1. Let x and y be integers such that $13 < x < 15$ and $-5 < y < -3$. Determine the value of $(2x + y)$.

2. Let (k, w) be the solution to the system $\begin{cases} x + 2y = 8 \\ 4x - y = 5 \end{cases}$. Determine the sum $(k + w)$.

3. Let $x^2 - 4y^2 = 30$ and $x - 2y = 5$. Determine the value of $(x + 2y)$.

4. The sum of twice a number and three times a second number is 16. The difference between the two numbers is 3. If the first number is greater than the second number, determine the sum of the two numbers.

5. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. If x is in set A and $x + 7$ represents an even integer, determine *the sum* of all distinct possibilities for the value of x.

6. **(Multiple Choice)** For your answer write the **capital letter(s)** which corresponds to the correct choice(s).

   Determine which of the following five statement(s) is sufficient to deduce that $x > y$.

   A) $x + 1 = y$ B) $x + 2.2 = y$ C) $x - 1.3 = y$ D) $xy > 0$ E) $xy < 0$

   Note: Be certain to write the correct capital letter(s) as your answer.

7. Determine the value(s) for k such that $kx^2 - 42xy + 49y^2$ factors to the square of a binomial.

8. Determine the value of $17x+17y$ when $13+x=47-y$.

9. Let b and c be integers with $g(x)=x^2+bx+c$ and $f(x)=x^2+cx+b$. Determine the sum $(b+c)$ when $g(c)=f(b)$ and $c\neq b$.

10. The ratio of $4a-3b$ to $2a+5b$ is $3:2$. Determine the ratio of a to b. Express your answer in the form $k:w$ where k is an integer, w is a positive integer and k and w have no common factors other than 1.

11. Determine the numerical area of the triangular region enclosed by the system $\begin{cases} y=4x-2 \\ y=-4x+14 \\ y=\frac{4}{3}x-2 \end{cases}$.

12. Fran's family has moved around a lot. Her street addresses have had the following numbers: 107, 213, 219, 512, 2118. Let S be the set of distinct prime divisors of at least one of these house numbers. In the equation $x^2-bx+c=0$, b is a number in S and $|c|$ is the product of two numbers in S. Determine the value of c.

13. Let a and b be such that $a^2b-3a-2ab^2+6b=32$. If a is 4 more than twice b, determine the product (ab).

14. Aune just completed a 4 day trip. On the first day she drove 531 miles; on the second day, 615 miles; on the third day, 704 miles; on the fourth day, k miles. If Aune drove an average of 622.5 miles per day for the 4 day trip, determine the value of k.

15. Let $k = \sqrt{110 + \sqrt{110 + \sqrt{110 + \sqrt{110 \cdots}}}}$. Determine the exact value of k.

16. Will has 52 coins in his Lego bank, all nickels or pennies. The total value of these coins is \$1.20. Determine how many of each type of coin Will has in his bank. Express your answer as an ordered pair $(\#\,nickels, \#\,pennies)$. Your answer should be in the form (x, y) without labels in the ordered pair.

17. Point P has coordinates $(4, -3)$ and is located a constant distance of 4 from all points on the graph of $y = Ax^2 + By^2 + Cx + D$. Determine the sum $(A + B + C + D)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

18. Determine *the sum* of all integers that satisfy the system $\begin{cases} 4x + 6 < x + 33 \\ 4(x-2) + 2x \geq 2(x+8) \end{cases}$.

19. For all x in the expression's natural domain, the expression $\left[\dfrac{2}{x+3} + \dfrac{x-1}{x+2} - \dfrac{4x+5}{x^2+5x+6}\right]^{(-1)}$ can be written as a single simplified rational expression. Determine the ***numerator*** of this simplified rational expression.

20. A square with area S and an isosceles right triangle with area T have the same perimeter. Determine the larger of the two ratios $\dfrac{S}{T}$ and $\dfrac{T}{S}$. Express your exact answer as a single reduced fraction.

2017 SA
Algebra I

Name ANSWERS

School ______________________
(Use full school name – no abbreviations)

_______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 24

2. 5

3. 6

4. 7

5. 36

6. C (Must be this capital letter only.)

7. 9 (Must be this value only.)

8. 578

9. -1

10. $21:2$ (Must be this ratio and in this form.)

11. 8

12. -5183

13. 11

14. 640 ('miles" or "mi." optional.)

15. 11

16. $(17,35)$ (Must be this ordered pair.)

17. $-\frac{1}{2}$ OR $\frac{-1}{2}$ (Must be this reduced common fraction.)

18. 21

19. $x+3$ OR $3+x$ (Must be this algebraic expression, use of parenthesis optional.)

20. $\frac{3+2\sqrt{2}}{4}$ OR $\frac{2\sqrt{2}+3}{4}$ (Must be this single reduced fraction

1. Determine the number of diagonals that can be drawn in a convex polygon that has 100 sides.

2. A rectangle is half as wide as it is long and has a perimeter of x units. The numeric area of this rectangle can be written as the expression kx^2. Determine the value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

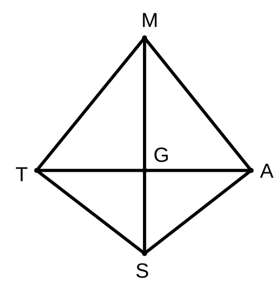
3. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never,** whichever is correct.

   The circumcenter and the centroid of an isosceles triangle are the same point.

4. Rectangle $RECT$ has diagonals that meet at point Z. $RZ = 650$, and the numeric area of rectangle $RECT$ is 739200. Determine the perimeter of rectangle $RECT$.

5. A square with numeric area k is inscribed in a semicircle. (That is, two vertices on the semicircle and the side opposite the two vertices lies on the diameter.) A second square with numeric area w is inscribed in a full congruent circle (same radius as the semicircle.). Determine the reduced and simplified ratio $k:w$.

6. $MAST$ is a kite with $TM = TS$. $TG = 2y-2$, $GA = y+3$, $MG = 2y$, and $GS = y+6$. Determine the length of $\overline{TS}$.



7. A circle is inscribed within the region that is the solution of $\begin{cases} x \geq 2 \\ x \leq 6 \\ y \geq 1 \\ y \leq 5 \end{cases}$. This circle can be represented algebraically by $(x-h)^2 + (y-k)^2 = r^2$. Determine the ordered triple (h, k, r^2).

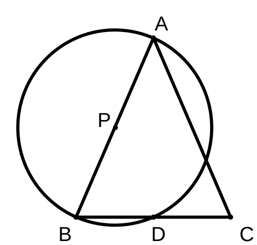
8. When the surface area of a sphere is doubled, the volume is increased so that the new volume is k times the original volume. Determine the exact value of k.

9. The perimeter of ΔABC is 62. The length of $\overline{AB}$ is $4x-2$, the length of $\overline{BC}$ is $5x-5$ and the length of $\overline{AC}$ is $3x+9$. Using one letter to name the vertex of the angle, determine the smallest angle in ΔABC. Write as your answer the **capital letter** of this angle.

10. Equilateral ΔXYZ is inscribed in a circle with diameter 6. Tangent segments through points X, Y, and Z meet to form another triangle circumscribing the circle. Determine the sum of the perimeters of this second triangle and ΔXYZ.

11. Line ℓ, with equation $3x-4y-4=0$, intersects $\odot O$, with equation $(x-1)^2+(y+k)^2=25$, in exactly one point P. Determine *the sum* of all possible value(s) of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

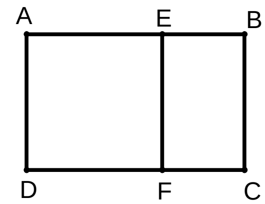
12. Circle P has diameter $\overline{AB}$. ΔABC is isosceles with base $\overline{BC}$ intersecting the circle at point D. $AC=4$ and $DC=1$. Determine the numeric area of ΔABC.



13. In ΔABC, $AB=6$, $BC=9$, D lies on $\overline{AC}$, $\angle ABD \cong \angle DBC$, and E is the midpoint of $\overline{BD}$. The numeric area of ΔABC is 25. Determine the numeric area of ΔABE.

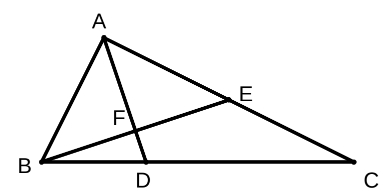
14. A segment of a circle with radius 12 is determined by a $120°$ arc. Determine the area of this segment of the circle. Express your answer as a decimal rounded to the nearest thousandth.

15. In rectangle $ABCD$, $\overline{EF} \perp \overline{AB}$, and $AE = AD$. Also $\frac{AB}{AD} = \frac{EF}{EB} = \frac{k + w\sqrt{p}}{q}$ in completely simplified and reduced radical form, k, w, p, and q integers with $q > 0$. Determine the sum $(k + w + p + q)$.



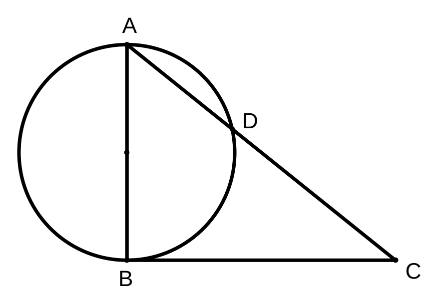
16. In a circle with center C, minor arc $\overset{\frown}{AB}$ has length $\frac{8\pi}{9}$. $\angle ACB = 40°$. Determine the radius of circle C.

17. ΔABC is a right triangle with right $\angle BAC$. E is the midpoint of $\overline{AC}$, D lies on $\overline{BC}$ such that $\overline{AD}$ is the perpendicular bisector of $\overline{BE}$ at point F. $AD = 4$ and $BE = 6$. Determine the exact perimeter of ΔABC.



18. Determine the length of the altitude of a regular tetrahedron with edges of length 6.

19. In the diagram, points A, B, and D lie on the circle with diameter $\overline{AB}$ and with D also on $\overline{AC}$. $\overline{BC}$ is tangent to the circle at point B, $BC = 224$, and $DC = \frac{12544}{65}$. Determine the sum of the areas of the regions that are inside the circle but outside ΔABC. Express your answer rounded to the nearest integer.



20. The apothem of a regular hexagonal based prism is $2\sqrt{3}$. The height of this prism is 10 times the length of one side of the base. Determine the numeric volume of this prism.

2017 SA
Geometry

Name ______________ **ANSWERS** ______

School ______________________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 4850 ("diagonals" optional.)

2. $\frac{1}{18}$ (Must be this reduced common fraction.)

3. Sometimes (Must be this whole word.)

4. 3560

5. $2:5$ (Must be this exact ratio.)

6. $2\sqrt{61}$ (Must be this exact answer.)

7. $(4,3,4)$ (Must be this ordered triple.)

8. $2\sqrt{2}$ (Must be this exact answer.)

9. C (Must be this capital letter.)

10. $27\sqrt{3}$ (Must be this exact answer.)

11. $\frac{1}{2}$ (Must be this reduced common fraction.)

12. $\sqrt{15}$ (Must be this exact answer.)

13. 5

14. 88.443 (Must be this decimal.)

15. 9

16. 4

17. $9\sqrt{2}+3\sqrt{10}$ OR $3\left(3\sqrt{2}+\sqrt{10}\right)$ OR $3\sqrt{10}+9\sqrt{2}$ OR $3\left(\sqrt{10}+3\sqrt{2}\right)$ (Must be this exact answer.)

18. $2\sqrt{6}$ (Must be this exact answer.)

19. 7257

20. $960\sqrt{3}$

1. Determine the ordered pair (x, y) that is the solution to $\begin{cases} 3x + y = 5 \\ 2x - y = 0 \end{cases}$. Express your answer as an ordered pair (x, y).

2. Determine the sum of all distinct value(s) for x such that $\sqrt{2x-12} = x-10$.

3. Determine the value of k such that $\frac{16^{(k)} \bullet 8^{(k+3)}}{32^{(k-1)} \bullet 4^{k} \bullet 2^{(4k)}} = 32^{(4)}$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. Let $y = \log_4 \left(8 \bullet 2^x\right)$. Then $y = k\left(x + w\right)$ for real numbers k and w. Determine the sum $\left(k + w\right)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. $\frac{147}{\sqrt{10}-\sqrt{3}} = k\sqrt{p} + w\sqrt{q}$ in simplified radical form and p and q are positive integers. Determine the exact sum $\left(k + w + p + q\right)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

6. Determine the sum of all real roots for the equation $6x^3 - 25x^2 - 76x + 60 = 0$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

7. Determine the area enclosed by the graph of the inequality $\left|3x-12\right| + \left|2y+4\right| \le 3$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

8. Three times the larger of two numbers is 6 less than ten times the smaller of the two numbers. The difference of the two numbers is 5. Determine the sum of these two numbers. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

9. Let $x = \sqrt{2017 + \sqrt{2017 + \sqrt{2017 + \cdots}}}$. Then $x = \dfrac{k + p\sqrt{w}}{q}$ in simplified and reduced radical form where k, w, p, and q are integers with $q > 0$. Determine the sum $(k + w + p + q)$.

10. Bob eats lunch at McDonalds or Burger King every week day according to the following pattern. When Bob eats at McDonalds, he eats at McDonalds the next day 70% of the time. When Bob eats at Burger King, he eats at Burger King the next day 80% of the time. Bob ate lunch at Burger King Monday. According to his pattern, determine the probability Bob will eat lunch at McDonalds on Friday, four days later. Express your answer as a decimal.

11. The graph of an ellipse with center at $(7,2)$ is tangent to the x-axis at point $(7,0)$ and the y-axis at point $(0,2)$. Determine the exact area of this ellipse.

12. $i = \sqrt{-1}$ and $A = i + i^2 + i^3 + i^4 + i^5 + \cdots + i^{2017}$. Determine the exact value of A.

13. Let m be a non-zero real number such that the positive difference between the roots of $mx^2 + 5x - 6 = 0$ is 1. Determine the sum of all possible value(s) for m. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

14. In a geometric sequence, $a_1 = \dfrac{5}{4}$ and $a_4 = -10$. Determine the value of a_3. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

15. Determine the value of $(3k - w)$ where (k, w) is a solution for x and y for the system $\begin{cases} \dfrac{4}{x} + \dfrac{6}{y} = 2 \\ \dfrac{6}{x} + \dfrac{3}{y} = 4 \end{cases}$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

16. Let $a^2 + ab + b^2 = \sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4}$. Determine the exact value of the positive difference $(a^3 - b^3)$.

17. Let n be a positive integer such that $n > 1000$. Let k be a positive integer. Let $(n+1)$ be a number that divides the group of $(n+k)$ positive integers into 2 groups such that the group of integers less than $(n+1)$ has the same sum as the other group of integers that is greater than $(n+1)$. Determine the smallest possible value of n.

18. The endpoints of the latus rectum (focal diameter) of a parabola are $(-2, 2)$ and $(10, 2)$ and the directrix is $y = k$. Determine (list) all possible value(s) of k.

19. Determine all possible value(s) for x such that $|x+2| + |x| = 8$. List all value(s) as an integer or as a common or improper fraction reduced to lowest terms.

20. The sum of the lengths of the two legs of an right triangle is 49. The square of the numeric length of the hypotenuse is 1225. Determine the numeric area of this triangle.

2017 SA
Algebra II

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $(1,2)$ (Must be this ordered pair.)

2. 14

3. $-\frac{3}{2}$ OR $\frac{-3}{2}$ (Must be this reduced common fraction.)

4. $\frac{7}{2}$ (Must be this reduced improper fraction.)

5. 55

6. $\frac{25}{6}$ (Must be this reduced improper fraction.)

7. 3

8. 11

9. 8073

10. 0.375 OR .375 (Must be this decimal.)

11. 14π (Must be this exact area.)

12. i OR $0+i$

13. 24

14. 5

15. 10

16. 3

17. 1188 or 1001

18. 8, –4 (Must have both answers in either order, using "or", "and" or sol'n sets ok.)

19. –5, 3 (Must have both answers in either order, using "or", "and" or sol'n sets ok.)

20. 294

1. $i=\sqrt{-1}$. Determine the value of k such that $k(-7+2i)=10i-35$.

2. Determine the constant term when the expansion of $\left(x-\frac{2}{x}\right)^4$ is expanded and completely simplified.

3. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ represent vectors such that $\vec{a}=(3,2)$, $\vec{b}=(-8,13)$ and $\vec{c}=(16,-24)$. Determine ***the sum*** of the two vectors that are perpendicular. Express your answer as the ordered pair representation of the sum of those two vectors.

4. Let $A=\log_2 3$, $B=\log_2 5$, $C=\log_2 7$ and $D=\log_2 11$. Then $\log_4\left(\frac{31500}{1331}\right)=aA+bB+cC+dD+f$ for real numbers a, b, c, d, and f. Determine the sum $(a+b+c+d+f)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. Determine the sum of the series $\sum_{n=1}^{\infty}\frac{1}{3^n}$, if it exists. Report your answer as an integer or as a common or improper fraction reduced to lowest terms, or "DNE" if the sum does not exist.

6. Let $f(x)=-3+2\cot 2\left(x+\frac{\pi}{3}\right)$. Determine the value of $f\left(\frac{\pi}{2}\right)$. Express your answer as a decimal rounded to the nearest thousandth.

7. Determine the remainder when 2^{2017} is divided by 1023.

8. Determine the sum of all possible distinct value(s) of $\sin x$, $\sin x<0$, when x satisfies $\cos 2x=2\cos x$. Express your answer as a decimal rounded to four significant digits.

9. Let $\vec{i}$, $\vec{j}$, and $\vec{k}$ be the mutually perpendicular unit vectors in 3-space. Let $\vec{u} = 2\vec{i} - 5\vec{j} + \vec{k}$, $\vec{v} = 5\vec{i} + 2\vec{j} - \vec{k}$, and $\vec{w} = a\vec{i} + b\vec{j} + c\vec{k}$. Determine $\vec{w}$ when $\vec{w} = \dfrac{\vec{u} \times \vec{v}}{\vec{u} \bullet \vec{v}}$. Express your answer as the ordered triple (a, b, c).

10. Let $0 \le \theta \le 2\pi$. This domain is redundant in that at least part of the figure graphed is traced more than once. Let $[0, k]$ be the least domain such that the figure is traced one time. Let L represent the exact length of the figure generated by graphing the parametric equations $\begin{cases} x = 2\lvert\cos(\theta)\rvert \\ y = 2\lvert\sin(\theta)\rvert \end{cases}$ with that domain. Determine the ordered pair (k, L). Express your answer as that ordered pair.

11. Let θ be an acute angle such that $\sin\left(\dfrac{\theta}{2}\right) = \sqrt{\dfrac{x-1}{2x}}$. Then $\tan^2\theta = kx^2 + wx + p$. Determine the sum $(k + w + p)$.

12. Let $\lim\limits_{n\to\infty}\left(\ln\left(\dfrac{n^2 + \cos n}{n^2 + 167}\right)\right) = k$. Determine the exact value of k if it exists. Express your answer as an integer or as a common or improper fraction reduced to lowest terms or "DNE" if the limit does not exist.

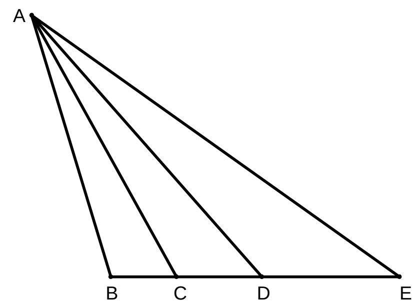
13. $i = \sqrt{-1}$. $f(x) = x^2 + kx + w$ has real coefficients k and w. The zeros of $f(x)$ are r_1 and r_2 such that $r_1 - r_2 = 2i$ and $\dfrac{1}{r_1} + \dfrac{1}{r_2} = \dfrac{3}{5}$. Determine the sum of all possible value(s) for k and w. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

14. $Arc\cos(x)$ represents the inverse cosine function expressed in radians. Determine the absolute value of the difference between the maximum and minimum values in the **range** of $f(x) = 2\left(Arc\cos(x)\right)$.

15. The bullet train traveled at r miles per hour for t hours and was 143 miles short of the train station at the estimated time of arrival. The next day, on the same run starting at the same time and same place, the engineer increased the rate of travel by 13 miles per hour and reached the station at the same estimated time of arrival. Determine the number of hours it took the train to reach the station on the second trip.

16. Consider the graph of the function $y = a\cos\left[b\left(x + c\pi\right)\right] + d$, x measured in radians and $b > 0$. The first minimum of the graph with positive abscissa occurs at $(k, 4)$ where $0 < k < 1$. The first maximum with positive abscissa occurs at $(3\pi, 12)$. The period is an odd multiple of $\left(\frac{\pi}{4}\right)$. Determine the value of b, Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

17. C and D lie on $\overline{BE}$ such that $\overrightarrow{AC}$ and $\overrightarrow{AD}$ trisect $\angle BAE$ in ΔABE. $BC = 2$, $CD = 3$, and $DE = 6$. Then the perimeter P of ΔABC may be expressed as $P = f + k\sqrt{w} + p\sqrt{q}$ in simplified and reduced radical form. Determine the sum $(f + k + w + p + q)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.



18. Let n be a positive integer such that $\frac{n!}{(n-2)!} + (2n-4)^2 - \frac{n!}{(n-1)!} = 0$. Determine the value(s) of n.

19. Determine the value(s) of k and w such that $(kx + w)(x^5 + 1) - (5x + 1)$ is exactly divisible by $x^2 + 1$. Express your answer(s) as ordered pair(s) (k, w).

20. Determine the geometric mean of 3, 27, and 72.

2017 SA

Pre-Calculus

Name ANSWERS

School ____________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 5 (Accept 5+0i.)

2. 24

3. $(19,-22)$ (Must be this ordered pair.)

4. $\frac{5}{2}$ (Must be this reduced improper fraction.)

5. $\frac{1}{2}$ (Must be this reduced common fraction.)

6. -4.155 (Must be this decimal.)

7. 128 OR 2^7

8. -0.9306 OR $-.9306$ (Must be this decimal.)

9. $(-3,-7,-29)$ (Must be this ordered triple.)

10. $\left(\frac{\pi}{2},\pi\right)$ (Must be this ordered pair with exact entries.)

11. 0 or zero

12. 0 or zero

13. $\frac{40}{9}$ (Must be this reduced improper fraction.)

14. 2π (Must be this exact answer.)

15. 11 ("hours" or "hrs. optional.)

16. $\frac{8}{23}$ (Must be this reduced common fraction.)

17. 23

18. 2

19. $(2,3)$ (Must be this ordered pair only.)

20. 18

NO CALCULATORS

1. Determine all value(s) for x such that $x^2+12x+35=0$.

2. Determine the simplified radical expression equivalent to $\left(-3\sqrt{20}\right)\left(\sqrt{15}\right)$.

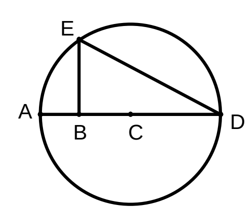
3. Determine the value of x so that $\frac{3x+1}{2}-(x+2)=\frac{5x}{4}$.

4. A rectangle has area $3\left(9^{\left(\frac{2}{3}\right)}\right)$ and has width $(27)^{\left(\frac{1}{2}\right)}$. The length can be expressed as $L=3^{(k)}$. Determine the value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. A 2-foot wide cement sidewalk encloses a circular pond that has a diameter of 12 feet. The numeric surface area of the cement walk is $k\pi$. Determine the value of k.

6. $\dfrac{1+\dfrac{1}{1-\dfrac{1}{a}}}{1-\dfrac{3}{1+\dfrac{1}{a}}}=\dfrac{ka+w}{pa+q}$ when completely simplified and reduced and where k, w, p, and q are integers with $p>0$. Determine the sum $(k+w+p+q)$.

7. $\overline{AD}$ is a diameter passing through the center C of a circle. E lies on the circle and $\overline{EB}\perp\overline{AD}$. $EB=5$ and $ED=13$. Determine the length of the diameter of this circle. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.



NO CALCULATORS

8. Determine the exact value(s) of x so that $2\sqrt{x}-8=0$.

9. The distance from the point $(6,3)$ to the line $y=2x-4$ can be expressed as $k\sqrt{w}$ in reduced and simplified radical form. Determine the value of $(k+w)$.

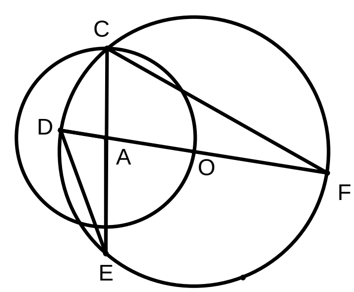
10. Determine *the number* of integer solutions for the inequality $|2x-5|<25$.

11. $A(-10,4)$, $B(2,-4)$, and $C(-4,6)$ form ΔABC. Determine the coordinates of the point that is the centroid of this triangle. Express your answer as an ordered pair (x,y).

12. Determine the exact product of the mean (arithmetic average) and the median for the set of data $\{14,12,19,33,22,29,11,16,13,16\}$.

13. Line ℓ has equation $2x+3y=18$, line m has equation $6x+ky=28$, and line n has equation $6x+wy=15$. Lines ℓ and n are parallel while line m is perpendicular to line ℓ. Lines ℓ and m intersect at point P and lines m and n intersect at point Q Determine the length of $\overline{PQ}$.

14. C, D, E, and F lie on the circle with center at O. A lies on $\overline{DF}$, a diameter of $\odot O$, and is between D and O. The circle with center at A passes through O and C. $\angle CFD=27°$. Determine the degree measure of $\angle EDF$.



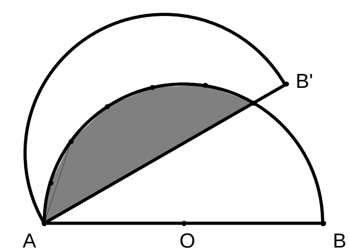
15. Xavier, Yves and Zach each walk at steady paces measured in meters per minute. Yves walks 40 meters further than Xavier in 5 minutes. Xavier walks 280 meters further than Zach in 10 minutes. Determine the positive difference between Yves and Zach's pace as measured in meters per minute.

NO CALCULATORS

16. Let $9^{\left(x^2-14x+50\right)}=81$. Determine the least common multiple of all solution(s) for x.

17. A chemist has a container that is filled with a thoroughly mixed liquid mixture consisting of 2 ounces of water and 8 ounces of acid. She pours out $\frac{1}{4}$ of this mixture, replaces it with pure water to completely refill the container and thoroughly mixes the resulting liquid. She then pours out $\frac{3}{4}$ of this new mixture, replaces it with pure acid to completely refill the container and again thoroughly mixes the resulting liquid. This final resulting liquid contains $k\%$ acid. Determine the value of k. Report the value of k only. Do not use "%" in your answer.

18. The semicircle with center at O, radius 1, and diameter $\overline{AB}$ is rotated $30°$ counter clockwise about point A to produce the semicircle with $\overline{AB'}$ as diameter. The overlapping (shaded) region has area that can be represented in simplified form as $\dfrac{k\pi + p\sqrt{q}}{n}$ where k, p, q, and n are integers with $n>0$. Determine the sum $(k+p+q+n)$.



19. The length of the shortest edge of a rectangular prism is an integer greater than 9, and the lengths of all edges, of at least one face diagonal, and of a main diagonal are integers. Find the smallest possible volume of the rectangular prism.

20. <u>*(Multiple Choice)*</u> For your answer write the <u>*capital letter(s)*</u> which corresponds to the correct choice(s).

The locus of points in a plane that are 5 inches from a given line is/are:

A) two parallel lines B) a line C) a point
D) two points E) three points F) four points G) a circle

<u>*Note: Be certain to write the correct capital letter as your answer.*</u>

2017 SA

School ANSWERS

Fr/So 8 Person

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-5, -7$ (Must have both values, either order, "and", "or", or sol'n set opt'l)

2. $-30\sqrt{3}$ (Must be this exact answer.)

3. -2

4. $\frac{5}{6}$ (Must be this reduced common fraction.)

5. 28

6. -2

7. $\frac{169}{12}$ (Must be this reduced improper fraction.)

8. 16 (Must be this value only.)

9. 6

10. 24 ("integers" optional.)

11. $(-4, 2)$ (Must be this ordered pair.)

12. 296

13. $\sqrt{13}$ (Must be this exact answer.)

14. 81 ("degrees" or "°" optional.)

15. 36 ("meters per minute" or "mpm" optional.)

16. 24

17. 90 (Must be this answer without "%" sign.)

18. 16

19. 2880

20. A (Must be this capital letter only.)

NO CALCULATORS

1. Let $k=\lim_{n\to\infty}\frac{4x^3-7x^2+2x-1}{2+5x+7x^2-9x^3}$. Determine the value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2. (Multiple Choice: Write the *capital letter* of the best answer from the choices below.) Given a and b are consecutive integers, $c=ab$, and $k=a^2+b^2+c^2$. Then $\sqrt{k}$ is

   (A) Always an even integer.
   (B) Sometimes an odd integer, sometimes not.
   (C) Always an odd integer.
   (D) Sometimes rational, sometimes not.
   (E) Always irrational.

3. Let $5^{(3x+1)}=625$. Determine the exact value(s) for x.

4. $A=2\begin{bmatrix}3 & -1\\ 9 & 6\end{bmatrix}+3\begin{bmatrix}8 & 2\\ 1 & -3\end{bmatrix}$. Determine the exact value for $2(\det A)$. (Twice the value of the determinant of A.)

5. Let $k=5+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1}}}}$. Determine the exact value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

6. In ΔABC, $AB=8$, $AC=14$, and $AD=7$ is the length of the median from A to $\overline{BC}$. Determine the exact length of side $\overline{BC}$.

7. Let n be an integer such that $n>7$. Determine the least possible value of n such that the three consecutive integers n, $n+1$, and $n+2$ are divisible, respectively, by 5, 6, and 7.

NO CALCULATORS

8. The sphere $x^2+y^2+z^2+6x-4y-8z+20=0$ has center (h,k,p) and radius r. Determine the exact values h, k, p, and r. Report your answer as the ordered quadruple (h,k,p,r).

9. For all Real numbers in its natural domain, $f(x)=\dfrac{4x^2-6x+7}{2x+3}$ has a slant asymptote in the form $y=kx+w$. Determine the ordered pair (k,w).

10. A bag contains 5 identical marbles except that 2 are blue and 3 are orange. Two marbles are selected at random without replacement. Determine the probability that one of each color marble is drawn. Express your answer as a common fraction reduced to lowest terms.

11. Let $k=\cos(0)+\cos\left(\dfrac{\pi}{8}\right)+\cos\left(\dfrac{\pi}{4}\right)+\cos\left(\dfrac{3\pi}{8}\right)+\cdots+\cos(250\pi)$. Determine the exact value of k.

12. The graph of a parabola in the form $y=ax^2+bx+c$ has its vertex at $(2,2)$ and passes through the point $(-1,29)$. Determine the exact values of a, b, and c. Report your answer as the ordered triple (a,b,c).

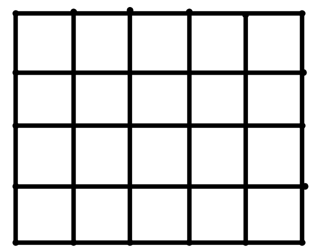
13. Let k and w represent the two (not necessarily distinct) roots of $x^2-7x+11=0$. Determine the exact value of (k^4+w^4).

14. Let $k=\sin^2 50°+2\sin 25°\sin 65°+\sin^2 40°-\cos 40°$. Determine the exact value of k.

NO CALCULATORS

15. The parametric equations $\begin{cases} x = -t - 3 \\ y = 3t^2 \end{cases}$ can be written in standard rectangular form as $y = a_0 + a_1 x + a_2 x^2 + a_3 x^4 + \cdots$. Determine the exact sum of all non-zero constants a_k, where $k = 0, 1, 2, 3 \cdots$ represents non-negative integers.

16. Consider a standard graph grid of 4 rows and 5 columns of unit squares. Determine the total number of distinct rectangles that can be found in this grid.



17. Let $k = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \cdots$. Determine the exact value for k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

18. Let $\log 7 = a$, $\log 3 = b$, $\log 2 = c$, $\log 5 = d$ and let N represent a real number. Then $\log 63 - \log 13122 + \log 343 = Aa + Bb + Cc + Dd + N$. Determine the ordered quintuple (A, B, C, D, N).

19. Given $\frac{8x - 42}{x^2 + 3x - 18} = \frac{A}{x + B} + \frac{C}{x + D}$ for all values of x in the natural domain. Determine the sum $(A + B + C + D)$.

20. The *__sum__* of all distinct real x, $0 \le x < 2\pi$ such that $\sin(5x) + \sin(3x) = 0$ is $k\pi$. Determine the exact value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2017 SA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = ☐

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-\frac{4}{9}$ OR $\frac{-4}{9}$ (Must be this reduced common fraction.)

2. C (Must be this capital letter only.)

3. 1 (Must be this value only.)

4. 12

5. $\frac{225}{43}$ (Must be this reduced improper fraction.)

6. 18

7. 215

8. $(-3, 2, 4, 3)$ (Must be this ordered quadruple.)

9. $(2, -6)$ (Must be this ordered pair.)

10. $\frac{3}{5}$ (Must be this reduced common fraction.)

11. 1

12. $(3, -12, 14)$ (Must be this ordered triple.)

13. 487

14. 1

15. 48

16. 150 ("rectangles" optional.)

17. $\frac{3}{4}$ (Must be this reduced common fraction.)

18. $(4, -6, -1, 0, 0)$ (Must be this ordered quintuple.)

19. 11

20. 7

Note: All answers must be written legibly. Exact answers are required unless otherwise specified in the question. Answers must be simplified and in the specific form if so stated. Except where noted, angles are in radians. No units of measurement are required.

1. A 72" smart TV was sold for $963.20 before tax. That price was 65% off the original list price. Determine the original list price of this TV.

2. $\log_2 k + \log_3 k = \log_{20} 17$. Determine the value of k. Express your answer as a decimal rounded to four significant digits.

3. At last year's pro football combine, a prospective quarterback ran a 40 yard dash in 4.56 seconds. Assume a steady rate. Determine this prospect's speed, in miles per hour. Express your answer as a decimal rounded to four significant digits.

4. A sphere is inscribed in a cube. The numeric volume of the sphere is 2017. Determine the numeric volume of the cube. Express your answer as a decimal rounded to the nearest thousandth.

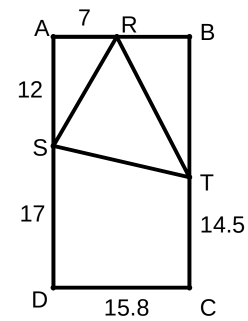
5. A triangle has sides of lengths 13, 15, and 18. Determine the length of the altitude to the longest side of this triangle. Express your answer as a decimal rounded to the nearest thousandth.

6. Determine the degree measure of the largest acute angle in a 5-12-13 triangle. Express your answer as a decimal rounded to four significant digits.

7. A circle with center (h,k) and radius of length r passes through the points $(2,9)$, $(-2,1)$, and $(-3,4)$. Determine the ordered triple (h,k,r^2). Report your answer as that ordered triple.

8. A coin is weighted so that the probability of any toss being a head is k. If the probability that the coin will turn up heads both times when this coin is tossed twice is less than 0.5222, determine the largest possible value of k. Express your answer as a decimal rounded to four significant digits.

9. The vertices of ΔRST lie on three sides of rectangle $ABCD$ as shown. $AR = 7$, $AS = 12$, $SD = 17$, $DC = 15.8$, and $CT = 14.5$. Determine the length of the altitude drawn from R to $\overline{ST}$. Express your answer as a decimal rounded to four significant digits.



10. An isosceles trapezoid has base angles measuring $78.47°$ each. The base that serves as a side of the $78.47°$ angle has a length of 7.998. One of the legs of the isosceles trapezoid has a length of 5.113. Determine the length of a diagonal of this isosceles trapezoid. Express your answer as a decimal rounded to four significant digits.

11. A cubic function of the form $y = x^3 + bx^2 + cx + d$ has zeros at 8, 15, and 17. Determine the sum of all possible values of y when x is a positive integer less than 27 and y is positive. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

12. $\log_{\left(\frac{3}{4}\right)} k + \log_{\left(\frac{4}{3}\right)}\left(\frac{1}{k}\right) = 20$. Determine the value of k. Express your answer rounded to four significant digits and in scientific notation.

13. Determine the numeric area of the largest rectangle inscribed under the curve $y = e^{\left(-x^2\right)}$ and in the first and second quadrant. (That is, two vertices on the curve and one side lies on the x-axis.) Express your answer as a decimal rounded to four significant digits.

14. Let $A = \{2.355,\ 4.617,\ 4.889,\ 5.812,\ 6.788,\ 9.411\}$. Determine the arithmetic mean of A. Express your answer as a decimal rounded to four significant digits.

15. The symbol $\prod a_n$ represents the product of terms. For example, $\prod_{n=1}^{3} a_n = (a_1)(a_2)(a_3)$.
Determine the value of $\prod_{n=1024}^{2017} \frac{\log n}{\log(n+2)}$. Express your answer as a decimal rounded to four significant digits.

16. A general equation of a plane is $Ax + By + Cz + D = 0$ where A, B, C, and D are relatively prime integers and $A > 0$. Determine the sum $(A + B + C + D)$ from the general equation for the plane containing points $(3,1,4)$, $(1,5,9)$, and $(2,6,5)$. Express your answer as an integer.

17. Let $a_1 = 0.0001$, $a_2 = 0.0002$, and $a_3 = 0.0003$. For integers $n \geq 4$, $a_n = a_{n-3} + a_{n-2} + a_{n-1}$.
Determine the value of $\sum_{n=1}^{350} a_n$. Express your answer rounded to four significant digits and in scientific notation.

18. Let $\vec{u} = (8,-2)$ and $\vec{v} = (9,3)$ represent two vectors in 2-space. Determine the degree measure of the angle between these two vectors. Express your answer as a decimal rounded to four significant digits.

19. A player plays a game that starts by flipping a fair coin. If 'heads' shows, the player tosses a fair standard cubical die and loses the dollar amount that shows on the top face of the die. If the player originally tosses a 'tails', the player draws a card from a standard 52-card deck of 4 suits and 13 ranks and wins the dollar amount of that card. (Ace=1, 2, 3, ..., Jack=11, Queen=12, and King=13.) Determine the dollar amount the player should pay in order to make this a fair game. Express your answer in dollars as an integer or as a common or improper fraction reduced to lowest terms.

20. In ΔABC, $AC = 8$, $BC = 6$ and $\angle ABC = 48°$. Determine the numeric area of ΔABC. Express your answer as a decimal rounded to four significant digits.

2017 SA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** ☐

Note: All answers must be written legibly. Exact answers are required unless otherwise specified in the question. Answers must be simplified and in the specific form if so stated. Except where noted, angles are in radians. No units of measurement are required.

1. 2752 OR 2752.00 ("dollars" or "$" optional.)

2. 1.495 (Must be this decimal.)

3. 17.94 (Must be this decimal, "miles per hour" or "mph" optional.)

4. 3852.186 (Must be this decimal.)

5. 10.657 (Must be this decimal.)

6. 67.38 (Must be this decimal, "degrees" or "°" optional.)

7. $(2, 4, 25)$ (Must be this ordered triple.)

8. 0.7226 OR .7226 (Must be this decimal.)

9. 12.95 (Must be this decimal.)

10. 8.589 (Must be this decimal.)

11. 6278 (Must be this integer.)

12. 5.631×10^{-2} (Must be this decimal and in scientific notation.)

13. 0.8578 OR .8578 (Must be this decimal.)

14. 5.645 (Must be this decimal.)

15. 0.8297 OR .8297 (Must be this decimal.)

16. –20 (Must be this integer.)

17. 4.822×10^{88} (Must be this decimal and in scientific notation.)

18. 32.47 (Must be this decimal, "degrees" or "°" optional.)

19. $\frac{7}{4}$ (Must be this reduced common fraction, "$" or "dollars" optional.)

20. 23.76 (Must be this decimal.)

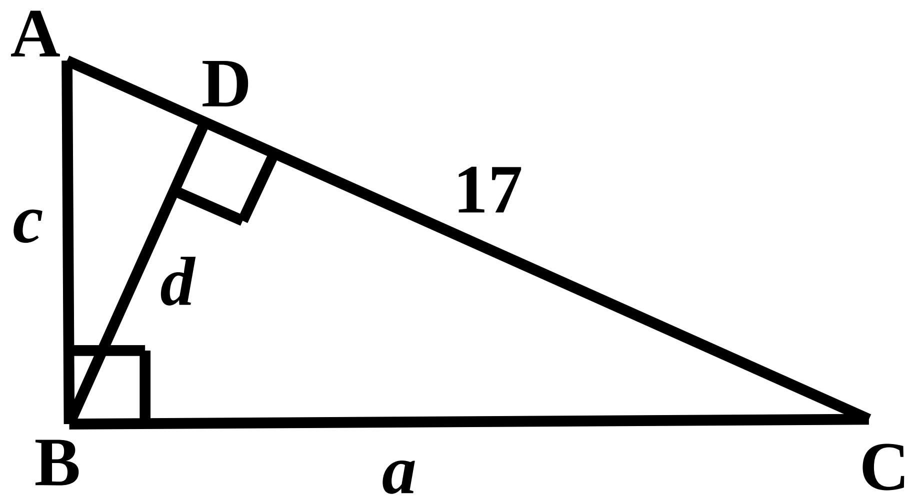
ICTM Math Contest

Freshman – Sophomore

2 Person Team

Division A

1. Find the sum of all distinct positive integers between 3 and 21 inclusive that ***cannot*** be expressed as the sum of at least two consecutive positive integers.



2. In right ΔABC with $\overline{AB} \perp \overline{BC}$ and D on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$, $CD = 17$, $AC = b = 20$ and $BD = d$. Determine the value of $\left(a^2 + b^2 + c^2 + d^2\right)$.

3. The volume of a sphere with a radius of 9 is $k\pi$. The slope of a line passing through the point $\left(\frac{1}{2}, 56\right)$ is two times b, the y-intercept of this line. Determine the sum $(k+b)$.

4. Let k be the number such that the reciprocal of half this number increased by half the reciprocal of this number is one-half. Monte Carlo travelled 120 miles in 105 minutes and then made the return trip along the same exact route at 60 miles per hour. Let w be Monte's average speed in miles per hour for the entire trip. Determine the sum $\left(k+w\right)$.

5. Let k be the number of distinct diagonals in a convex hexagon and let w be the measure of each exterior angle in a regular decagon. Determine the geometric mean between k and w.

6. One day Christie and Liz each shot some free throws. Christie made exactly 45% of her attempts while Liz made exactly $70.8\overline{3}\%$ (only the 3 repeats). The two girls **made** a total of k free throws that day. Determine the smallest possible value for k.

7. One of the following four quadratic polynomials is chosen at random. Determine the probability that it could be expressed in the form $(x+a)^2$ where a is a rational number. Express your answer as a common fraction reduced to lowest terms.

$$x^2+25.2x+158.76$$
$$x^2+57.4x+748.022$$
$$x^2+62.6x+979.69$$
$$x^2-14.4x+51.84$$

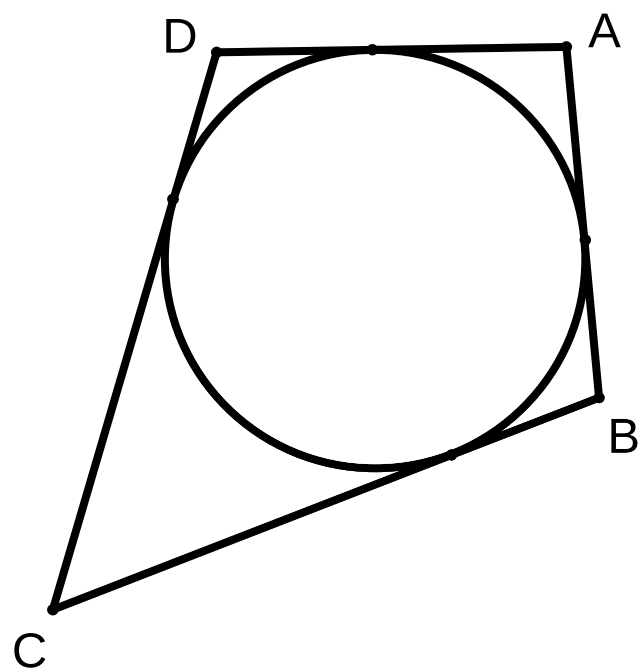
8. Let k be the greatest common factor and w the least common multiple of 655,200 and 317,520. Determine the quotient $\frac{w}{k}$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

9. A dozen more than a dozen squared,
Is added to three times the square root of four.
Divide that by seven,
Add six times eleven,
And get nine squared and just a bit more.

Determine how much more. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

10. Quad $ABCD$ is circumscribed about a circle.

$BC = 20$ and $AD = 17$. Determine the perimeter of quad $ABCD$.



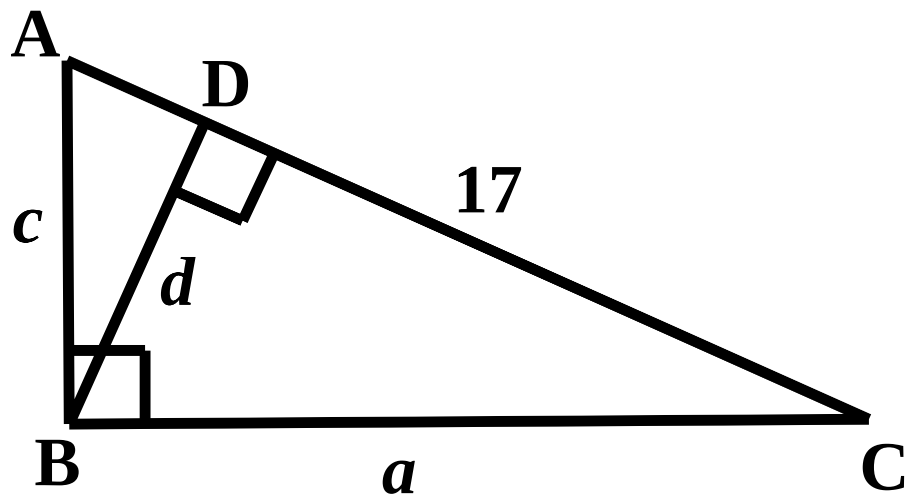
ICTM Math Contest

Freshman – Sophomore

2 Person Team

Division A

1. Find the sum of all distinct positive integers between 3 and 21 inclusive that ***cannot*** be expressed as the sum of at least two consecutive positive integers.



2. In right ΔABC with $\overline{AB} \perp \overline{BC}$ and D on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$, $CD = 17,\ AC = b = 20$ and $BD = d$. Determine the value of $\left(a^2 + b^2 + c^2 + d^2\right)$.

3. The volume of a sphere with a radius of 9 is $k\pi$. The slope of a line passing through the point $\left(\frac{1}{2}, 56\right)$ is two times b, the y-intercept of this line. Determine the sum $(k+b)$.

4. Let k be the number such that the reciprocal of half this number increased by half the reciprocal of this number is one-half. Monte Carlo travelled 120 miles in 105 minutes and then made the return trip along the same exact route at 60 miles per hour. Let w be Monte's average speed in miles per hour for the entire trip. Determine the sum $\left(k+w\right)$.

5. Let k be the number of distinct diagonals in a convex hexagon and let w be the measure of each exterior angle in a regular decagon. Determine the geometric mean between k and w.

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7. One of the following four quadratic polynomials is chosen at random. Determine the probability that it could be expressed in the form $(x+a)^2$ where a is a rational number. Express your answer as a common fraction reduced to lowest terms.

$$x^2+25.2x+158.76$$
$$x^2+57.4x+748.022$$
$$x^2+62.6x+979.69$$
$$x^2-14.4x+51.84$$

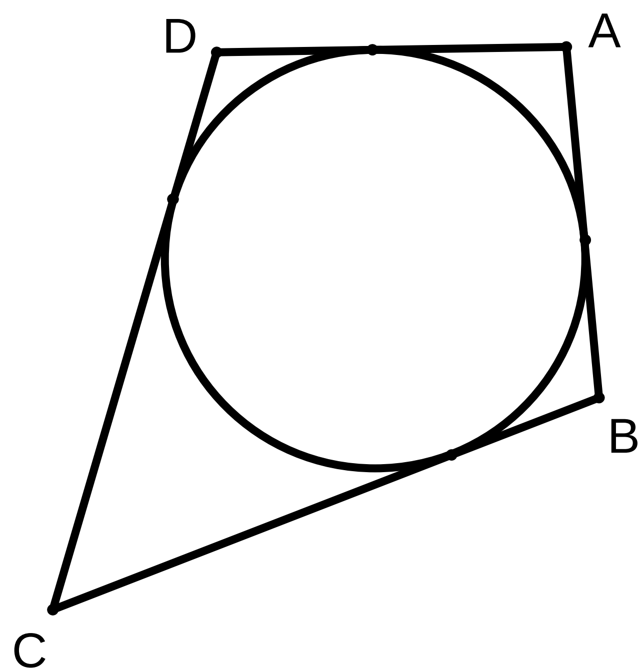
8. Let k be the greatest common factor and w the least common multiple of 655,200 and 317,520. Determine the quotient $\frac{w}{k}$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

9. A dozen more than a dozen squared,
Is added to three times the square root of four.
Divide that by seven,
Add six times eleven,
And get nine squared and just a bit more.

Determine how much more.
Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

10. Quad $ABCD$ is circumscribed about a circle.

$BC = 20$ and $AD = 17$. Determine the perimeter of quad $ABCD$.



2017 SA

School ANSWERS

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		Score (to be filled in by proctor)
1. 28		______
2. 851		______
3. 1000		______
4. 69		______
5. 18		______
6. 26	("free throws", "shots", etc. optional.)	______
7. $\frac{3}{4}$	(Must be this reduced common fraction.)	______
8. 8190	(Must be this integer, comma usage optional.)	______
9. $\frac{57}{7}$	(Must be this reduced improper fraction.)	______
10. 74		______

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. ______

12. ______

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

ICTM Math Contest

Junior – Senior

2 Person Team

Division A

1. Let $k = \langle 2,3 \rangle \bullet \langle 4,-1 \rangle$ (the dot product of two vectors) and $w = \log_7 7203 - \log_7 3$. Determine the sum $(k+w)$.

2. Consider the graph of $f(x) = 8\cos(4x-3)-5.$ Let k be the amplitude, w the phase shift, $p\pi$ the period, and q the vertical shift for the graph of this function. Determine the sum $(k+w+p+q)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

3. The Farm Bureau conducted an experiment in the month of June. They found, on average, a rat ate 10 pounds of corn in 6 days and that a raccoon ate $41\frac{2}{3}$ pounds of corn in 10 days. For this month, let k represent the rate, in pounds of corn per month, at which a rat ate corn and w represent the rate, in pounds of corn per month, at which a raccoon ate corn. Determine the ratio $k:w$. Express your answer as the ratio $p:q$ where p and q are relatively prime positive integers.

4. For the system of equations $\begin{cases} 3x - Ay = 3 \\ -4x + 5y = -1 \end{cases}$, let k be the value of A such that the system is inconsistent. Let w be the value of A such that this system represents perpendicular lines. Determine the product (kw).

Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. Let k represent the sum of the absolute values of all the distinct values of x such that $x^{2017} = 1$.
Let w represent the sum of the cubes of the roots of $x^2 - 4x + 7 = 0$.
Determine the value of the sum $(k + w)$.

6. Let k and w be positive integers such that

$$1+2+3+\cdots+k=k^2-15$$

and

$$1+3+5+7+\cdots+(2w-1)$$

$$=2w^2+3w-340$$

Determine the sum $(k+w)$.

7. Aune and Henrik each toss a fair coin 5 times. Determine the probability that Aune tossed at least 3 more heads than Henrik. Express your answer as a common fraction reduced to lowest terms.

8. Determine the sum of all real solution(s) for the equation $x^2 = 2\left[\ln\left(x+2\right)\right]$. Express your answer as a decimal rounded to four significant digits.

9. Consider the expansion and simplification of $(2x+3y+4z)^5$. Let k represent the <u>*sum*</u> of the numerical coefficients of this expansion.
Let w represent the numerical coefficient of the term that contains both y^2 and z^2.
Determine the difference $(k-w)$.

10. In degree mode, let $s(x) = Sin^{-1}x$ (the inverse sine function) and $t\left(x\right) = \tan x$.
Determine the exact value of $s\left(t\left(s\left(t\left(s\left(t(30)\right)\right)\right)\right)\right)$.

2017 SA

Jr/Sr 2 Person Team

School ____ANSWERS____

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

NOTE: Questions 1-5 only are ***NO CALCULATOR***

Note: **All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.**

Answer		Score (to be filled in by proctor)
1. 9		____
2. $\frac{17}{4}$	(Must be this reduced improper fraction.)	____
3. $2:5$	(Must be this exact ratio.)	____
4. -9		____
5. 1997		____
6. 23		____
7. $\frac{7}{128}$	(Must be this reduced common fraction.)	____
8. 0.8890 OR .8890	(Must be this decimal, trailing zero necessary.)	____
9. 50409		____
10. 90	("degrees" or "°" optional.)	____

TOTAL SCORE: ____

(*enter in box above)

Extra Questions:

11. ____

12. ____

13. ____

14. ____

15. ____

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Let (k, w) be the coordinates of the vertex of $y = 4x^2 + 5$. Determine the sum $(k + w)$.

2. Let $k = ANS$. Determine the median of the set $\{k, 1, 7, 9, 4, 12, 4\}$.

3. Let ANS be the distance from the point $(2,9)$ to the point $(-1,y)$. Determine the least possible value of y.

4. Let $k = ANS$. An angle has measure $(7k)°$. Determine the degree measure of the supplement of the complement of this angle.

1. The average score on a test in an Algebra class was 81. The two lowest scores of 34 and 48 are removed, and the remaining scores now average 85. Determine the number of students in this class.

2. Let $k = ANS$. Simplify the expression $\left(\frac{8c-(k+2)}{c}\right) \div \left(\frac{c^2-9}{c^2+3c}\right)$.

3. Let $k = ANS$. A trapezoid has a numeric area of 52 and an altitude of length k. One base has length 10. Determine the length of the other base.

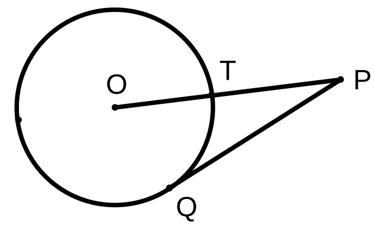
4. Let $k = ANS$. Determine the equation of the line passing through the points $P(2, k)$ and $Q(-1, 5)$. Write your equation in general linear form $Ax + By + C = 0$ where A, B, and C are relatively prime integral coefficients and $A > 0$. Express your answer as the ordered triple (A, B, C).

1. Solve for x when $3(x-4)=7(5-x)$.

2. Let $k = ANS$. Determine the value of x when $x \geq 0$ and $\frac{18.8}{k} + x^2 = 2x^2 - 5$.

3. Let $k = ANS$. In a rhombus the radius of the inscribed circle is (*ANS*) and the length of one of the diagonals is 12. Determine the exact perimeter of the rhombus.

4. Let $k = ANS$. $\overline{PQ}$ is tangent to circle O at point Q. $PQ = k$. $PT = 12$. Determine the exact numeric area of ΔPOQ.



1. It takes 5 minutes for Will to work 2 similar calculus problems. At this rate, determine the number *of seconds* it will take him to work 8 similar calculus problems.

2. Let $k = ANS$. Let $100x^2 + 700x + k = 0$. Determine the sum and the product of the solutions for this equation. Report as your answer the smaller of these two quantities.

3. Let $k = |ANS|$. k is the difference between the squares of the lengths of the two legs of a right triangle. k is also the sum of lengths of these same two legs. Determine the numerical area of this triangle.

4. Let $k = ANS$. The apothem of a regular hexagon is k units long. Determine the exact numeric area of this hexagon.

1. Sally's rectangular garden is twice as long as it is wide. The width of the garden is 6 feet. Determine the number of feet in the perimeter of the garden.

2. Let $k = \sqrt{ANS}$. Determine the solution (x, y) for the system $\begin{cases} 40x - 3y = 21 \\ ky - 76x = 18 \end{cases}$.

Report as your answer the y-coordinate of this ordered pair (x, y).

3. Let $k = ANS + 16$. A certain convex polygon has k diagonals. Determine the number of sides in this polygon.

4. Let $k = ANS$. In an equilateral triangle with sides of length k, the radius of the inscribed circle is $\frac{x\sqrt{y}}{z}$ when completely reduced and written in simplified radical form where x, y, and z are integers. Determine the sum $(x + y + z)$.

2017 SA FR/SO RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. 5
2. 5
3. 5
4. 125 ("degrees" or "°" optional.)

ROUND 2

1. 22 ("students" optional.)
2. 8
3. 3
4. $(2, 3, -13)$ (Must be this ordered triple.

ROUND 3

1. 4.7 OF $\frac{47}{10}$ OR $4\frac{7}{10}$
2. 3
3. $16\sqrt{3}$ (Must be this exact answer.)
4. $208\sqrt{3}$ (Must be this exact answer.)

EXTRA ROUND 4

1. 1200 ("seconds" or "sec." optional.)
2. -7
3. 6
4. $72\sqrt{3}$ (Must be this exact answer.)

EXTRA ROUND 5

1. 36
2. 193
3. 22
4. 17

1. Let $(2x+1)(x-1) = ax^2 + bx + c$ where a, b, and c are integers. Determine the value of b.

2. $i=\sqrt{-1}$. Let $k = ANS$. The complex expression $\frac{-1+ki}{4+3i} = a+bi$ where a and b are real numbers. Determine the sum $(a+b)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

3. Let $\left|ANS\right| = \frac{k}{w}$. The point of intersection of the diagonals of a rectangle is $(0,0)$ and one side of the rectangle is parallel to the *x*-axis. One vertex of the rectangle is (k, w). Determine the area of this rectangle.

4. Let $k = ANS$. Sam, an amateur Lego builder, is building 8 Lego structures in 2017. So far he has made structures of 915 Legos, 1215 Legos, k Legos, and 754 Legos. Determine the average number of Legos Sam must use in the remaining four Lego structures in order to average 1000 Legos per structure in 2017.

1. In the relation $x^2 + y^2 = 144$, determine the largest possible value of y.

2. Let $k = ANS$. Let $f(x) = \left(\sqrt{2x-27}\right)\left(\sqrt{x-k}\right)$ be a real-valued function. Determine the smallest integer in the domain of this function.

3. Let $k = ANS$. Zhuan is selecting students for the Homecoming Committee. He can choose from 10 boys and k girls. Determine the number of distinct committees of 5 students that can be formed if the committee needs to have more girls than boys.

4. Let $k = ANS$. The area of a triangle is k square feet. Two of its sides have lengths of 250 feet and 320 feet. Determine the degree measure of the largest possible angle that lies between these two sides. Express your answer as that degree measure rounded to the nearest degree.

1. Determine the remainder when $5x^3-7x^2+2x+9$ is divided by $(x-1)$.

2. Let $k = ANS$. Let x be a positive real number. $\log_3(x+24)-\log_3 x=\log_3|k|$. Determine the value of x.

3. Let $k = ANS$ and let θ measured in radians be such that $0 \le \theta < \pi$. The largest exact solution for the equation $4\left[\sin^2(k\theta)\right]+1=4\left[\sin(k\theta)\right]$ is $w\pi$. Determine the value of w. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. ANS should be in form $\frac{a}{b}$, where a and b are integers. Let $k = |a+b|$. There are 60 marbles in a brown cloth bag. Of the 60 marbles, k marbles are red, 10 marbles are green, and the rest of the marbles are yellow. Tom draws three marbles, without replacement. Determine the probability that the three marbles will be different colors. Express your answer as a decimal rounded to four significant digits.

1. Aune, Barb, Cindy and Ella are going to sit at a large, empty, circular table for math team practice. Determine the number of distinct ways they can be seated around the table.

2. Let $w = ANS$. $f(x) = \sqrt{(x+1)(x+2)(x+3)+25}$ and $g(x) = x(x-3)+(x+1)(x-2)$.

Determine the value of the expression $\dfrac{22\left(f(w)\right)}{g(w)}$.

3. Let $k = ANS$. Determine the value of x if matrix $A = 3\begin{bmatrix} k & 1 \\ 2 & -3 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ 2 & x \end{bmatrix}$ and the determinant of A is 37 (alternately written $\det A = 37$.) Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. Let $w = |ANS|$. Determine the value of the sum $\sum_{x=10}^{100}(4x - w)$.

1. Let $A = \begin{bmatrix} 2.5 & -1.5 \\ 2 & -1 \end{bmatrix}$. Determine the value of the determinant of A^{-1}. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2. Let $k = ANS$. Determine the ordered triple that is the solution for the system: $\begin{cases} x - y + 2z = 7 \\ 3x + 2y - z = -10 \\ -x + 3y + z = -(k) \end{cases}$.

3. ANS will be an ordered triple of real numbers, (a,b,c). Point P has coordinates $(2,3,4)$, point A has coordinates $(x,4,-2)$, point O has coordinates $(0,0,0)$, and point B has coordinates (a,b,c). Determine the value of x such that the vectors equivalent to $\overrightarrow{PA}$ and $\overrightarrow{OB}$ are perpendicular. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. Let $k = \left|ANS\right|$. All angles are in radians and all inverses are the respective inverse functions. Determine the exact value of the expression

$$k\left[\left(\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)-\left(\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)+\left(\text{Tan}^{-1}(-1)\right)-\left(\text{Cot}^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)\right].$$

2017 SA JR/SR RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. -1
2. $-\frac{8}{25}$ OR $\frac{-8}{25}$ (Must be this reduced common fraction.)
3. 800
4. 1079 (Comma usage and "Legos" optional.)

ROUND 2

1. 12
2. 14
3. 28392 (Comma usage and "committees" optional.
4. 135 (Must be this integer, "°" or "degrees" optional.)

ROUND 3

1. 9
2. 3
3. $\frac{17}{18}$ (Must be this reduced common fraction.)
4. 0.1534 OR $.1534$ (Must be this decimal.)

EXTRA ROUND 4

1. 6 ("ways" optional.)
2. 11
3. -5 (Must be this integer.)
4. 19565

EXTRA ROUND 5

1. 2
2. $(-1,-2,3)$ (Must be this ordered triple.
3. -18
4. -18π (Must be this exact value.)