1. The equations $4(x+3)-2=2(x-4)+12$ and $ax+6=21$ both have the same solution when solved for x. Find the value of a.

2. Find the greatest odd three-digit number with hundreds digit equal to twice its ones digit and an even tens digit.

3. The expressions $a(x-2)^2+(3x+1)^2$ and $(2x-5)^2+12(x^2+kx)+c$ are equivalent for all values of x. Determine the product (akc). Express your answer as a common or improper fraction reduced to lowest terms.

4. Given $f(x)=3x+14$ and $g(x)=2x^2-15$, find the value(s) of x for which $f(2x-1)=g(x+3)$.

5. When $x^3+kx^2-6x-17$ is divided by $x-3$ the remainder is 10. Find the value of k.

6. In the following number base problem, if $\frac{1}{6}$ of 40_x is 8 (base 10), then find $\frac{1}{7}$ of 36_x. Express your answer as a base 10 number, written without the base.

7. A fraction F has a value such that $\frac{5}{8}<F<\frac{2}{3}$. The denominator of F is an integer that is 7 more than the numerator of F. Find the largest possible denominator of F.

8. Let d represent the absolute value of the difference of the squares of two consecutive positive odd integers, and let m represent the arithmetic mean of those two consecutive positive odd integers. If $d = km$, find the value of k.

9. Find the largest value of x for which $(8x+4)^2 = 12(2x+1)+10$.

10. Express the repeating decimal $2.01\overline{4}$ (where only the 4 repeats) as an improper fraction reduced to lowest terms.

11. An odd and an even number are picked at random from the first ten positive integers. Find the probability that their sum is 11. Express your answer as a common fraction reduced to lowest terms.

12. The points $(6,3)$, $(-1,2)$, and $(9,k)$ are collinear, and $k = \frac{m}{n}$, where m and n are relatively prime positive integers. Find the value of $\sqrt{m^2+n^2}$.

13. Kellie rides her bike 3 miles each day to school. The first mile is uphill and she rides at a constant rate of 4 mph. The second mile is downhill, where she coasts at a constant rate that is an integer, is faster than her uphill rate and is less than 60 mph. For the final mile, she rides at a constant rate of 6 mph. Her average speed, in mph, for the entire trip to school is an integer. Find the speed, in mph, that she coasts down the hill.

14. There are 20 married couples in a group. Each couple is one man and one woman. Find the number of ways two people can be selected consisting of a woman and a man who is not her husband.

15. Solve for t if $y = \frac{4}{x}\left(\frac{a}{t}\right) + 1$ and $xt \neq 0$, $y \neq 1$. Express your answer as a single simplified (reduced) rational expression.

16. Find the value(s) of x for which $x + 5 - 2\sqrt{x+5} = 15$.

17. Find the smallest positive integer greater than 1000 that leaves the same positive integral remainder when divided by 30, 36, and 50.

18. The number 5760 can be expressed as $4^a 5^b 6^c$, where a, b and c are rational numbers. Find the value of $(a + b + c)$.

19. Find the value(s) of x so that $1^{(x)} = 2^{(x+1)} = 0^{(x+2)} + 1$.

20. If $x + y = 7$ and $\frac{1}{x} + \frac{1}{y} = \frac{3}{8}$ with $xy \neq 0$, find the value of $\left(x^2 + xy + y^2\right)$. Express your answer as a common or improper fraction reduced to lowest terms.

2014 SA
Algebra I

Name **ANSWERS**

School ______

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. -5

2. 683

3. $-\frac{14}{3}$ OR $\frac{-14}{3}$

4. $-4, 1$ (Must have both answers in either order.)

5. 2

6. 6

7. 20

8. 4

9. $\frac{1}{8}$ OR 0.125 $.125$

10. $\frac{1813}{900}$ (Must be this reduced improper fraction.)

11. $\frac{1}{5}$ (Must be this reduced common fraction.)

12. 25

13. 12 (mph optional.)

14. 380

(#15: Or reduced exact equivalent single rat'l expression.)

15. $\frac{4a}{yx-x}$ OR $\frac{-4a}{x-yx}$ OR $\frac{4a}{x(y-1)}$

16. 20

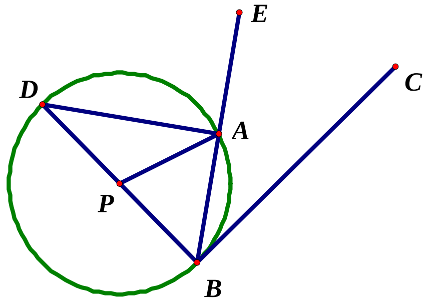
17. 1801

18. $\frac{11}{2}$ OR $5\frac{1}{2}$ OR 5.5

19. -1

20. $\frac{91}{3}$ (Must be this reduced improper fraction.)

1. Given circle P with tangent $\overline{BC}$, secant $\overline{BE}$, and diameter $\overline{DB}$. If the measure of $\angle ABC = 35°$, find the number of degrees in the measure of arc $\overset{\frown}{AD}$.



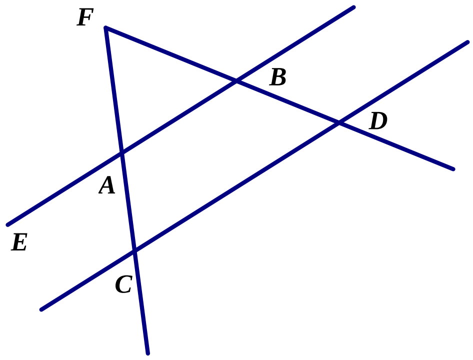
2. Given two triangles, find the largest **number** of corresponding parts (corresponding pairs of segments or angles) that may be congruent without the two triangles themselves being congruent.

3. Define a "mid-diagonal" to be a segment connecting a midpoint of one side of a polygon to a vertex of the polygon other than the two vertices that are endpoints of the side on which the midpoint is located. Find the number of mid-diagonals of an octagon.

4. In ΔABC the measure of $\angle A$ is $\frac{1}{3}$ the measure of $\angle C$ and $\frac{1}{2}$ the measure of $\angle B$. $AC = k\sqrt{w}$, with k and w being integers, and the sum of the lengths of sides $\overline{AB}$ and $\overline{BC}$ is an integer less than 30. If w is as small as possible, find the sum of all possible values of k.

5. The ratio $width : length : height$ of a rectangular solid is $8:15:144$. If the length of a diagonal of the rectangular solid is 108.75, find the width of the solid.

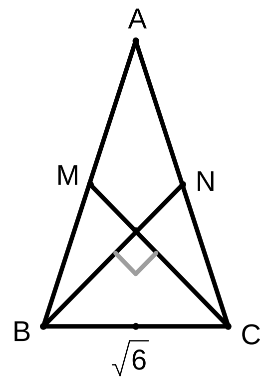
6. In the given diagram, $\overline{AB} \parallel \overline{CD}$, $\angle BDC = x°$, $\angle ACD = (x+10)°$, and $\angle EAF = 140°$. Find the degree measure of $\angle ABD$.



7. Two points are chosen on the circumference of a circle. Find the probability that the chord connecting these two points is longer than the radius of the circle. Express your answer as a common fraction reduced to lowest terms.

8. A regular hexagon is inscribed in a circle with circumference 24π. If a point is chosen at random from inside this circle, find the probability that it also lies inside the hexagon. Give your answer as a decimal, rounded to the nearest hundredth.

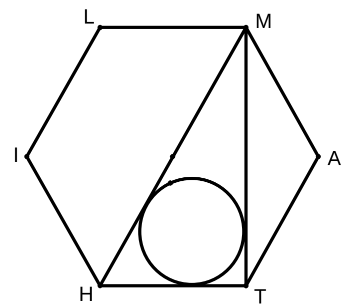
9. ΔABC is isosceles with $AB = AC$. $\overline{CM}$ and $\overline{BN}$ are medians such that $\overline{CM} \perp \overline{BN}$. The base of ΔABC has a length of $\sqrt{6}$. Find the perimeter of ΔABC.



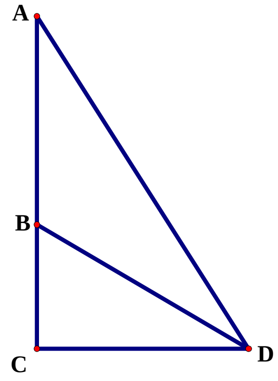
10. A circle is inscribed in ΔACE. Angle E is a right angle and points B, D and F are the circle's points of tangency on sides $\overline{AC}$, $\overline{CE}$ and $\overline{AE}$ respectively. If $CD = \sqrt{45}$ and $AB = \sqrt{20}$, find the area of the inscribed circle.

11. Find the perimeter of a rhombus with diagonals of lengths 9 and 16.

12. $ILMATH$ is a regular hexagon with perimeter of 54. Find the length of the radius of the inscribed circle in ΔMTH. Express your answer in reduced simplified radical form.

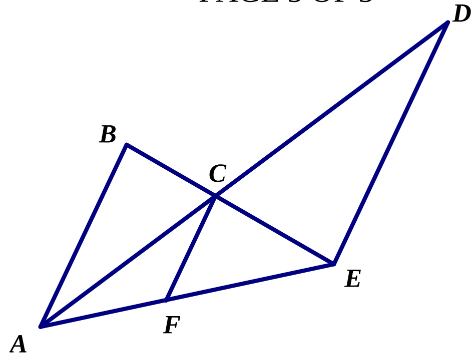


13. In ΔADC, $\overline{BD}$ bisects $\angle ADC$, and $AD = 2(CD)$. Find the ratio of the length of $\overline{BC}$ to $\overline{AC}$. Write your answer in the form $k : w$, where k and w are integers with no common factors.

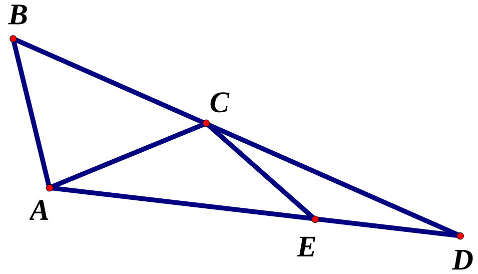


14. Circles of radii 4, 5 and 6 are mutually tangent externally. Find the area of the triangle formed by connecting the centers of these three circles.

15. If $\overline{AB} \parallel \overline{CF} \parallel \overline{DE}$ where $AB = 16$ and $DE = 24$, find the length of CF. Express your answer as an exact decimal.



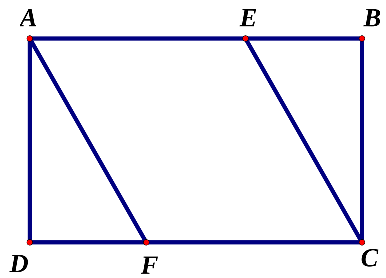
16. In the given diagram, $\overline{BA} \cong \overline{AC} \cong \overline{CE} \cong \overline{ED}$ and $\angle DAB = 110°$. Find the degree measure of $\angle CED$.



17. A triangle is selected at random from the set of triangles with integral degree measure angles and for which the ratio of the degree measures of the angles is $a:b:c$, where a, b and c are consecutive positive integers. Find the probability that the degree measure of the largest angle is an odd number. Express your answer as a common fraction reduced to lowest terms.

18. Two triangles are similar, but not congruent. The lengths of all sides of both triangles are integers. The first triangle has sides of 9 and 32 and a perimeter of 68. If one of the sides of the second triangle is the same length as one of the sides of the first triangle, find the perimeter of the second triangle.

19. $ABCD$ is a rectangle with $AB = 20$ and $BC = 12$. Points E and F are located such that $AECF$ is rhombus. Find the length EF. Give your answer as a decimal value rounded to 4 significant digits.



20. A rectangular sheet of paper 40 cm by 70 cm is folded once so that opposing corners are coincident. Find the number of cm in the length of the crease in the paper.

2014 SA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 110 (Degrees optional.)

2. 3 (Parts optional.)

3. 48 (Mid-diagonals optional.)

4. 45

5. 6

6. 150 (Degrees optional.)

7. $\frac{2}{3}$ (Must be this reduced common fraction.)

8. 0.83 OR .83 (Must be this decimal.)

(#9 must be this exact answer.)

9. $2\sqrt{15}+\sqrt{6}$ OR $\sqrt{6}+2\sqrt{15}$

10. 5π (Must be this exact answer.)

11. $2\sqrt{337}$ (Must be this exact answer.)

12. $\frac{9\sqrt{3}-9}{2}$ OR $\frac{1}{2}\left(-9+9\sqrt{3}\right)$ (Or reduced exact equivalent.)

13. $1:3$ (Must be this ratio and in this form.)

14. $30\sqrt{2}$ (Must be this exact answer.)

15. 9.6 (Must be this exact decimal.)

16. 145 (Degrees optional.)

17. $\frac{4}{11}$ (Must be this reduced common fraction.)

18. 204

19. 13.99 (Must be this decimal.)

20. $\frac{40\sqrt{65}}{7}$ (Must be this exact answer.)

1. On the planet Skyron, there are eleven cities. Each pair of cities is connected by exactly one road. Find the number of roads on Skyron.

2. If $m=\dfrac{pqr}{p-q}$, solve the equation for q. Assume no denominator is equal to zero. Express your answer as a single rational expression.

3. Find the ordered pair of positive prime integers (a,b) with $a \le b$, for which $a^b \bullet b^a = 6272$.

4. Find the ordered pair (a,b), $a > b > 0$ where a and b are integers, such that $(a+bi)(a-bi)=13$.

5. Find all values of x such that $x = f(x)$ in the function $f(x)=|3x-1|$.

6. The circle $x^2 + y^2 + Ax + By + C = 0$ has center at $(2,-1)$ and is tangent to the line $x = -3$. Find the value of C.

7. Find the number of 4-digit numbers between 1000 and 9999, inclusive, that can be formed from the digits in the set $\{0, 2, 3, 4, 6, 8, 9\}$ using the following rules:
   I. No digit may be repeated, and
   II. The digits must be in increasing order from left to right.

8. $|\mathrm{M}|$ represents the determinant of matrix M. Solve for x given $\begin{vmatrix} 2 & x & 1 \\ 1 & x & 2 \\ 3 & 4 & 0 \end{vmatrix} = 9$.

9. Find the sum of all values of x for which $9 \cdot 2^{2x+3} - 4^{2x} = 512$.

10. The point $(3,5)$ is on the graph of $y = f(x)$. Find the exact coordinates of point(s) that must be on the graph of $y = 2f\left(x^2\right)$.

11. Solve for x: $\left|(x+3)^2 - 26\right| = 10$.

12. A rectangular solid box has two faces each of area 12, two faces each of area 16, and two faces each of area 8. Find the exact volume of the box.

13. There exist two arithmetic sequences, A and B. Both have the same first and same last term (p and q respectively), where p and q are positive. The sum of A and the sum of B differ by $(p+q)$. Determine how many more terms the sequence with the larger sum has than the sequence with the smaller sum.

14. Given $\log_4 3 = a$ and $\log_4 5 = c$. Write the value of $\log_2 7.5$ in terms of a and c.

15. Three of the four vertices of a parallelogram are (not necessarily in order) the points $(4,2)$, $(10,4)$ and $(6,8)$. The fourth vertex is on one of the coordinate axes. Find the coordinates of this fourth vertex. Express your answer as an ordered pair (x,y).

16. Find the largest integer x for which another integer n exists, with $nx = n + 12x$.

17. Determine the sum of all natural numbers less than 115 that are not divisible by 7.

18. Given that f and g are real, non-constant functions such that for all x, y, $f(x+y) = f(x)g(y) + g(x)f(y)$ and $g(x+y) = g(x)g(y) - f(x)f(y)$, find all possible ordered pairs $(f(0), g(0))$. Express your answer(s) as **ordered pair(s).**

19. Let $d(n)$ represent the largest odd divisor of n, where n is a positive integer. Find the **number** of values of n, $n < 1000$, for which $d(n) = 3$.

20. Find all positive integers n for which $\frac{n^2+4}{n+5}$ will be a positive integer.

2014 SA
Algebra II

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 55

2. $\frac{mp}{m+pr}$ (Must be a single rational expression, this or exact equiv.)

3. $(2, 7)$ (Must be this ordered pair.)

4. $(3, 2)$ (Must be this ordered pair.)

5. $\frac{1}{4}, \frac{1}{2}$ OR $0.25, 0.5$ (Must have both, either order.)

6. -20

7. 15

8. 7

9. $\frac{9}{2}$ OR $4\frac{1}{2}$ OR 4.5

10. $(\sqrt{3}, 10), (-\sqrt{3}, 10)$ (Must have both ordered pairs in either order)

11. $-9, -7, 1, 3$ (Must have all 4 answers in any order.)

12. $16\sqrt{6}$ (Must be this exact answer.)

13. 2

14. $2(a+c)-1$ OR $2a+2c-1$ (This answer or exact equivalent.)

15. $(0, 6)$ (Must be this ordered pair.)

16. 13

17. 5603

18. $(0,0), (0,1)$ (Must have both ordered pairs in either order.)

19. 9 (Values optional.)

20. 24

1. If $\sin\theta = \dfrac{7}{25}$ and $\tan\theta > 0$, find the value of $\cos\theta$. Express your answer as a common fraction reduced to lowest terms.

2. Find the positive geometric mean between 12 and 75.

3. If $\dfrac{\sin(2x)}{\sin(x)} = \dfrac{k}{w}$ for $0 < x < \dfrac{\pi}{2}$, write $\cos x$ as a single simplified rational expression in terms of k and w.

4. Find the absolute value of the exact difference between the two roots for x in the equation $\left(1+\sqrt{3}\right)x^2 + \left(2-\sqrt{3}\right)x - 1 = 0$.

5. If $2^{x^2+2x} = 7$, then the exact value of $(x+1)^2$ is $\log_2 k$ where k is an integer. Find the value of k.

6. If $2x^3 - 3x^2 + kx + w$ is divisible by $(x-2)$ and $(x+1)$, find the value of the sum $(k+w)$.

7. The equation of a parabola with vertex at $(7,0)$ and focus at $(-4,0)$ can be written as $y^2 = k(x+w)$. Find the value of $(k+w)$.

8. $i=\sqrt{-1}$. If $x=4+i$, $y=-3+5i$, and $z=7+4i$, find the value of $\frac{xy-yz}{yx}$. Express your answer as a complex number in the form $a+bi$ where a and b are common or improper fractions reduced to lowest terms.

9. $4082400x^2y^5$ is a term in the expansion of $(kx+wy)^p$, where k, w, and p are integers. Find the sum $(k+w+p)$.

10. If $\frac{a}{c}-1=6\left(\frac{c}{a}\right)$, then $a=kc$. Find the value(s) of k.

11. A cubic equation with rational coefficients has $1-3i$ as one of its roots, 1 as its leading coefficient, and no quadratic term. Find the constant term.

12. A sequence is defined recursively as follows: $a_1=5$, $a_2=9$, $a_n=2a_{n-1}-a_{n-2}$. Find the value of a_{2014}.

13. $\sin^2 15°+\cos^2 75°=\frac{k+p\sqrt{w}}{q}$ when completely simplified and reduced and where k, w, p, and q are integers. Find the sum $(k+w+p+q)$.

14. Find the largest integer value of x satisfying $\binom{2013}{1000}+\binom{2013}{1001}=\binom{2014}{x}$. (Note: $\binom{n}{r}=nCr=C(n,r)$ is the combinatoric combination.)

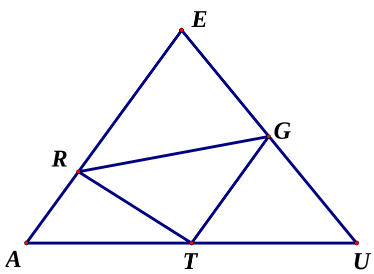
15. Find the sum of the numerical coefficients of all terms in the expansion of $(3x-y)^{12}$.

16. Two distinct numbers are chosen without replacement from the set $\{1,2,3,...,20\}$. Find the probability that the sum of the two numbers is odd. Express your answer as a common fraction reduced to lowest terms.

17. Given $\left(x+\frac{1}{x}\right)^2=8$ and $x>0$, find the exact value of $x^3+\frac{1}{x^3}$.

18. The line with equation $y=k$ is a horizontal asymptote of the graph of $y=\frac{3x-2x^2}{5x^2+7}$. Find the value of k.

19. ΔAEU is equilateral, ΔRTG has a right angle at T, $RA=1$, $AT=4$, and $UT=2$. Find the exact length of UG. Express your answer as a common or improper fraction reduced to lowest terms.



20. Find the value of the indicated sum: $\sum_{n=2}^{7}\left(5n+2^n\right)$

2014 SA PreCalculus

Name ANSWERS

School ______________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{24}{25}$ (Must be this reduced common fraction.)

2. 30

3. $\frac{k}{2w}$ (Must be this single rational expression.)

4. $\frac{\sqrt{33}-\sqrt{11}}{2}$ OR $\frac{1}{2}\sqrt{33}-0.5\sqrt{11}$ (Or exact equivalent.)

5. 14 (Must be this integer.)

6. -1

7. -51

8. $-\frac{15}{17}-\frac{9}{17}i$ OR $\frac{-15}{17}+\frac{-9}{17}i$

9. 18

10. $3, -2$ (Must have both values in either order.)

11. 20

12. 8057

13. 6

14. 1013

15. 4096

16. $\frac{10}{19}$ (Must be this reduced common fraction.)

17. $10\sqrt{2}$ (Must be this exact answer.)

18. $-\frac{2}{5}$ OR $\frac{-2}{5}$ OR -0.4 OR $-.4$

19. $\frac{14}{5}$ (Must be this reduced improper fraction.)

20. 387

NO CALCULATORS

1. The ratio $k:w$ is $3:4$. The ratio $m:p$ is $15:14$. The ratio $w:m$ is $8:5$. Find the ratio $k:p$. Express your answer in the form $k:p$ where k and p are integers with no common factors.

2. Five of the interior angles in a hexagon form a pattern such that each angle is $20°$ more than the previous angle. If the largest of the angles is $160°$, find the degree measure of the sixth angle.

3. Find the sum of the squares of the roots of the equation $2x^2+5x=1$. Express your answer as a common or improper fraction reduced to lowest terms.

4. The slope of a line is equal to $\frac{1}{3}$ of its $y-\text{intercept}$. If this line contains the point $(9,8)$, find the $x-\text{intercept}$ of this line. Express your answer as the exact x-intercept, not the coordinates of the point representing the x-intercept on a graph.

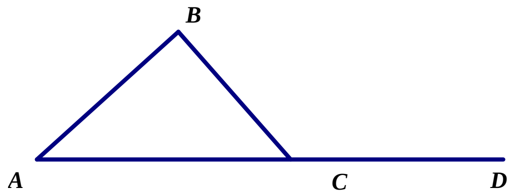
5. If x is an integer such that $-6<x<-1$ and if $\frac{(x^2)^2 x}{k+3}=x^2$, find the least possible value of k.

6. A line has an $x-\text{intercept}$ of 4 and makes a $60°$ angle of inclination with the $x-\text{axis}$. The point $(6,k)$ lies on the line in the first quadrant. Find the exact value of k.

7. The corners of a square, with side length of 5, are cut off, producing a regular octagon. Find the exact length of a side of the octagon.

8. A test is given to nine students. Their scores are integers between 0 and 100, inclusive. Find the greatest possible range of the test scores if the median of the scores is 80 and the mean is 85.

9. If the degree measure of $\angle BCD$ is three times the degree measure of $\angle A$, and if ΔABC is isosceles, find the smallest possible degree measure of $\angle B$. (Note: Drawing is not necessarily to scale.)



10. Find the units digit of the product $\left(2^{47}\right)\left(3^{85}\right)\left(7^{117}\right)\left(11^{268}\right)$.

11. Given ΔABC and ΔDEF such that $AC = 8$, $AB = 12$, $BC = 12$, $\angle D \cong \angle B$, $\angle F \cong \angle E$ and $\overline{EF} \cong \overline{BC}$. Find the exact area of ΔDEF.

12. Circle O has center at $(3,1)$. $\overleftrightarrow{AB}$ is tangent to circle O at point A $(0,5)$. The distance along $\overleftrightarrow{AB}$ between points A and B is 12 units. Find the shortest distance from point B to circle O.

13. Solve for k if $\dfrac{\frac{x+3}{2}+x}{x-\frac{k}{x}} = \dfrac{3x}{2x-2}$ for all values of $x > 1$.

14. Andy buys a piece of land. Barry buys the same land from Andy for at least one hundred percent more than what Andy paid for it. Carol buys the same land from Barry for at least three hundred percent more than what Barry paid for it. David buys the same land from Carol for at least fifty percent more than what Carol paid for it. If David paid 4.8 million dollars for the land, find the largest number of dollars that Andy could have paid for the land.

NO CALCULATORS

15. If $0 < 2^n < k < 20$ where n and k are integers, determine the **number** of ordered pairs (n,k) for which the expression $2^n + k$ represents a prime integer.

16. A circle is inscribed in quadrilateral $MATH$, where $MA = AT = 12$, $TH = HM = 9$, $\overline{MA} \perp \overline{HM}$ and $\overline{AT} \perp \overline{TH}$. Find the radius of this circle. Express your answer as a common or improper fraction reduced to lowest terms.

17. In the Mathletes Retirement Village, the following facts are true:
- No two inhabitants have exactly the same number of gray hairs.
- No inhabitant has exactly 85,124 gray hairs.
- There are more inhabitants than there are gray hairs on the head of any one inhabitant.

Find the largest possible number of inhabitants of the Mathlete Retirement Village.

18. A triangle has vertices at the points $(2,1)$, $(4,3)$, and $(7,2)$. Find the exact length of the shortest median of this triangle.

19. Points D, E and F are located on sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$, respectively, of equilateral ΔABC. If $AD = BE = CF = 4$ and the perimeter of ΔABC is $6\left(2+\sqrt{3}\right)$, find the exact area of ΔDEF.

20. $P(x)$ and $Q(x)$ are polynomial functions such that when $P(x)$ is divided by $Q(x)$, the quotient is $x+4$ and there is a remainder of $x-6$. When $P(x)$ is divided by $Q(x)+3$, the quotient is $x+2$ and there is no remainder. Find the value of $P(-1)$.

2014 SA

Fr/So 8 Person Team

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $9:7$ (Must be this ratio in this form.)

2. 120 (Degrees optional.)

3. $\frac{29}{4}$ (Must be this reduced improper fraction.)

4. -3 (Must be this value, not ordered pair.)

5. -128

6. $2\sqrt{3}$ (Must be this exact answer.)

7. $5\sqrt{2}-5$ OR $-5+5\sqrt{2}$

8. 55

9. 72 (Degrees optional.)

10. 8

11. $72\sqrt{2}$ (Must be this exact answer.)

12. 8

13. 1

14. $400,000$ (Dollars or \$ optional.)

15. 21 ("Ordered pairs" optional.)

16. $\frac{36}{7}$ (Must be this reduced improper fraction.)

17. 85124 (Inhabitants optional.)

18. $\frac{\sqrt{10}}{2}$ OR $\frac{1}{2}\sqrt{10}$ OR $0.5\sqrt{10}$

(EXACT ANSWERS REQUIRED IN 18 & 19.)

19. $7\sqrt{3}-6$ OR $-6+7\sqrt{3}$

20. 8

1. Given $a-2b=8-c$ and $b+4=5c-a$, find the value of $(b+2c)^2-8bc$.

2. A contest question required an equation of the form $x^2+bx+c=0$ to be solved. John miscopied only the value of c and got $4\pm i\sqrt{3}$ as the roots. Jamie miscopied only the value of b, resulting in roots of $\pm 2i$. Find the roots Jim should have found if he had copied and solved the problem correctly. Note: Both John and Jamie correctly solved the problem they had copied.

3. The product of Lauren's age 21 years ago and her age 21 years from now is the cube of a prime. Find Lauren's current age. (Assume all ages are in whole numbers of years.)

4. Evaluate $\log 4^5+\log 5^{10}$.

5. Find the exact **sum** of all x, $0\le x<2\pi$, such that $2\cos(2x)-1=0$.

6. $i=\sqrt{-1}$. If $(2+3i)x-2=(11+2y)i-4y$, find the sum $(x+y)$.

7. Let f be a function such that for all valid real x, $f(x)+2f\left(\dfrac{x+2012}{x-1}\right)=2016-x$. Find $f(2014)$.

8. If $16, a, b, c, d, 81$ is an arithmetic sequence and $16, e, f, g, 81$ is a geometric sequence, find the value of $(b+c-f)$.

9. A positive integer leaves a remainder of 7 when divided by 8, a remainder of 5 when divided by 6, a remainder of 3 when divided by 4, and a remainder of 1 when divided by 2. Find the least such positive integer.

10. David has a bag containing six fair coins and four double-headed coins. He takes a coin at random from the bag and tosses it in the air. Given the outcome of the toss was heads, find the probability that David picked a double-headed coin. Express your answer as a common fraction reduced to lowest terms.

11. Find all ordered pairs of positive integers (k, w) for which $k^2 - w^2 = 77$.

12. The polar graphs $r = 1$ and $r = 1 - 2\cos\theta$ intersect in k distinct points. Find k. Count only the points of intersection, not the number of times the graphs cross in a particular domain.)

13. Find the exact point(s) of intersection of $\frac{x^2}{18} + \frac{y^2}{12} = 1$ and $\frac{x^2}{7} - \frac{y^2}{21} = 1$ which is/are in the first quadrant. Express your answer(s) as ordered pair(s).

14. Determine the **sum** of all the natural numbers from 1 to 100 inclusive that are divisible by two but not divisible by three.

15. Find the probability that the product of three consecutive integers is a multiple of 12. Express your answer as a common fraction reduced to lowest terms.

16. Find the value of the base 6 expression $4_6\left(503_6+2014_6\right)$, writing your answer as a base 6 number.

17. The graph of $9x^2-4y^2-36x-8y=4$ has an asymptote with a positive slope. Give the $y-$intercept of this asymptote. Express your answer as the $y-$intercept only, NOT the ordered pair representing the graph of the point.

18. Box A contains 4 red marbles and 5 green marbles. Box B contains 2 red marbles and 6 green marbles. One marble is randomly chosen from Box A and put into Box B. Then one marble is randomly chosen from Box B. Find the probability that this marble is red. Express your answer as a common fraction reduced to lowest terms.

19. Given $\dfrac{2x^2-4x-6}{3x^3-21x-18}=\dfrac{a}{bx+c}$, where a and b are relatively prime integers. Find the value of c.

20. The lengths of the sides of a triangle are a, b and c, and A is the angle opposite side a. If $b^2+c^2=a^2+4$ and $bc=\dfrac{a^3}{\cos A}$ and $\angle A\neq 90°$, the exact sum b^2+c^2 can be expressed as $k+w^{(1/p)}$ where k, w, and p are positive integers. Express your answer as the ordered triple (k,w,p).

2014 SA

School ANSWERS

Jr/Sr 8 Person Team

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 16

2. $4 \pm 2\sqrt{3}$ (Must have both roots in combined form or separately.)

3. 28 ("Years old" optional.)

4. 10

5. 4π (Must be exact answer.)

6. 2

7. 1342

8. 61

9. 23

10. $\frac{4}{7}$ (Must be this reduced common fraction.)

11. $(39,38)$, $(9,2)$ (Must have both in either order.)

12. 3

13. $\left(3, \sqrt{6}\right)$ (Must be this ordered pair only with exact entries.)

14. 1734

15. $\frac{3}{4}$ (Must be this reduced common fraction.)

16. 15324 OR 15324_6

17. -4 (Must be this answer only, not ordered pair.)

18. $\frac{22}{81}$ (Must be this reduced common fraction.)

19. 6

20. $(4,4,3)$ (Must be this ordered triple.)

Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. The decimal representation of the rational number $\frac{1}{17}$ repeats in a sequential block of 16 digits on the right of the decimal point. Find the sixteenth digit in the sequence.

2. A model rocket is launched vertically from level ground with an initial velocity, v, of 50 meters per second. Acceleration, a, due to gravity is 9.8 meters per second squared. Find the amount of time, t, in seconds the model rocket will travel before landing on the ground after the launch. Express your answer as a decimal rounded to the nearest hundredth. (Note: The formula for the distance an object travels can be given as $d = \frac{1}{2}at^2 + vt$.)

3. Find the area of the triangle with sides of length 8, 9, and 10.

4. Find the units digit of 7^{2014}.

5. Find the last 3 digits of the sum of factorials $1! + 2! + 3! + \ldots + 2014!$. Express your answer as the three digit integer representing the last three digits.

6. Find the sum of all solution(s) for x when $2\sqrt{x-1} + \sqrt{x+4} = 7$.

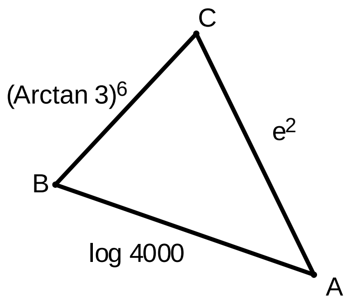
7. The Magic Math Candy Company recorded sales (in thousands of dollars) for the first eleven months of the year as shown in the table. Find the sum of the slope and y-intercept of the line of best fit for this data table.

Month	1	2	3	4	5	6	7	8	9	10	11
Sales	31	42	60	100	82	85	91	103	111	107	114

8. Let $x_n = \sqrt[(x_{n-1})]{(x_{n-1})^4 + 2}$ for $n > 0$ and $x_0 = 1$. Find x_{2014}.

9. Find the **number** of solutions for θ, $-2\pi \le \theta \le 2\pi$, that exist if $\sec^2\theta = 3\cos\theta$.

10. ΔABC has sides of length $(Arc\tan 3)^6$, e^2, and $\log 4000$ as shown. Find the **radian** measure of $\angle A$



11. The Illinois State Fair held a "Beans in a Barrel" contest. The clues were: (1) There is more than one bean in the barrel, and (2) The number of beans in the barrel leaves a remainder of 1 when divided by each positive integer between 2 and 20, inclusive. Find the least number of beans in the barrel. Report your answer as an exact integer.

12. Let x be a positive real number. $\log(x+1)$ is at least one one-thousandth greater than $\log x$. Find the largest possible value of x.

13. On the day his grandchild was born, Grandpa invested $10000 for 18 years at 2.5% interest compounded monthly. Find the dollar amount that will be available for the grandchild on that grandchild's 18th birthday. Express your answer as a decimal rounded to the nearest hundredth.

14. Find the value of x so that $x^5 = 2014^2$.

15. It is known that Wanda will have a hot fudge sundae while watching a baseball game 90% of the time. It is known that Paul will have a hot fudge sundae while watching a baseball game 80% of the time. It is known that Richard will have a hot fudge sundae while watching a baseball game 80% of the time. Find the probability that **at least two** of the three will have a hot fudge sundae if each of them watches a baseball game today. Express your answer as an **exact decimal**. **Do not use 4 significant digits. Do not use scientific notation.**

16. The adjacent sides of a parallelogram have length 7 and 11. One of the angles of this parallelogram measures exactly 1 radian. Find the area of this parallelogram.

17. $A = \{1, 3, 11, 8, x\}$ where A consists of five **distinct** positive integers. Let k be the total population standard deviation (σ_X) of A. Let w be the median of A. Find the **number** of distinct values of x such that $|k - w| < 1$. Express your answer as an integer.

18. Amy's Eats can seat exactly three times as many people as the maximum capacity of Bob's Banquets. Bob's Banquets can seat exactly four times as many people as the maximum capacity of Chuck's Catering. All together, the three sites can accommodate almost 1200 people at one time. Find the largest number of people that can be seated at Amy's Eats.

19. Find the sum of the first 50 terms of the series $-2 + \frac{8}{2} - \frac{26}{6} + \frac{80}{24} - \frac{242}{120} + \ldots$.

20. At the Deluxe Dining Restaurant, it has been determined that 7% of all people who make reservations at the Restaurant will not show. Rounded to the nearest whole person, find the maximum number of people who can make reservations at the Restaurant and still have the proprietor be at least 84% sure that no more than 1 person who made a reservation will fail to show.

2014 SA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.

1. 7

2. 10.20 (Must be this exact decimal, seconds optional.)

3. 34.20 OR 3.420×10 OR 3.420×10^{1} (Trailing 0 necessary.)

4. 9

5. 313 (Must be this integer.)

6. 5 OR 5.000 5.000×10^{0} (3 trailing zeros necessary in decimal answers.)

7. 45.86 OR 4.586×10 OR 4.586×10^{1}

8. 4.006 OR 4.006×10^{0}

9. 4 (Solutions optional.)

10. 0.05408 OR 5.408×10^{-2} (Radians optional.)

11. 232792561 (Must be this integer, may use commas.)

12. 433.8 OR 4.338×10^{2}

13. 15675.78 (Must be this decimal, $ optional.)

14. 20.97 OR 2.097×10 OR 2.097×10^{1}

15. 0.928 OR .928 (Must be this decimal.)

16. 64.79 OR 6.479×10 OR 6.479×10^{1}

17. 8 (Must be this integer.)

18. 840 (People or persons optional.)

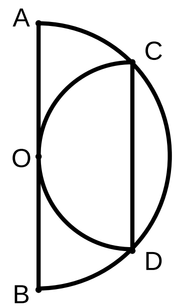
19. –0.3181 OR –.3181 OR -3.181×10^{-1}

20. 10 (People or persons optional.)

1. Let $k = \frac{x}{y}$ if $3^{x+4y} = \left(\frac{1}{9}\right)^{y-2x}$. Let $w = ab$ if $(a+b)^2 = 25$ and $a^2 + b^2 = 13$. Find the reduced, simplified sum $(k+w)$.

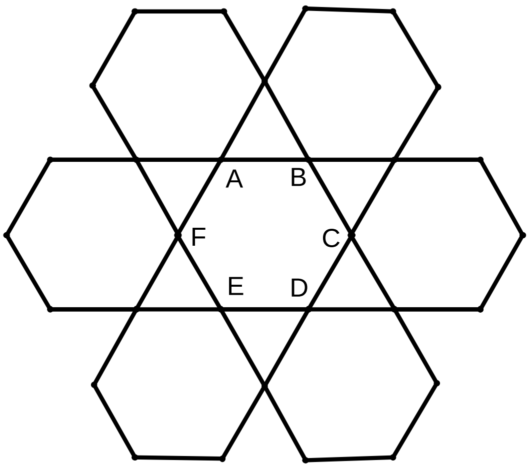
2. A rhombus has sides and diagonals with positive integral lengths. The numerical area of this rhombus is 240. Find the numerical perimeter of this rhombus.

3. Arc $\overset{\frown}{ACB}$ is a semicircle with diameter $\overline{AB}$ and center O. Chord $\overline{CD}$ is parallel to $\overline{AB}$ and is the diameter of the semicircle formed by $\overset{\frown}{COD}$. $\overset{\frown}{COD}$ is tangent to $\overline{AB}$ at O. $AO = 8$. Determine the exact area of the semicircle formed by $\overset{\frown}{COD}$.

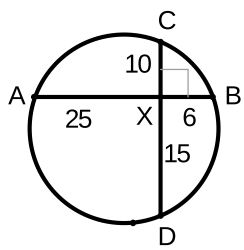


4. The universal set is $U = \{0,1,2,3,4,5,6,7,8,9\}$. Let set $A = \{\text{possible unit digits in the square of any integer}\}$. Let set $B = \{\text{possible unit digits in the cube of any integer}\}$. Find the set $\overline{A \cap B}$ (the complement of A and B in U). Express your answer using set notation.

5. Seven ***regular*** hexagons are shown in the diagram, including hexagon $ABCDEF$ that has numerical area $24\sqrt{3}$. Find the exact area enclosed in the diagram.



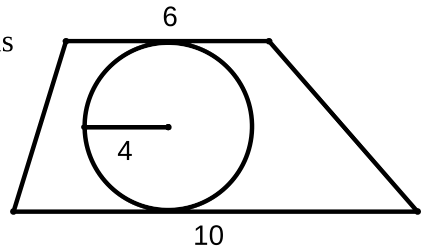
6. In the circle shown (not drawn to scale), chords $\overline{AB} \perp \overline{CD}$ and intersect at X. $AX = 25$, $XB = 6$, $CX = 10$, and $XD = 15$. Find the exact radius of the circle.



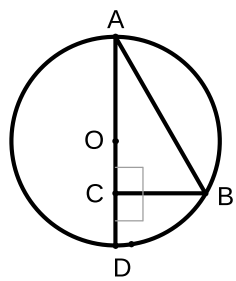
7. A palindromic integer is an integer which has the same value when the digits are written in the reverse order. For example, 12821 is a palindromic integer. Find the smallest ***non-palindromic number with 2 or more digits*** whose square ***IS*** palindromic.

8. The values $2x+4$, $3x-2$, $3x$, $4x-6$, $3x+4$, $4x-2$, $5x-8$, and $4x+2$ have a mean of 27. If 4 is added to each of the 3 smallest values, and 4 is subtracted from each of the remaining 5 values, find the new resulting mean.

9. In the given figure, not drawn to scale, the circle with radius 4 is tangent to both bases of the trapezoid. The bases have lengths 6 and 10. Find the area inside the trapezoid but outside the circle. Write your answer rounded to the nearest hundredth of a unit.



10. In circle O, $\overline{AD}$ is a diameter and $\overline{CB} \perp \overline{AD}$. $BC = 6$ and $CD = 2$. Find the exact numerical length of $\overline{AB}$.



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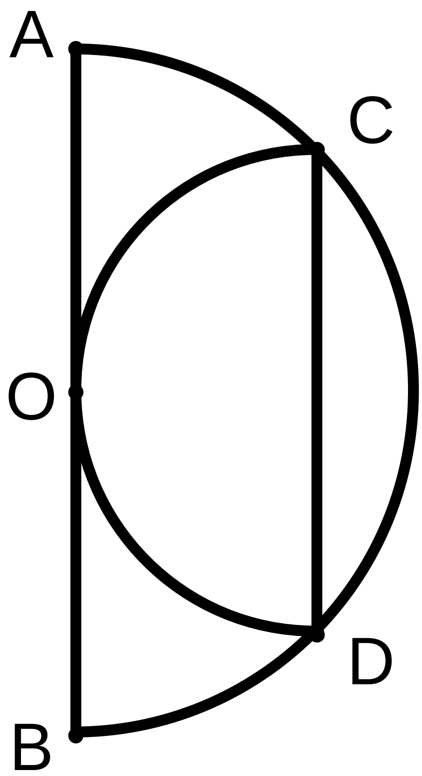

3. Arc $\overset{\frown}{ACB}$ is a semicircle with diameter $\overline{AB}$ and center O. Chord $\overline{CD}$ is parallel to $\overline{AB}$ and is the diameter of the semicircle formed by $\overset{\frown}{COD}$. $\overset{\frown}{COD}$ is tangent to $\overline{AB}$ at O. $AO = 8$. Determine the exact area of the semicircle formed by $\overset{\frown}{COD}$.

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Let set

$$A = \left\{ \begin{array}{l} \text{possible unit digits in} \\ \text{the square of any integer} \end{array} \right\}.$$

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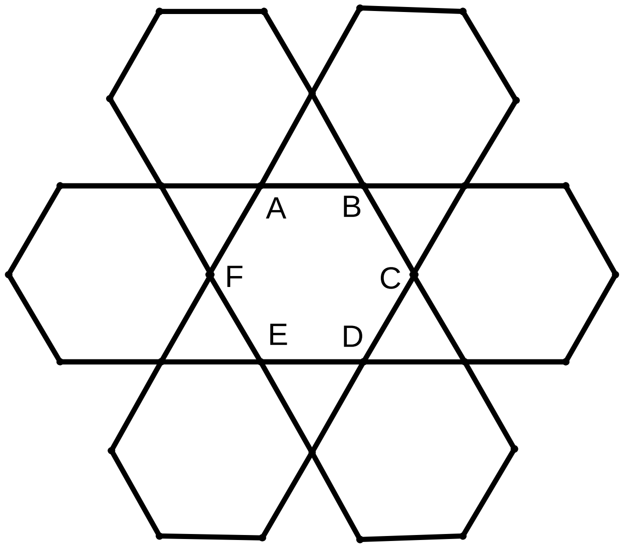
Find the set $\overline{A \cap B}$ (the complement of A and B in U). Express your answer using set notation.



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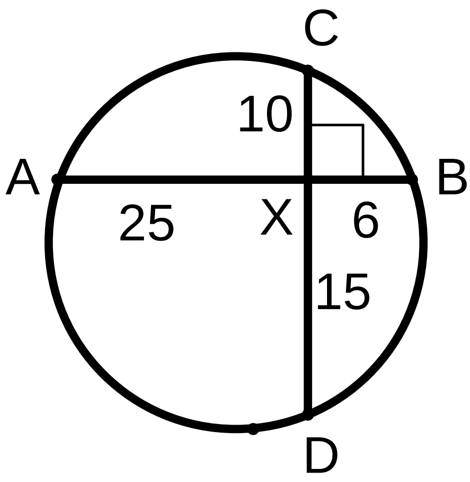

6. In the circle shown (not drawn to scale), chords $\overline{AB} \perp \overline{CD}$ and intersect at X. $AX = 25$, $XB = 6$, $CX = 10$, and $XD = 15$. Find the exact radius of the circle.

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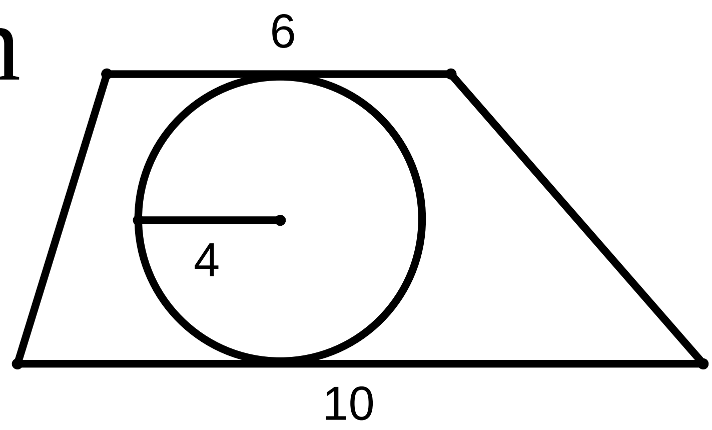

9. In the given figure, not drawn to scale, the circle with radius 4 is tangent to both bases of the trapezoid. The bases have lengths 6 and 10. Find the area inside the trapezoid but outside the circle. Write your answer rounded to the nearest hundredth of a unit.

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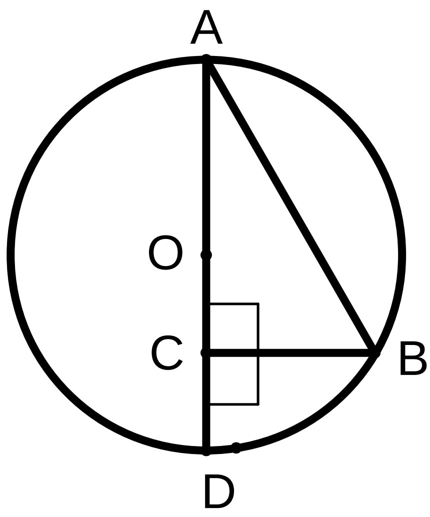

2014 SA

School ___ANSWERS___

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer		**Score** (to be filled in by proctor)
1.	8		______
2.	68		______
3.	16π	(Must be this exact answer.)	______
4.	$\{2,3,7,8\}$	(Must be this set notation, elements in any order.)	______
5.	$192\sqrt{3}$	(Must be this exact answer.)	______
6.	$\frac{\sqrt{986}}{2}$	(Must be this exact answer.)	______
7.	26		______
8.	26		______
9.	13.73	(Must be this decimal.)	______
10.	$6\sqrt{10}$	(Must be this exact answer.)	______

TOTAL SCORE: ______
(*enter in box above)

Extra Questions:

11. ______

12. ______

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let 2 and 10 be the first and third terms respectively of a sequence. k represents the 7th term if the sequence is arithmetic. w represents the 7th term if the sequence is geometric. Find the sum $(k+w)$.

2. The function $f(x)$ has a maximum value at coordinates $(2,12)$. Find the coordinates of the minimum of the function $-2f(x+3)$. Express your answer as an ordered pair (x,y).

3. Let $A=\{3,4,5,\cdots,n,\cdots\}$. Let w be the smallest number in A such that $w=ab=cd=ef$ where a, b, c, d, e, and f are 6 distinct numbers in A. Let p be the perimeter of an equilateral triangle if the length of its inradius is $3\sqrt{3}$. Find the value of $(w+p)$.

4. ΔABC is a right triangle with hypotenuse of length 17.5, one leg of length 10.5, and the other leg of length k. $\begin{vmatrix} 3 & w & 5 \\ 2 & 0 & 3 \\ 4 & 1 & 2 \end{vmatrix} = 65$. Find the sum $(k+w)$.

5. If $p \le q \le r$ are the lengths of the three sides of a triangle whose area is 1, let k be the exact minimum value for q. The line $y=3x+b$ is tangent to the circle $x^2+y^2=10$. Let w be the larger of the two possible values for b. Find the exact product (kw).

6. Let $k=\left|7-3\sqrt{6}\right|-\left|2\sqrt{6}-4\right|$. Let $w=\left[\sqrt{18}-2\pi\right]$. Find the exact sum $(k+w)$. (Note: $[x]$ represents the greatest integer function of x.)

7. The conic with equation $4x^2+9y^2-16x+54y-11=0$ can be expressed in the form $\frac{(x-h)^2}{a}+\frac{(y-k)^2}{b}=1$. Find the value of $C(a,b)$. (Note: The notation $C(a,b)={}_aC_b=\binom{a}{b}$ is the combinatoric combination.)

8. Find the value of $\frac{-4x^2-9xy-2y^2}{x+2y}\times\frac{-1}{4x^2-3xy-y^2}$ when $x=2014$ and $y=1999$. Express your answer as a common or improper fraction reduced to lowest terms.

9. Let k be the number of digits in the product $4^{125}\left(5^{254}\right)$. Cook County has an 11.75% sales tax. Across Lake Cook Road in Lake County, the sales tax is only 6.25%. A new golf set costs k dollars in both counties. How much more tax does a person buying in Cook County pay than a person in Lake County for the same set of clubs. Report your answer in dollars rounded to the nearest cent.

10. A triangle with sides of lengths 12, 35, and 37 is inscribed in a circle. Let k be the numerical area inside the circle but outside the triangle. Let w be the numerical length of the altitude to the longest side of the triangle. Find the numerical sum $(k+w)$. Express your answer as a decimal rounded to four significant figures.

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Find the sum $(k + w)$.

5. If $p \le q \le r$ are the lengths of the three sides of a triangle whose area is 1, let k be the exact minimum value for q. The line $y = 3x + b$ is tangent to the circle $x^2 + y^2 = 10$. Let w be the larger of the two possible values for b. Find the exact product (kw).

6. Let $k = \left|7 - 3\sqrt{6}\right| - \left|2\sqrt{6} - 4\right|$. Let $w = \left[\sqrt{18} - 2\pi\right]$. Find the exact sum $(k + w)$. (Note: $[x]$ represents the greatest integer function of x.)

7. The conic with equation $4x^2+9y^2-16x+54y-11=0$ can be expressed in the form

$$\frac{(x-h)^2}{a}+\frac{(y-k)^2}{b}=1.$$

Find the value of $C(a,b)$.

(Note: The notation $C(a,b)={}_aC_b=\binom{a}{b}$ is the combinatoric combination.)

8. Find the value of

$$\frac{-4x^2 - 9xy - 2y^2}{x + 2y} \times \frac{-1}{4x^2 - 3xy - y^2}$$

when $x = 2014$ and $y = 1999$. Express your answer as a common or improper fraction reduced to lowest terms.

9. Let k be the number of digits in the product $4^{125}\left(5^{254}\right)$. Cook County has an 11.75% sales tax. Across Lake Cook Road in Lake County, the sales tax is only 6.25%. A new golf set costs k dollars in both counties. How much more tax does a person buying in Cook County pay than a person in Lake County for the same set of clubs. Report your answer in dollars rounded to the nearest cent.

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2014 SA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		Score (to be filled in by proctor)
1. 276		
2. $(-1,-24)$	(Must be this ordered pair.)	
3. 102		
4. 22		
5. $10\sqrt{2}$	(Must be this exact answer.)	
6. $\sqrt{6}-6$ OR $-6+\sqrt{6}$	(Must be this exact answer.)	
7. 17383860		
8. $\frac{1}{15}$	(Must be this reduced common fraction.)	
9. 13.92	(Must be this decimal, $ optional.)	
10. 876.6	(Must be this decimal.)	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. ______

12. ______

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first
In round with correct answer

1. If $\frac{7}{3}x = 35$, find the value of x.

2. Find the value of y such that $\frac{y-4}{2}-\frac{2y-7}{6}=ANS$.

3. A kite has one diagonal of length ANS and the other diagonal of length $2\frac{18}{95}$. Find the area of this kite.

4. A chord of length 96 is drawn in a circle with diameter ANS. Find the shortest distance from the chord to the center of the circle.

1. a, b, and c are digits in the three-digit numbers in the subtraction problem

$$\begin{array}{cccc} & 7 & a & 2 \\ - & 4 & 8 & b \\ \hline & c & 7 & 3 \end{array}$$

. Determine the value of $(a+b+c)$.

2. A total of 120 people, teens and adults, attended a jazz concert and paid a total of $1836. The adults paid $\$ANS$ each and the teens paid $5 each. Find the number of adults that attended the concert.

3. Seven interior angles in an octagon are $115°$, $117°$, $140°$, $145°$, $155°$, $170°$ and $ANS°$. Find the measure, in degrees, of the remaining interior angle.

4. A circular can is wedged into a rectangular corner. The can has a radius of *ANS* cm. Find the exact distance, in cm, from the corner point to the can.

1. Find the value of $\left(x^x\right)^{\left(x^x\right)}$ when $x = 2$.

2. A board of ANS meters in length is divided into three parts proportional to 2, 4, and 6. Find the length, in meters, of the longest part.

3. ΔABC is a right triangle. Hypotenuse $\overline{BC}$ is ANS units in length and leg $\overline{AB}$ is 64 units in length. Find the exact length of leg $\overline{AC}$.

4. A rectangle has one side of length ANS and a diagonal of length 128. Find the exact area of this rectangle in square units.

1. Find the 2014^{th} digit to the right of the decimal point in the decimal representation of $\frac{1}{7}$.

2. One marble is drawn at random from a bag containing ANS orange, 18 blue, and 13 green marbles. Find the probability the marble chosen was orange. Express your answer as a common fraction reduced to lowest terms.

3. ANS will be in the form of a common fraction. k is the numerator of that fraction. The diagonals of an isosceles trapezoid each measure 17 units, the altitude measures k units and the upper base measures 9 units. Find the perimeter of the trapezoid.

4. Two of the angles of a convex quadrilateral have respective degree measures of $143°$ and $ANS°$. One of the two remaining angles of the quadrilateral has a measure that is 5 more than twice the degree measure of the other remaining angle of the quadrilateral. Find the degree measure of the larger of these two remaining angles.

1. Claire, who is four years older than her sister Amy, is now three times as old as Amy was two years ago. Find the number of years old Amy is now.

2. For real numbers x and y, define $x \otimes y = (x+y)(x-y)$. Find the value of $3 \otimes (4 \otimes ANS)$.

3. A regular hexagon with a perimeter of $\left(-\frac{1}{3}ANS\right)$ is inscribed in a circle. Find the exact distance from the center of the circle to one side of the hexagon.

4. Let $k = (ANS)^2$. Two telephone poles are perpendicular to level ground, are 100 feet apart, as measured on the ground, and have heights of k feet and 48 feet. There is a line from the top of each pole to the bottom of the other pole. Find the height above the ground, in feet, for the point where the two lines intersect. Express your answer as an exact decimal.

2014 SA FR/SO RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. 15
2. 95
3. 104
4. 20

ROUND 2

1. 17
2. 103 (Adults optional.)
3. 135 (Degrees optional.)
4. $135\sqrt{2} - 135$ OR $135\left(\sqrt{2} - 1\right)$ (Must be this exact answer or exact equivalent, cm optional.)

ROUND 3

1. 256
2. 128 (Meters optional.)
3. $64\sqrt{3}$ (Must be this exact answer.)
4. $4096\sqrt{3}$ (Must be this exact answer, square units optional.)

EXTRA ROUND 4

1. 8
2. $\frac{8}{39}$ (Must be this reduced common fraction.)
3. 50
4. 113 (Degrees optional.)

EXTRA ROUND 5

1. 5 (Years optional.)
2. -72
3. $2\sqrt{3}$ (Must be this exact answer.)
4. 9.6 (Must be this decimal, feet optional.)

1. In the quadratic equation $x^2 - 193x + k = 0$ if one of the solutions for x is 1, find the value of the other solution for x.

2. The area of an equilateral triangle is $\sqrt{4\,(ANS)}$. Find the length of one side.

3. In right ΔABC, $\angle C$ is the right angle. If $AC = ANS$ and $BC = 2.036(ANS) - 1.288$, find $\sin \angle A$. Report your answer as a reduced common fraction.

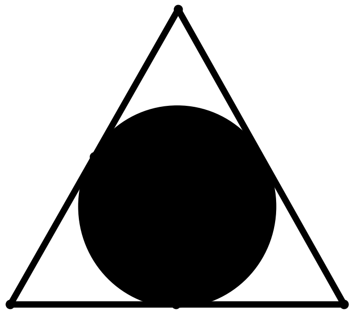
4. $ANS = \frac{k}{w}$. In a survey of 50 high school seniors, k of the seniors surveyed said they were taking AP Statistics and w of the seniors surveyed said they were taking Calculus. Six of the seniors surveyed said they were taking both Calculus and AP Statistics. How many of the 50 seniors surveyed were not taking either Calculus or AP Statistics?

1. Find the sum of all the integral values of x in the solution set for $(3x+1)(x-2) \le 0$.

2. Find the radius of the circle whose equation is $x^2 + 20x - 1 = 22y - y^2 + ANS$.

3. Sam has a generous grandfather who gives gifts in the pattern of an arithmetic sequence. One day, Sam's grandfather gave Sam one dollar. The next day his grandfather gave Sam three dollars, and the next day he gave Sam five dollars. At the end of the $(ANS)^{\text{th}}$ day how many dollars in all had Sam's grandfather given Sam?

4. The area of an equilateral triangle is ANS. The inscribed circle of the equilateral triangle is cut out of the triangle. Find the area of the part of the equilateral triangle that remains (the ***non-shaded*** area). Report your answer as a decimal rounded to the nearest hundredth.



1. Given the following information: b is 6 less than a, c is 20 less than b and 7 more than d, e is 12 more than d and 7 more than f. By how much does e exceed c?

2. If x and y are integers, solve for y: $(3^y)(4^{(x+ANS)}) = (2^{(3y)})(9^{(x+1)})$.

3. Let $k = ANS$. Find the focus of the parabola whose equation is $y^2 + 2ky = 8x$. Report your answer as an ordered pair.

4. ANS is an ordered pair (a,b). Let $k = |a-b|$. A circular target is formed by 3 concentric circles with radii of k, $k+3$, and $k+5$. If a dart hits the target, what is the probability the dart will fall in the middle ring? Report your answer as a reduced common fraction.

1. Find the minimum value of $f(x)$ for the function $f(x) = x^2 + 4x - 6$.

2. Let $k = |ANS|$. Find the number of distinct 5-person committees that can be formed from a group of 6 men and k women if 2 men and 3 women must be selected.

3. Let k be the sum of the digits in ANS. In a raffle there are 3 tickets, each worth \$50, 6 tickets each worth \$25 and k tickets worth \$10. If all the tickets are sold and each ticket sold wins one of these prizes, find the probability a ticket drawn at random is one of the 6 tickets that are each worth \$25. Report your answer as a reduced common fraction.

4. Let $k = ANS$. Find, in terms of π, the period of the function
$f(x) = -3\tan\left(2\left(kx - \frac{\pi}{4}\right)\right)$.

1. Find the product of the roots of $(x+5)(x-3)=(2x+5)(3x-4)$.

2. Find x where $\log_3\left(2x+(ANS)\right)=3$.

3. Let $k = ANS$. Solve this system of equations for x: $\begin{cases} 2^{2x} - 4\left(2^{y}\right) = 25 \\ 2\left(2^{2x}\right) + 2^{y} = k \end{cases}$. Report your answer as a decimal rounded to the nearest hundredth.

4. Let $k = 100(ANS)$. The area of an isosceles trapezoid with base angles of $60°$ and a shorter base of 6 inches is $k\sqrt{3}$ square inches. Find the length, in inches, of the longer base. Report your answer as a decimal rounded to the nearest hundredth.

2014 SA JR/SR RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. 192
2. 8 (Units optional.)
3. $\frac{15}{17}$ (Must be this reduced common fraction.)
4. 24 (Students optional.)

ROUND 2

1. 3
2. 15
3. 225 ($ or dollars optional.)
4. 88.97 (Must be this decimal.)

ROUND 3

1. 5
2. 4
3. $(0,-4)$ (Must be this ordered pair.)
4. $\frac{11}{27}$ (Must be this reduced common fraction.)

EXTRA ROUND 4

1. -10
2. 1800 (Committees optional.)
3. $\frac{1}{3}$ (Must be this common fraction.)
4. $\frac{3\pi}{2}$ OR 1.5π (Must be exact answer.)

EXTRA ROUND 5

1. -1
2. 14
3. 1.58 (Must be this decimal.)
4. 25.85 (Must be this decimal, inches optional.)