{3}{4})+\cdots+\log(\frac{n}{n+1})+\cdots+\log(\frac{99}{100})$. The length k of a side of an equilateral triangle is also a root of the quartic equation $k^4-474k^2-4840=0$. Find the value of $(S+k)$.

9. The quartic equation $x^4-ax^3+bx^2+cx+2262=0$ has four distinct positive integral roots for x. If a is NOT a positive prime number, find the sum of all possible values of a.

10. Let g be the number of distinct permutations of the letters in the word “geometry.” Let p be the probability of drawing 2 hearts if 2 cards are selected (without replacement) at random from 4 hearts and 2 spades. Find the value of the product (gp).

1. Let w be a positive integer less than 200 such that $\sqrt{w}=\sqrt{k}+\sqrt{k}+\sqrt{k}$ where k is a positive integer. Find the sum of all distinct possible values of w.

2. Let $i=\sqrt{-1}$ and $|a+bi|=\sqrt{a^2+b^2}$.

Find the value of $\log_8|8+8i|$.

Express your answer as an improper fraction reduced to lowest terms.

3. It is given that

$$f(x)=\frac{1-2x}{3} \quad \text{and}$$

$$g(x)=\frac{1-3x}{2}. \quad \text{Find}$$

$$f\left(g(2)\right)+g\left(f(3)\right)+ f\left(g(4)\right)+g\left(f(5)\right).$$

4. Let x be a positive integer such that $1 < x < 20$. Find the sum of all possible distinct values of x such that the sum of the degree measures of the interior angles of some convex polygon is $(x!)$ degrees.

5. The bases of a trapezoid have lengths of 13 and 34, and the non-parallel sides have lengths of 10 and 17. Find the length of the altitude of the trapezoid.

6. The roots for x of $x^3 + kx^2 + 935x - 847 = 0$ are three distinct positive integers. Let a be a positive integer greater than one. Let w be the sum of all distinct possible values for a such that $\log_a(4096)$ is a positive integer. Find the value of $(k + w)$.

7. Let k be the length of the third side of a triangle if the angle included between the sides of lengths 11 and 35 is 60°. Let the **focus** of the parabola whose equation is $4y = x^2 + 8x + 8$ be (h, w). Find the value of $(k + h + w)$.

8. Let

$$S = \log(\frac{1}{2}) + \log(\frac{2}{3}) + \log(\frac{3}{4}) + \cdots + \log(\frac{n}{n+1}) + \cdots + \log(\frac{99}{100}).$$

The length k of a side of an equilateral triangle is also a root of the quartic equation

$$k^4 - 474k^2 - 4840 = 0.$$

Find the value of $(S + k)$.

9. The quartic equation

$$x^4 - ax^3 + bx^2 + cx + 2262 = 0$$

has four distinct positive integral roots for x.

If a is NOT a positive prime number, find the sum of all possible values of a.

10. Let g be the number of distinct permutations of the letters in the word "geometry." Let p be the probability of drawing 2 hearts if 2 cards are selected (without replacement) at random from 4 hearts and 2 spades. Find the value of the product (gp).

2013 SA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. 2277	
2. $\frac{7}{6}$ (Must be this reduced improper fraction.)	
3. 14	
4. 175	
5. 8	
6. 4101	
7. 26	
8. 20	
9. 124	
10. 8064	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. ANS
12. ANS
13. ANS
14. ANS
15. ANS

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 1
ICTM 2013 DIVISION A STATE FINALS

1. Let $R = gS - 4$ where g remains constant. When $S = 8$, then $R = 12$. Find the value of R when $S = 12$.

2. Successive discounts of $ANS\%$ and 10% are equivalent to a single discount of $x\%$. Find the value of x. Give your answer as the value of x only (without the % symbol).

3. The diameters of two circles are 8 inches and ANS inches respectively. Find the ratio of the area of the smaller circle to the area of the larger circle, written as a reduced common fraction $\frac{m}{n}$ with m and n integers. Give as your answer the value of n.

4. A triangle and a trapezoid have the same number of square inches in their areas. They also have the same altitude. If the base of the triangle is ANS inches, find the length, in inches, of the median of the trapezoid. Give your answer as an exact decimal.

ANSWERS:

1. 20

2. 28

3. 49

4. 24.5 (Must be this decimal, inches optional.)

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 2
ICTM 2013 DIVISION A STATE FINALS

1. Liz travels 360 miles due north at 30 **miles per hour**. She returns 360 miles due south to her starting point at 2 **miles per minute**. Find her average rate in **miles per hour** for her entire trip?

2. The arithmetic mean (average) of a set of ANS scores is 38. If the scores of 45, 55, 65, and 75 are discarded, find the mean of the remaining set of scores.

3. A running track is formed by two concentric circles. The track is ANS feet wide. Find the difference, in feet, of the circumferences of the two circles. Give your answer rounded to the nearest foot.

4. A regular octagon is formed by cutting congruent isosceles right triangles from each of the corners of a square with sides of length ANS. Find the exact length of each side of the octagon.

ANSWERS:

1. 48 (mph optional.)

2. 36

3. 226 (Feet optional.)

4. $226\left(\sqrt{2}-1\right)$ OR $226\sqrt{2}-226$ OR exact equivalent.

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 3
ICTM 2013 DIVISION A STATE FINALS

1. In a group of cows and chickens, the number of legs was 20 more than twice the number of heads. How many cows are in the group?

2. A person born in the first half of the ANS^{th} century was x years old in the year x^2. Find the year in which that person was born. (Note: For example, the 3rd century includes the years 201 through 300.)

3. ANS should be a three digit number. The number in the hundreds place represents the hours and the number formed by the tens and ones digit represents the minutes. Find the smaller degree measured angle formed by the hour and minute hands at this time shown on a clock.

4. A circle is inscribed in a triangle with sides of length ANS, 56, and 119. Find the length of the radius.

ANSWERS:

1. 10 (Cows optional.)

2. 930

3. 105 (Degrees optional.)

4. 21

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 4
ICTM 2013 DIVISION A STATE FINALS

1. k and w are the roots of $x^2 + kx + w = 0$, $k \neq 0$ and $w \neq 0$. Find the sum $(k + w)$.

2. a and b are single digit integers used as the ten's digit in the following problem. The three-digit number $2a3$ is added to the number $(327 + ANS)$ to give the three-digit number $5b9$. If $5b9$ is divisible by 9, find the value of $(a + b)$.

3. Let $k = ANS - 3.5$. The area of a rectangle remains unchanged when it is made k inches longer and $\frac{2}{3}$ of an inch narrower, or when it is made k inches shorter and $\frac{4}{3}$ inch wider. Find the numerical area, in square inches, of the rectangle.

4. Two high school classes took the same test. One class of ANS students scored a mean grade of 80% while another class of 30 students scored a mean grade of 70%. Find the mean grade for all students in both classes.

ANSWERS:

1. -1

2. 6

3. 20 (Square inches optional.)

4. 74 (% optional.)

FRESHMAN-SOPHOMORE RELAY COMPETITION ROUND 5
ICTM 2013 DIVISION A STATE FINALS

1. At his usual rate Sean rows 15 miles downstream in five hours less time than it takes him to return. If he doubles his usual rate, the time downstream is only one hour less than the time upstream. Find the rate of the stream's current in miles per hour. (Assume the stream flows at a constant rate.)

2. Amanda and Daoud working together can complete a job in ANS days. Daoud and Muhsen can do the same job in four days; while Amanda and Muhsen can complete the job working together in $2\frac{2}{5}$ days. In how many days can Amanda complete the job if she works alone?

3. A cube is made by using twelve pieces of wire, each ANS inches long, for the twelve edges of the cube. An ant starts at one vertex and then walks along the edges and returns to its starting point without passing through any other vertex more than once and without retracing any edge (or part of an edge.) Find the maximum possible distance the ant could walk under these conditions. Give your answer measured in inches.

4. The lateral surface area of a right circular cone is $(ANS)(10\pi)$ square feet. If the radius of the cone's base is 12 feet, find the length of the altitude of the cone measured in feet.

ANSWERS:

1. 2 (mph optional.)

2. 3 (Days optional.)

3. 24 (Inches optional.)

4. 16 (Feet optional.)

1. Let $R = gS - 4$ where g remains constant. When $S = 8$, then $R = 12$. Find the value of R when $S = 12$.

2. Successive discounts of $ANS\%$ and 10% are equivalent to a single discount of $x\%$. Find the value of x. Give your answer as the value of x only (without the % symbol).

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4. The lateral surface area of a right circular cone is $(ANS)(10\pi)$ square feet. If the radius of the cone's base is 12 feet, find the length of the altitude of the cone measured in feet.

2013 SA FR/SO RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. 20
2. 28
3. 49
4. 24.5 (Must be this decimal, inches optional.

ROUND 2

1. 48 (mph optional.)
2. 36
3. 226 (Feet optional.)
4. $226\left(\sqrt{2}-1\right)$ OR $226\sqrt{2}-226$ OR exact equivalent.

ROUND 3

1. 10 (Cows optional.)
2. 930
3. 105 (Degrees optional.)
4. 21

EXTRA ROUND 4

1. -1
2. 6
3. 20 (Square inches optional.)
4. 74 (% optional.)

EXTRA ROUND 5

1. 2 (mph optional.)
2. 3 (Days optional.)
3. 24 (Inches optional.)
4. 16 (Feet optional.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 1
ICTM 2013 DIVISION A STATE FINALS

1. Let (k, w) be the solution for the system of equations $\begin{cases} 3x-2y=6 \\ 2x-3y=-6 \end{cases}$. Give as your answer the value of k.

2. Let $2<|x-ANS|<12$. Find the smallest integral value for x for which this inequality is true.

3. At Honeybees, making independent choices, the probability of Simon ordering a Coke with his meal is $\frac{3}{8}$, and the probability of Suzanne ordering a Coke with her meal is $\frac{3}{|ANS|}$. Find the probability of either Simon or Suzanne ordering a Coke with their meal. Give your answer as a reduced common fraction.

4. If $0 \leq \theta \leq \frac{\pi}{2}$ and $\tan\theta = ANS$, find the value of $\sin(36°+\theta) - \cos(54°-\theta) + \sin(2\theta)$. Give your answer as a reduced common or improper fraction.

ANSWERS:

1. 6

2. -5

3. $\frac{3}{4}$ (Must be this reduced common fraction.)

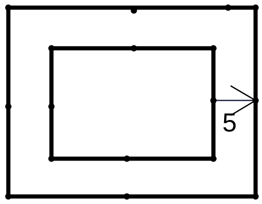
4. $\frac{24}{25}$ (Must be this reduced common fraction.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 2
ICTM 2013 DIVISION A STATE FINALS

1. Find the smallest integral value for x that is a solution of $-4x+17<6x-8$.

2. Ruby has 7 tee shirts, identical in every way except color. *ANS* of these shirts are the same color while the other shirts are all different colors. In how many ways can she arrange these 7 tee shirts in a single row on a table?

3. The total area (walk and garden together) of a rectangular garden surrounded by a uniform 5 foot wide walk, as shown in the diagram, is *ANS* square feet. The length of the actual garden is 2 feet longer than the width. Find the length, in feet, of the garden (not including the walk).



4. If $f(x)=5x^2-3x+7$, find the value of the constant term of $f(x+ANS)$.

ANSWERS:

1. 3

2. 840

3. 20 (Feet optional.)

4. 1947

JUNIOR-SENIOR RELAY COMPETITION ROUND 3
ICTM 2013 DIVISION A STATE FINALS

1. (k, w) is the solution for the matrix equation $\begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$. Find the value of $(k + w)$.

2. A flat circular ring is formed by two concentric circles. The outer radius is $6 + |ANS|$ centimeters and inner radius is 6 centimeters. The area between the two circles is $k\pi$ square centimeters. Find the value of k.

3. From a starting point in Death Valley, two hikers take separate straight line paths to different check points. Assume a flat surface. One check point is 6 miles from the starting point, and the other check point is 8 miles from the starting point. The angle between the paths is $(ANS)°$. Find the number of miles between the two checkpoints. Give your answer as an integer rounded to the nearest whole mile.

4. The probability of Diana making a 3 point shot in basketball is $\frac{3}{2(ANS)}$. What is the probability of Diana making at least four 3 point shots in 7 tries? Give your answer as a decimal rounded to the nearest hundredth.

ANSWERS:

1. -2

2. 28

3. 4 (Must be this integer, miles optional.)

4. 0.24 OR .24 (Must be this decimal.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 4
ICTM 2013 DIVISION A STATE FINALS

1. Find the value of x that is a solution for $\left(3^{(x+2)}\right)\left(9^{2}\right)=\left(9^{(x-4)}\right)\left(27^{\frac{4}{3}}\right)$.

2. Points $(3,5)$, $(-1, ANS)$, and $(6,k)$ are collinear. Find the value of k. Give your answer as a reduced common or improper fraction.

3. Find the integral value of x that is a solution for $\log_{16}|2x-26|-\log_{16}|x+2| = ANS$.

4. How many integral solutions does $|(ANS)x+5|<17$ have?

ANSWERS:

1. 10

2. $\frac{5}{4}$ (Must be this reduced improper fraction.)

3. -3

4. 11 (Integers or solutions optional.)

JUNIOR-SENIOR RELAY COMPETITION ROUND 5
ICTM 2013 DIVISION A STATE FINALS

1. Find the largest negative integer in the domain of the function $f(x)=\sqrt{3x^2+7x-6}$.

2. A circular stained glass window is divided into 5 sectors (not necessarily equal). The radius of the window is $|ANS|$ feet. The two yellow sectors have a total area of $\frac{21\pi}{4}$ square feet. Find the sum total of the degree measure of the central angles of these two sectors.

3. On the same day, Maria invested ANS dollars at 1.2% per year compounded continuously for 10 years at Bank A and invested \$250 at 1.1% per year compounded quarterly for 10 years at Bank B. At the end of the 10 years, she cashed in each investment separately. Find the total amount, rounded to the nearest cent, Maria received when she cashed in her two investments. Give your answer in normal dollar and cent decimal notation. (**Note:** Banks round down to the nearest cent when determining the cash value of an investment.)

4. Let $k=100(ANS)$. The area of a residential lot is k square feet. The lot is shaped like a trapezoid with a height of 160 feet. The longer parallel side is 52 feet longer than the shorter parallel side. Find the length, rounded to the nearest foot, of the longer parallel side.

ANSWERS:

1. -3

2. 210 (Degrees optional.)

3. 515.79 (Must be this decimal, \$ or dollars optional.)

4. 348 (Must be this integer, feet optional.)

1. Let (k, w) be the solution for the system of equations $\begin{cases} 3x - 2y = 6 \\ 2x - 3y = -6 \end{cases}$. Give as your answer the value of k.

2. Let $2 < |x - ANS| < 12$. Find the smallest integral value for x for which this inequality is true.

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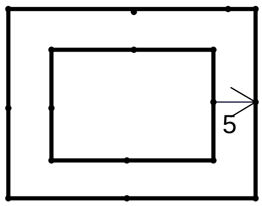

4. If $f(x) = 5x^2 - 3x + 7$, find the value of the constant term of $f(x + ANS)$.

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4. Let $k = 100(ANS)$. The area of a residential lot is k square feet. The lot is shaped like a trapezoid with a height of 160 feet. The longer parallel side is 52 feet longer than the shorter parallel side. Find the length, rounded to the nearest foot, of the longer parallel side.

2013 SA JR/SR RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. 6
2. -5
3. $\frac{3}{4}$ (Must be this reduced common fraction.)
4. $\frac{24}{25}$ (Must be this reduced common fraction.)

ROUND 2

1. 3
2. 840
3. 20 (Feet optional.)
4. 1947

ROUND 3

1. -2
2. 28
3. 4 (Must be this integer, miles optional.)
4. 0.24 OR .24 (Must be this decimal.)

EXTRA ROUND 4

1. 10
2. $\frac{5}{4}$ (Must be this reduced improper fraction.)
3. -3
4. 11 (Integers or solutions optional.)

EXTRA ROUND 5

1. -3
2. 210 (Degrees optional.)
3. 515.79 (Must be this decimal, $ or dollars optional.)
4. 348 (Must be this integer, feet optional.)

ORAL COMPETITION
ICTM STATE 2013 DIVISION A

1. Let O, A and B have coordinates (0, 0, 0), (3, 2, 1) and (2, -1, 2), respectively. Let $\vec{A}$ and $\vec{B}$ be short notations for $\overrightarrow{OA}$ and $\overrightarrow{OB}$, respectively. Find the distance between the terminus of $\vec{A}\times\vec{B}$ and the terminus of $\vec{B}\times\vec{A}$.

2. Let A = (2, 3, 0), B = (5, 1, 0) and C = (8, 6, 0).

   (i) Use Theorem 9 (reference, page 137) to find the area of triangle ABC.

   (ii) Evaluate the determinant $D=\begin{vmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 1 & 8 & 6 \end{vmatrix}$.

   (iii) Use your results to suggest (but not prove) a formula for the area of a triangle in the xy-plane with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3).

3. Prove or disprove: If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are non-zero vectors in three-space and $\vec{A}\times\vec{B}=\vec{A}\times\vec{C}$, then $\vec{B}=\vec{C}$.

ORAL COMPETITION
ICTM STATE 2013 DIVISION A

EXTEMPORANEOUS STATE QUESTIONS

Judges: Give this sheet to the students at the beginning of the extemporaneous question period.

STUDENTS: You will have a maximum of 3 minutes TOTAL to solve and present your solution to these problems. Either or both the presenter and the oral assistant may present the solutions.

1. If $\vec{G} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ and $\vec{H} = 6\vec{i} - 7\vec{j} + 8\vec{k}$ and $\vec{Q} = 20\vec{i} - 30\vec{j} + 40\vec{k}$, find the length of $\vec{H} \times (\vec{Q} \times \vec{G})$.

2. If you are given all of the vertices of a parallelepiped, how can you compute its volume? Draw a picture to clarify your explanation.

ORAL COMPETITION
ICTM STATE 2013 DIVISION A

SOLUTIONS FOR JUDGES

1. Let *O*, *A* and *B* have coordinates (0, 0, 0), (3, 2, 1) and (2, -1, 2), respectively. Let $\vec{A}$ and $\vec{B}$ be short notations for $\overrightarrow{OA}$ and $\overrightarrow{OB}$, respectively. Find the distance between the terminus of $\vec{A}\times\vec{B}$ and the terminus of $\vec{B}\times\vec{A}$.

SOLUTION

The two cross-product vectors are negatives of each other, so we need only find the length of one of them and then double the result.

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 2 & 1\\ 2 & -1 & 2\end{vmatrix}=5\vec{i}-4\vec{j}-7\vec{k}.$$

The length of this vector is $3\sqrt{10}$. Report $6\sqrt{10}$.

2. Let *A* = (2, 3, 0), *B* = (5, 1, 0) and *C* = (8, 6, 0).

(i) Use Theorem 9 (reference, page 137) to find the area of triangle *ABC*.

(ii) Evaluate the determinant $D=\begin{vmatrix}1 & 2 & 3\\ 1 & 5 & 1\\ 1 & 8 & 6\end{vmatrix}$.

(iii) Use your results to suggest (but not prove) a formula for the area of a triangle in the *xy*-plane with vertices $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3).

SOLUTION

(i) $Area=\frac{1}{2}\bullet\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\bullet\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & -2 & 0\\ 6 & 3 & 0\end{vmatrix}=\frac{1}{2}\bullet\left|21\vec{k}\right|=\frac{21}{2}.$

(ii) $D=\begin{vmatrix}1 & 2 & 3\\ 1 & 5 & 1\\ 1 & 8 & 6\end{vmatrix}=1(30-8)-2(6-1)+3(8-5)=21.$

(iii) Looks like $Area=\left|\frac{1}{2}\bullet\begin{vmatrix}1 & x_1 & y_1\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}\right|$. (*D* could have been negative.)

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SOLUTIONS FOR JUDGES

3. Prove or disprove: If $\vec{A}, \vec{B}$ and $\vec{C}$ are non-zero vectors in three-space and $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$, then $\vec{B} = \vec{C}$.

SOLUTION

The assertion is false.
A simple counter-example is $\vec{A} = 5\vec{i}, \vec{B} = 6\vec{i}, \vec{C} = 8\vec{i}$.
Both cross-products are $\vec{0}$ but $\vec{B} \neq \vec{C}$.
Another example is $\vec{A} = 6\vec{i} + 10\vec{j} - 6\vec{k}$, $\vec{B} = 2\vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{C} = 5\vec{i} + 8\vec{j} + \vec{k}$.
Then both cross-products are $58\vec{i} - 36\vec{j} - 2\vec{k}$ but $\vec{B} \neq \vec{C}$.
What does follow from $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$ is that $\vec{A} \times (\vec{B} - \vec{C}) = \vec{0}$, implying that $\vec{A}$ and $\vec{B} - \vec{C}$ are parallel.

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EXTEMPORANEOUS STATE QUESTIONS

SOLUTIONS FOR JUDGES

1. If $\vec{G} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ and $\vec{H} = 6\vec{i} - 7\vec{j} + 8\vec{k}$ and $\vec{Q} = 20\vec{i} - 30\vec{j} + 40\vec{k}$, find the length of $\vec{H} \times (\vec{Q} \times \vec{G})$.

SOLUTION

$\vec{Q}$ and $\vec{G}$ are parallel, so $\vec{Q} \times \vec{G} = \vec{0}$ and $\vec{H} \times (\vec{Q} \times \vec{G}) = \vec{0}$, so report 0.

2. If you are given all of the vertices of a parallelepiped, how can you compute its volume? Draw a picture to clarify your explanation.

SOLUTION

With the notation shown, all faces being parallelograms, one formula for the volume is $\left| \overrightarrow{AE} \bullet (\overrightarrow{AB} \times \overrightarrow{AD}) \right|$. (Parentheses are not necessary.) More generally, choose one vertex and the three vectors tailed there and form a scalar triple product as illustrated here. Having selected three vectors, the choice of which two to cross is not important; the numerical result is the same for all permutations.