1. Determine the exact value of $\left[7^2 - 18 \times 5 \div 2\right]^{\left(-\frac{3}{2}\right)}$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2. Determine ***the number*** of integers that are solutions for the inequality $\left|\frac{x}{2}+3\right| \le 4$.

3. Let $x^2 - 4y^2 = 30$ and $x - 2y = 5$. Determine the value of $(x+2y)$.

4. In order to test High Efficiency solar vehicles. the State Police closed one lane of an interstate highway so these test cars could drive unobstructed. One test car starts and drives at a constant rate of 37 miles per hour. One hour later, a second test car starts driving from the same starting point on the same interstate and in the same direction at a constant rate of 40 miles per hour. Determine the number of miles the second car will need to travel to catch up to the first car. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. Let $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$. If x is in set A and $x+7$ represents an even integer, determine ***the sum*** of all distinct possibilities for the value of x.

6. Let $18 \le x \le 52$, $5 \le y \le 15$, and $42 \le z \le 78$. Determine the least possible value of $\frac{x+z}{y}$.

7. Determine the value(s) for k such that $kx^2 - 42xy + 49y^2$ factors to the square of a binomial.

8. The graphs of the equations $xy = 10$ and $x^2 + y^2 = 101$ meet in four points that are corners of a rectangle. Determine the area of this rectangle.

9. In each of the bases 6, 7, and 8, the three-digit number $1a4$, where 1, a, and 4 are positive digits, represents a perfect square number in base 10. Determine the base 10 sum of these three perfect squares.

10. If $|x| = x$ and $|y| = -y$, then the graph of the equation $|x| + |y| = 4$ has a total length L. Determine the value of L.

11. Determine the numerical area of the triangular region enclosed by the system $\begin{cases} y = 4x - 2 \\ y = -4x + 14 \\ y = \frac{4}{3}x - 2 \end{cases}$.

12. (All ages are in whole number of years.) Tom asked Kay to marry him. Kay said, “Are you serious? You’re twice as old as I am.” Some years later Tom proposed a second time, saying, “I’m now only a third again as old as you are.” Kay softened a bit but refused and said, “Ask me again when you’re only ten per cent older than I am.” “Forget it,” Tom replied, “you’ll be seventy then.” Determine Tom's age, in years, when he proposed to Kay the second time.

13. Let a and b be such that $a^2b - 3a - 2ab^2 + 6b = 32$. If a is 4 more than twice b, determine the product (ab).

14. Let $k = 0.\overline{12468}$ be a decimal that repeats in the block of five digits. Determine the digit that is in the 2017th place to the right of the decimal point.

15. Determine *the sum* of all integral values of x for which $x+6 \le 2|x+3|+1 \le x+10$.

16. When the positive integer *A* is divided by the positive integer *B* the remainder is 5. When the positive integer *B* is divided by the positive integer *A* the remainder is 6. Determine the sum of the three smallest possible values of *A*.

17. Point P has coordinates $(4,-3)$ and is located a constant distance of 4 from all points on the graph of $y = Ax^2 + By^2 + Cx + D$. Determine the sum $(A+B+C+D)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

18. Let (k,w) be the point on the line $4x - y = 5$ that is equidistant from both the x-axis and y-axis. Determine the exact distance from (k,w) to the origin.

19. Evie can finish a job in 12 days if Xavier helps her for 7 of those days. Xavier can finish the same job in 14 days if Evie helps him for 7 of those days. Determine the number of days it would take Evie to finish the same job working alone. Assume both Evie and Xavier always work at their respective steady rates.

20. Let $k = 5 + 11 \times 3 - 7 + 6^2$. One pair of parentheses is inserted and the value of this new expression is 49. Then the 7 is changed to 8 in the second expression to form a third expression which has value w. Determine the value of $|w-k|$, the absolute difference between the value of the third expression and the value of the original expression.

2017 SAA
Algebra I

Name ___ANSWERS___

School ______________________

(Use full school name – no abbreviations)

______ Correct X 2 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $\frac{1}{8}$ (Must be this reduced common fraction.)

2. 17 ("integers" optional.)

3. 6

4. $\frac{1480}{3}$ (Must be this reduced improper fraction, "mi." or "miles" optional.)

5. 36

6. 4

7. 9 (Must be this value only.)

8. 198

9. 245 OR 245_{10} OR 245_{ten}

10. $4\sqrt{2}$ (Must be this exact answer.)

11. 8

12. 28 ("years" or "yrs. optional.)

13. 11

14. 2

15. –4

16. 51

17. $-\frac{1}{2}$ OR $\frac{-1}{2}$ (Must be this reduced common fraction.)

18. $\frac{5}{3}\sqrt{2}$ OR $\frac{5\sqrt{2}}{3}$ Also accept $\sqrt{2}$

19. 17 ("days" optional.)

20. 51

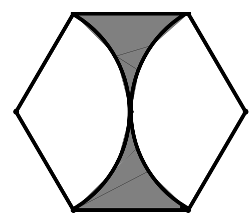
1. Determine the area of square $ABCD$ when $BD = 3\sqrt{2}$.

2. A rectangular solid has width (depth) that is twice its height and a length that is four times its height. A diagonal of this solid measures 84 units. Determine the numeric volume of this rectagonal solid.

3. **(Always, Sometimes, or Never)** For your answer, write the *whole word* **Always, Sometimes,** or **Never,** whichever is correct.

   The circumcenter and the centroid of an isosceles triangle are the same point.

4. In the regular hexagon shown, two arcs of circles are drawn from opposite vertices using the length of a side as radius. This length is 12. Determine the shaded area of this hexagon. (The area inside the hexagon but exterior to the sectors of the circles.)



5. A square with numeric area k is inscribed in a semicircle. (That is, two vertices on the semicircle and the side opposite the two vertices lies on the diameter.) A second square with numeric area w is inscribed in a full congruent circle (same radius as the semicircle.). Determine the reduced and simplified ratio $k:w$. Report your answer as a ratio of two relatively prime integers.

6. Determine the number of 1-inch diameter circular cross section pipes that are needed to carry the same amount of water as a 6-inch diameter circular cross section pipe. (Assume the water flow is the same in both.)

7. A circle is inscribed within the region that is the solution of $\begin{cases} x \geq 2 \\ x \leq 6 \\ y \geq 1 \\ y \leq 5 \end{cases}$. This circle can be represented algebraically by $(x-h)^2 + (y-k)^2 = r^2$. Determine the ordered triple (h, k, r^2).

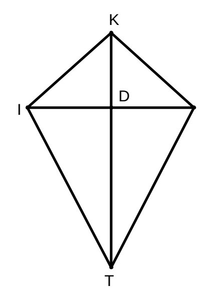
8. Two sides of a triangle have lengths 6 and 10. The area is 18. Determine the smallest possible exact length of the third side.

9. (All points are coplanar.) Square $SQUA$ has a side of length 5 units. Square $AQBZ$ is drawn such that point B is closer to point U than it is to point S. Square $ABCD$ is drawn such that point A is between point S and point D. Determine the exact length of $\overline{SC}$.

10. The ratio of the lateral surface areas of two similar cones is $20:9$. The ratio of their volumes is $k:1$. Determine the value of k expressed as a decimal rounded to the nearest thousandth.

11. Line ℓ, with equation $3x-4y-4=0$, intersects $\odot O$, with equation $(x-1)^2+(y+k)^2=25$, in exactly one point P. Determine *__the sum__* of all possible value(s) of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

12. Kite $KITE$ has $KI=KE$, $EI=10$, and $DT=3(DK)$. Kite $KITE$ has numeric area 40. Determine the length of $\overline{KE}$.

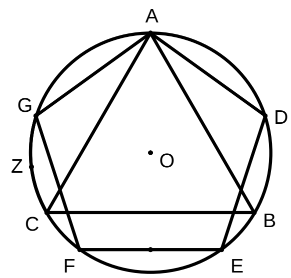


13. In ΔABC, $AB=6$, $BC=9$, D lies on $\overline{AC}$, $\angle ABD \cong \angle DBC$, and E is the midpoint of $\overline{BD}$. The numeric area of ΔABC is 25. Determine the numeric area of ΔABE.

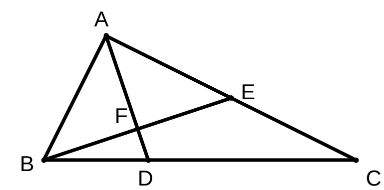
14. A line segment is drawn from point $P(4,5)$ to a point A on the y-axis. Then a segment is drawn from A to point B that lies on the x-axis. Finally, a segment is drawn from B to point $Q(11,3)$. Determine the least total sum of the lengths of these three segments.

15. In rhombus $ABCD$, $\angle CDA = 60°$. A circle passes through points D, A, and B. A point is selected at random inside the circle. Determine the probability that the point selected lies inside rhombus $ABCD$. Express your answer as a decimal rounded to four significant digits.

16. ΔABC is an equilateral triangle inscribed in circle O. $ADEFG$ is a regular pentagon inscribed in circle O, with $\overline{CB} \parallel \overline{EF}$. Z is the midpoint of arc $\overset{\frown}{CG}$. Then $\overline{ZG}$ is a side of a regular $n-gon$ inscribed in the circle. Determine the value of n.



17. ΔABC is a right triangle with right $\angle BAC$. E is the midpoint of $\overline{AC}$, D lies on $\overline{BC}$ such that $\overline{AD}$ is the perpendicular bisector of $\overline{BE}$ at point F. $AD = 4$ and $BE = 6$. Determine the exact perimeter of ΔABC.



18. Given $\overline{DE}$ has endpoints $D(-7,-5)$ and $E(-1,-2)$. $\overline{DE}$ is rotated $90°$ counter-clockwise about the origin and then reflected over the x-axis, resulting in the image $\overline{D'E'}$. Determine the coordinates of the endpoints $D'(x_1, y_1)$ and $E'(x_2, y_2)$. Express your answer as the ordered quadruple (x_1, y_1, x_2, y_2).

19. The length of the radius of the inscribed circle of a right triangle is an *integer* r such that $16.3 < r < 23.3$. The lengths of each of the *legs* of the right triangle are also *integers*. If one of the legs has a length of 60, determine the sum of all possible distinct lengths for the other leg.

20. Square $ABCD$ has sides of length 12. P lies on $\overline{BC}$ and $BP = 8$. Q lies on $\overline{CD}$ and $CQ = 4$. Square $ABCD$ is reflected across $\overleftrightarrow{PQ}$ to get square $A'B'C'D'$ Determine the length of $\overline{AA'}$.

2017 SAA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 9

2. $10752\sqrt{21}$ (Must be this exact answer.)

3. Sometimes (Must be this whole word.)

4. $216\sqrt{3}-96\pi$ OR $24\left(9\sqrt{3}-4\pi\right)$ (Must be this exact answer.)

5. $2:5$ (Must be this exact ratio.)

6. 36 ("pipes" optional.)

7. $\left(4,3,4\right)$ (Must be this ordered triple.)

8. $2\sqrt{10}$ (Must be this exact answer.)

9. $5\sqrt{13}$ (Must be this exact answer.)

10. 3.313 (Must be this exact decimal.)

11. $\frac{1}{2}$ (Must be this reduced common fraction.)

12. $\sqrt{29}$ (Must be this exact answer.)

13. 5

14. 17

15. 0.2757 OR $.2757$ (Must be this exact decimal.)

16. 15 ("15-gon", "n-gon", or "sides" optional.)

17. $9\sqrt{2}+3\sqrt{10}$ OR $3\left(3\sqrt{2}+\sqrt{10}\right)$ OR $3\sqrt{10}+9\sqrt{2}$ OR $3\left(\sqrt{10}+3\sqrt{2}\right)$ (Must be this exact answer.)

18. $\left(5,7,2,1\right)$ (Must be this ordered quadruple.)

19. 234

20. $20\sqrt{2}$ (Must be this exact answer.)

1. Determine the value of k such that -3 is a root of $3x^4 - kx^3 + 2x^2 + kx - 1 = 0$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2. Determine the value(s) for x such that $\sqrt[3]{x\sqrt{x}} = 4$.

3. Determine the value of k such that $\dfrac{16^{(k)} \bullet 8^{(k+3)}}{32^{(k-1)} \bullet 4^{k} \bullet 2^{(4k)}} = 32^{(4)}$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. Let a and b be real constants for the polynomial $x^3 + ax^2 + b$. This polynomial has linear factors $(x-3)$, $(x+1)$, and $(x-k)$. Determine the value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. $\dfrac{147}{\sqrt{10} - \sqrt{3}} = k\sqrt{p} + w\sqrt{q}$ in simplified radical form and p and q are positive integers. Determine the exact sum $(k + w + p + q)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

6. Determine all real value(s) for x such that $\log(2x) + \log(x-5) = 2$.

7. Determine the area enclosed by the graph of the inequality $|3x - 12| + |2y + 4| \le 3$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

8. The length of a rectangle is 65 more than 7 times the width of the rectangle. Determine the minimum integral length of this rectangle.

9. Let x, y, and represent positive integers that satisfy the system $\begin{cases} 2x+3y+7z=1311 \\ 3x+2y+5z=1718 \end{cases}$.

Determine the *smallest* possible value of $(x+y+z)$.

10. Determine the sum of all positive solution(s) for x for the equation $2\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)-5=0$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

11. The graph of an ellipse with center at $(7,2)$ is tangent to the x-axis at point $(7,0)$ and the y-axis at point $(0,2)$. Determine the exact area of this ellipse.

12. $i=\sqrt{-1}$. Determine the value of $\left(\frac{3}{8}\right)^{\left(i^{(966)}\right)}$. Express your answer using an integer or a common or improper fraction reduced to lowest terms.

13. Let m be a non-zero real number such that the positive difference between the roots of $mx^2+5x-6=0$ is 1. Determine the sum of all possible value(s) for m. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

14. Let $5^{(2x)}-4\bullet5^{(x+1)}+2^{(6)}=0$. Determine all possible value(s) of k when $x=\log_5 k$.

15. In a bridge game using a standard 52 card deck of 13 ranks in each of four suits, each of 4 persons is dealt 13 cards at random. If Jerry is one of these 4 persons, determine the probability that Jerry was dealt exactly 4 cards of one suit, exactly 4 cards of a second suit, exactly 3 cards of a third suit, and exactly 2 cards of the remaining suit. Express your answer as a decimal rounded to 4 significant digits.

16. Determine the value of k such that the graph of $y = 16x + 2x^2 + k$ is tangent to the x-axis. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

17. Let n be a positive integer such that $n > 1000$. Let k be a positive integer. Let $(n+1)$ be a number that divides the group of $(n+k)$ positive integers into 2 groups such that the group of integers less than $(n+1)$ has the same sum as the other group of integers that is greater than $(n+1)$. Determine the smallest possible value of n.

18. Determine the exact value(s) of k when $k = \cfrac{4}{3 + \cfrac{4}{3 + \cfrac{4}{3 + \cdots}}}$.

19. Determine the value(s) for x such that x is a real number and such that $\log\left(\sqrt{x + \frac{x}{2} + \frac{x}{4} + \cdots + \frac{x}{2^{(n-1)}} + \cdots}\right) = \log(4x - 15)$. Express your answer(s) as an integer or as a common or improper fraction reduced to lowest terms.

20. Determine the number of distinct ways to arrange three nickels and four quarters in a line so that quarters are on both ends.

2017 SAA

Algebra II

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. $-\frac{65}{6}$ OR $\frac{-65}{6}$ (Must be this reduced improper fraction.)

2. 16

3. $-\frac{3}{2}$ OR $\frac{-3}{2}$ (Must be this reduced common fraction.)

4. $\frac{3}{2}$ (Must be this reduced improper fraction.)

5. 55

6. 10 (Must be this value only.)

7. 3

8. 66

9. 547

10. $\frac{5}{2}$ (Must be this reduced improper fraction.)

11. 14π (Must be this exact area.)

12. $\frac{8}{3}$ (Must be this reduced improper fraction.)

13. 24

14. 4,16 (Must have both answers, either order, "and", "or", or solution sets ok.)

15. 0.2155 OR .2155 (Must be this decimal.)

16. 32

17. 1188 or 1001

18. 1

19. $\frac{9}{2}$ (Must be this reduced improper fraction.)

20. 10 ("arrangements" or "ways" optional.)

1. (Always, Sometimes, or Never) For your answer, write the *whole word* "Always", "Sometimes", or "Never"—whichever is correct.

   $i = \sqrt{-1}$. Let a, b, c, and d represent real numbers. Then
   $\left|(a+bi)(c+di)\right| = \left(\left|a+bi\right|\right)\left(\left|c+di\right|\right)$.

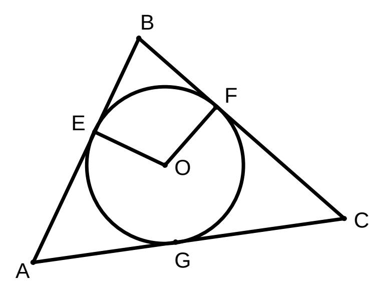
2. Let $k = \left(\log_{\pi} 8\right)\left(\log_{\sqrt{3}} \pi\right)\left(\frac{1}{2}\log_2 3\right)$. Determine the exact value of k.

3. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ represent vectors such that $\vec{a} = (3,2)$, $\vec{b} = (-8,13)$ and $\vec{c} = (16,-24)$. Determine *the sum* of the two vectors that are perpendicular. Express your answer as the ordered pair representation of the sum of those two vectors.

4. Let $f_0(x) = \frac{1}{1-x}$ and $f_n(x) = f_0\left(f_{(n-1)}(x)\right)$ for integers $n \geq 1$. Determine the value of $f_{2017}(2017)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. Determine the sum of the series $\sum_{n=1}^{\infty} \frac{1}{3^n}$, if it exists. Report your answer as an integer or as a common or improper fraction reduced to lowest terms, or "DNE" if the sum does not exist.

6. Circle O is inscribed in ΔABC with E, F, and G the respective points of tangency. $AB = 8$, $BC = 10$, and $CA = 12$. Determine the radian measure of $\angle EOF$. Express your answer as a decimal rounded to the nearest thousandth.



7. Determine the remainder when 2^{2017} is divided by 1023.

8. $Arc\tan(x)$ is expressed in radians and represents the inverse tangent function. Determine the smallest value in the *range* of $f(x) = 8\left(Arc\tan(x^2+1)\right)$.

9. At a party of married couples where every man is married to a woman, every woman is married to a man, and all are present, every man shakes hands with every other man and that man's wife. However, none of the wives shake each other's hands and no man shakes his own wife's hand. 459 handshakes occur. Determine how many people are at the party.

10. $f\left(x\right)=\begin{cases} ax^2+bx+1 & , \quad x<3 \\ 4 & , \quad x=3 \\ ax-\dfrac{b}{2} & , \quad x>3 \end{cases}$. Determine the ordered pair $\left(a,b\right)$ such that $f\left(x\right)$ is continuous at $x=3$.

11. Let θ be an acute angle such that $\sin\left(\dfrac{\theta}{2}\right)=\sqrt{\dfrac{x-1}{2x}}$. Then $\tan^2\theta=kx^2+wx+p$. Determine the sum $\left(k+w+p\right)$.

12. $k=\lim\limits_{x\to 0}\left(\dfrac{x+\tan x}{x}\right)$. Determine the value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms..

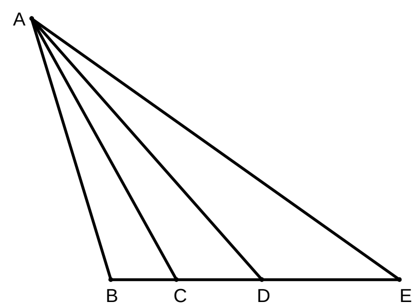
13. $i=\sqrt{-1}$. $f\left(x\right)=x^2+kx+w$ has real coefficients k and w. The zeros of $f\left(x\right)$ are r_1 and r_2 such that $r_1-r_2=2i$ and $\dfrac{1}{r_1}+\dfrac{1}{r_2}=\dfrac{3}{5}$. Determine the sum of all possible value(s) for k and w Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

14. A circular carnival Ferris wheel has a diameter of 20 meters with the lowest point at a ramp 5 meters off level ground. The Ferris wheel makes a complete revolution in 30 seconds. At time $t=0$, a point on the outside of the Ferris wheel is at its lowest height above ground. Determine, in meters, the height above the ground of that point on the wheel at 17 seconds. Express your answer as a decimal rounded to the nearest thousandth.

15. $5\sin(4x)+2\cos(4x)=a\sin(bx+c)$. Determine the exact value of the sum $(a+b)$.

16. Let n represent a positive integer such that $\frac{720}{n!}=6$. Determine the value of n.

17. C and D lie on $\overline{BE}$ such that $\overrightarrow{AC}$ and $\overrightarrow{AD}$ trisect $\angle BAE$ in ΔABE. $BC=2$, $CD=3$, and $DE=6$. Then the perimeter P of ΔABC may be expressed as $P=f+k\sqrt{w}+p\sqrt{q}$ in simplified and reduced radical form. Determine the sum $(f+k+w+p+q)$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.



18. Determine the length of the polar curve $r=\frac{4}{\sin\theta+\cos\theta}$ when $0\le\theta\le\frac{\pi}{2}$.

19. The equations of two planes in 3-space are given by $2x-4y+2z=-8$ and $x+y-z=0$. A parametric representation of the line of intersection of these two planes is $x(t)=-2+t$, $y(t)=at$, and $z(t)=b+ct$ for all real values of t. Determine the sum $(a+b+c)$.

20. Your Math Team Coach probably often says, “There is no i in ‘team’.” Since Coach is also a math teacher, Coach might say, “There is no $e^{ki\pi}$ in ‘team’.” (To a math teacher, $i=\sqrt{-1}$.) Given Euler’s formula: $e^{i\theta}=\cos\theta+i\sin\theta$, determine the smallest positive value of $(12k-100)$.

2017 SAA
Pre-Calculus

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. Always (Must be this whole word, capital A optional.)

2. 3

3. $(19,-22)$ (Must be this ordered pair.)

4. $\frac{2016}{2017}$ (Must be this reduced common fraction.)

5. $\frac{1}{2}$ (Must be this reduced common fraction.)

6. 1.696 (Must be this decimal, "radians" or "r" optional.)

7. 128 OR 2^7

8. 2π (Must be this exact answer.)

9. 36 ("people" optional.)

10. $(1,-2)$ (Must be this ordered pair.)

11. 0 or zero

12. 2

13. $\frac{40}{9}$ (Must be this reduced improper fraction.)

14. 24.135 (Must be this decimal, "meters" or "m" optional.)

15. $4+\sqrt{29}$ OR $\sqrt{29}+4$ (Must be this exact answer.)

16. 5

17. 23

18. $4\sqrt{2}$ (Must be this exact answer.)

19. 3

20. 2

NO CALCULATORS

1. The sum of the measures of the interior angles of a regular convex polygon is 3240 degrees. Determine the degree measure of one of the exterior angles of this polygon.

2. A plane parallel to the base of a right circular cone passes through the cone and divides the cone into a smaller cone and a frustum of a cone. The ratio of the numeric lateral surface area of the small cone to the numeric lateral surface area of the large cone is $4:9$. Determine the ratio of the numeric volume of the frustum to the numeric volume of the larger cone. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

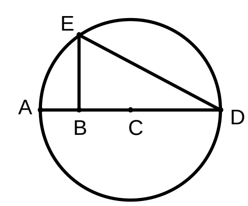
3. Determine the value of x so that $\frac{3x+1}{2}-(x+2)=\frac{5x}{4}$.

4. One print machine, working alone, can do a certain job twice as fast as another machine, working alone. The same job can be done in 16 minutes if the two machines work together at their own rates. Determine the number of minutes it takes the slower machine to complete the job alone.

5. A 2-foot wide cement sidewalk encloses a circular pond that has a diameter of 12 feet. The numeric surface area of the cement walk is $k\pi$. Determine the value of k.

6. $\frac{2\sqrt{3}}{3\sqrt{3}-1}$ can be written in the form $\frac{k+w\sqrt{p}}{q}$ when reduced and in simplest radical form with integers k, w, p and q and with $q>0$. Determine the sum $(k+w+p+q)$.

7. $\overline{AD}$ is a diameter passing through the center C of a circle. E lies on the circle and $\overline{EB}\perp\overline{AD}$. $EB=5$ and $ED=13$. Determine the length of the diameter of this circle. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.



8. Determine *__the sum__* of the solution(s) for x when $4x^2+14x+12=0$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

NO CALCULATORS

9. Let $R=\{a,b,c,d,e,f,g\}$. Determine the number of distinct subsets of R that exist in which a is a member and either b or c or both are members.

10. Determine the value of k when $\frac{8^{k+1}\left(4^{3k-1}\right)}{2^{2k-1}}=1$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

11. $A(-10,4)$, $B(2,-4)$, and $C(-4,6)$ form ΔABC. Determine the coordinates of the point that is the centroid of this triangle. Express your answer as an ordered pair (x,y).

12. Three segments have lengths $x+5$, $2x-3$, and 12. A value for x is chosen from the interval $[-10,30]$ at random. Determine the probability the three segment lengths will form a triangle. Express your answer as a common fraction reduced to lowest terms. (Note: x in $[-10,30]$ means $-10\le x\le 30$.)

13. Line ℓ has equation $2x+3y=18$, line m has equation $6x+ky=28$, and line n has equation $6x+wy=15$. Lines ℓ and n are parallel while line m is perpendicular to line ℓ. Lines ℓ and m intersect at point P and lines m and n intersect at point Q Determine the length of $\overline{PQ}$.

14. The first 25 terms of the Fibonacci sequence are 1, 1, 2, 3, 5, ..., 28657, 46368, 75205. (The sequence where the first two terms are each one and each succeeding term is the sum of the previous two terms.) Determine the sum of the first 23 terms of the Fibonacci sequence.

15. If $4^{42}(2^{95}-1)+(2^{43}-3)$ is computed and the answer is written in base two, compute the number of 0's that would appear in the base two representation of the answer. (Report your answer as a base 10 integer.)

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

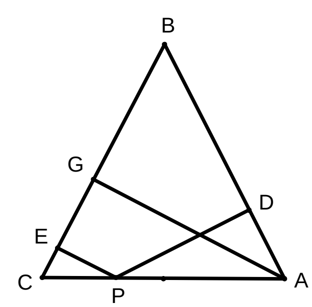
NO CALCULATORS

16. Alfred and Dan are friends and live in houses in a city whose surface is completely flat. For Dan to get home from Alfred's house, he must ride his bike 960 feet due north and then 400 feet due west. Find the number of feet in the straight line distance from Dan's house to Alfred's house.

17. A chemist has a container that is filled with a thoroughly mixed liquid mixture consisting of 2 ounces of water and 8 ounces of acid. She pours out $\frac{1}{4}$ of this mixture, replaces it with pure water to completely refill the container and thoroughly mixes the resulting liquid. She then pours out $\frac{3}{4}$ of this new mixture, replaces it with pure acid to completely refill the container and again thoroughly mixes the resulting liquid. This final resulting liquid contains $k\%$ acid. Determine the value of k. Report the value of k only. Do not use "%" in your answer.

18. The equation of the line tangent to $x^2 - 2x + y^2 + 6y - 15 = 0$ at the point $(-2,1)$ can be written in general form as $Ax + By + C = 0$ where A, B, and C are relatively prime integers and $A > 0$. Determine the sum $(A + B + C)$.

19. In isosceles triangle ΔABC with $\overline{AB} \cong \overline{BC}$, P lies on $\overline{AC}$, E lies on $\overline{BC}$ and D lies on $\overline{AB}$ such that $\overline{PE} \perp \overline{BC}$ and $\overline{PD} \perp \overline{AB}$. G lies on $\overline{BC}$ such that $\overline{AG}$ is an altitude of ΔABC. $EG = 8$, $PA = 17$, and $AG = 18$. Determine the length of $\overline{EP}$.



20. ***(Multiple Choice)*** For your answer write the *capital letter* which corresponds to the correct choice(s).

The locus of points in a plane that are equidistant from two parallel lines is/are:

A) two parallel lines B) a line C) a point D) two points
E) three points F) four points G) a circle

Note: Be certain to write the correct capital letter(s) as your answer.

NO CALCULATORS NO CALCULATORS NO CALCULATORS

2017 SAA

Fr/So 8 Person

School _____ANSWERS_____

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 18 ("degrees or "°" optional.)

2. $\frac{19}{27}$ (Must be this reduced common fraction.)

3. -2

4. 48 ("minutes" or "min." optional.)

5. 28

6. 26

7. $\frac{169}{12}$ (Must be this reduced improper fraction.)

8. $-\frac{7}{2}$ OR $\frac{-7}{2}$ (Must be this reduced improper fraction.)

9. 48 ("subsets" optional.)

10. $-\frac{2}{7}$ OR $\frac{-2}{7}$ (Must be this reduced common fraction.)

11. $(-4, 2)$ (Must be this ordered pair.)

12. $\frac{5}{12}$ (Must be this reduced common fraction.)

13. $\sqrt{13}$ (Must be this exact answer.)

14. 75024 or 75204

15. 42 OR 42_{ten} OR 42_{10}

16. 1040

17. 90 (Must be this answer without "%" sign.)

18. 9

19. 3

20. B (Must be this capital letter only.)

NO CALCULATORS

1. Let $f(x) = 20x + 17$ and $g(x) = 17x + 20$. Determine the value of $f \circ g(5)$.

2. Patty Pixel has 15 distinct pictures to print, one copy per picture. She has a maximum of $5 to spend. Large pictures cost $0.80 per print and small pictures cost $0.20 to print. Patty will print as many of the pictures in the large format as possible. Determine the greatest number of large prints Patty can print while staying within her budget. (Ignore sales tax.)

3. Let $5^{(3x+1)} = 625$. Determine the exact value(s) for x.

4. Let $k = \lim\limits_{x \to 0} \dfrac{\sqrt{3} - \sqrt{3 - x}}{x}$. Determine the exact value of k.

5. Let $k = 5 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1}}}}$. Determine the exact value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

6. Let A represent the area common to the polar curves $r = 2$ and $r = 4\cos\theta$ over the domain $[0, 2\pi]$ for θ. Then $A = \dfrac{k\pi + w\sqrt{p}}{q}$ when written as a single rational expression in reduced and in simplified radical form with k, w, p, and q integers, and $q > 0$. Determine the value of A. Express your answer as $\dfrac{k\pi + w\sqrt{p}}{q}$ in simplified radical form and completely reduced.

7. Let n be an integer such that $n > 7$. Determine the least possible value of n such that the three consecutive integers n, $n+1$, and $n+2$ are divisible, respectively, by 5, 6, and 7.

NO CALCULATORS

8. $\dfrac{7x^2+x+2}{x^3+x^2+x+1}=\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x+1}$ where A, B, and C are integers. Determine the ordered triple (A,B,C). Express your answer as that ordered triple.

9. The exact value for $\sin(-195°)$ may be written as a single reduced rational expression in simplified radical form as $\dfrac{k\sqrt{w}+p\sqrt{q}}{f}$ where k, w, p, q, and f are integers with $f>0$. Determine the sum $(k+w+p+q+f)$.

10. Let $A=a_1,a_2,a_3,a_{4,\ldots}$ represent an arithmetic sequence. Let $F = f_1, f_2, f_3, f_4,\cdots$ represent a Fibonacci-type sequence in which every term after the first two is the sum of the previous two terms. $S=10,15,23,33,48,70,104,\cdots$ is a sequence where each term is the sum of the corresponding sequence terms from A and F. That is, for all positive integers n, $s_n = a_n + f_n$ is an element of the sequence S. Determine a_{20}, the 20th term of the arithmetic sequence A.

11. Let $k=\cos(0)+\cos\left(\dfrac{\pi}{8}\right)+\cos\left(\dfrac{\pi}{4}\right)+\cos\left(\dfrac{3\pi}{8}\right)+\cdots+\cos(250\pi)$. Determine the exact value of k.

12. Let $k=\lim\limits_{x\to 0}\dfrac{\tan\left(\frac{2}{3}x\right)}{\left(\frac{4}{5}x\right)}$. Determine the exact value of k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

13. Let k and w represent the two (not necessarily distinct) roots of $x^2-7x+11=0$. Determine the exact value of (k^4+w^4).

14. Let $f(x)=3x^4-8x^3+6x^2+2$. Determine the <u>*sum*</u> of the absolute minimum and the absolute maximum for this function over the interval $[-1,2]$.

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

NO CALCULATORS

15. Let (k,w) represent a point of intersection of the graphs of $y = x^3 - 9$ and $y = -2x^2 + 5x + 1$. Determine the sum of all possible value(s) of w.

16. One of the following 8 expressions below is chosen at random. Determine the probability the chosen expression is equivalent to the expression $\log_4(6x^2)$ for all $x > 0$. Express your answer as a common fraction reduced to lowest terms. The possible 8 expressions are:

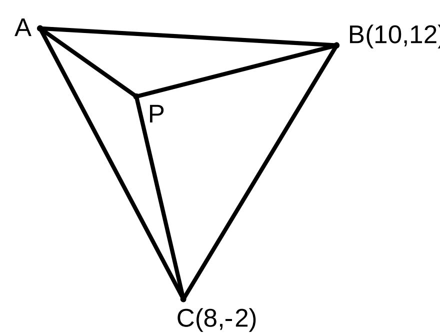
(A) $\log_4 6 + 2\log_4 x$ (B) $2\log_4(6x)$ (C) $\log_4 24 + \log_4 x^2 - 1$

(D) $\frac{1}{2}\log_2(6x^2)$ (E) $\log_4 2 + \log_4 3 + 2\log_4 x$ (F) $\log_4 3 + 2\log_4 x + \frac{1}{2}$

(G) $\log_4 3 + 1 + 2\log_4 x^2$ (H) $2(\log_4 x + \log_4 6) - \log_4 3$

17. Let $k = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \cdots$. Determine the exact value for k. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

18. In the diagram (not necessarily drawn to scale) with ΔABC and coordinates $B(10,12)$, $C(8,-2)$, and point P in the interior of the triangle, the ratio of the area of ΔPAB to the area of ΔPBC to the area of ΔPAC is $1:4:5$. Determine the **ordered pair** that represents point A. when the **ordered pair** that represents point P is $(7,11)$.



19. Water is pumped into a right cylindrical tank with a radius of 10 feet and a height greater than 2 feet. The water is pumped in at a rate of 25π cubic feet per minute. At the instant the height of the water in the tank is 2 feet, determine the rate of change, in feet per minute, of the height of the water in the tank. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

20. A quadratic equation has roots such that the sum of the roots is 2 and the sum of the squares of the roots is 52. Determine the exact product of these roots.

NO CALCULATORS **NO CALCULATORS** **NO CALCULATORS**

2017 SAA

School ANSWERS

Jr/Sr 8 Person

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 2117
2. 3 ("pictures" or "prints" optional.)
3. 1 (Must be this value only.)
4. $\frac{\sqrt{3}}{6}$ OR $\frac{1}{6}\sqrt{3}$ (Must be this exact value.)
5. $\frac{225}{43}$ (Must be this reduced improper fraction.)
6. $\frac{8\pi - 6\sqrt{3}}{3}$ (Must be this exact answer.)
7. 215
8. $(3,-2,4)$ (Must be this ordered triple.)
9. 12
10. 62
11. 1
12. $\frac{5}{6}$ (Must be this reduced common fraction.)
13. 487
14. 21
15. -35
16. $\frac{5}{8}$ (Must be this reduced common fraction.)
17. $\frac{3}{4}$ (Must be this reduced common fraction.)
18. $(3,13)$ (Must be this ordered pair only.)
19. $\frac{1}{4}$ ("feet per minute" or ft/min optional.)
20. -24

Note: All answers must be written legibly. Exact answers are required unless otherwise specified in the question. Answers must be simplified and in the specific form if so stated. Except where noted, angles are in radians. No units of measurement are required.

1. A particular automobile depreciates in value at the rate of 32% the first year and half as much each year following. (That is, 16% of the remaining value for year two, 8% of the remaining value for year three, and so forth.) This auto had an original value of $27,500.00. Determine the value that remains after 5 years depreciation. Express your answer in dollars rounded to the nearest cent.

2. Determine the value of k when $\log_{20} k + \log_{17} k = \log_{20} 17$. Express your answer as a decimal rounded to the nearest thousandth.

3. At last year's pro football combine, a prospective quarterback ran a 40 yard dash in 4.56 seconds. Assume a steady rate. Determine this prospect's speed, in miles per hour. Express your answer as a decimal rounded to four significant digits.

4. Determine the numeric volume of the solid formed by revolving the region enclosed by the graph of $y = 17 - |8 + 2x|$ and the x-axis about the x-axis. Express your answer as a decimal rounded to the nearest thousandth.

5. A triangle has sides of lengths 13, 15, and 18. Determine the length of the altitude to the longest side of this triangle. Express your answer as a decimal rounded to the nearest thousandth.

6. Determine the largest solution for x when $(\ln x)^2 - 3\ln x = \ln 10$. Express your answer as a decimal rounded to four significant digits.

7. A circle with center (h,k) and radius of length r passes through the points $(2,9)$, $(-2,1)$, and $(-3,4)$. Determine the ordered triple (h,k,r^2). Report your answer as that ordered triple.

8. A coin is weighted so that the probability of any toss being a head is k. If the probability that the coin will turn up heads all three times when this coin is tossed three times is less than 0.4688, determine the largest possible value of k. Express your answer as a decimal rounded to four significant digits.

9. Let $f(x) = -3\times10^{-9}x^4 + 1.3\times10^{-5}x^3 - 0.016x^2 + 4.62x + 850$ and $g(x) = 4000 - \frac{8,000,000}{x+2000}$. Determine *the number of integers* x for which $f(x) > g(x)$. Express your answer as an integer.

10. Determine the area of a regular polygon with 31 sides if each side has a length of 5. Express your answer as a decimal rounded to the nearest thousandth.

11. A cubic function of the form $y = x^3 + bx^2 + cx + d$ has zeros at 8, 15, and 17. Determine the sum of all possible values of y when x is a positive integer less than 27 and y is positive. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

12. The angle of elevation from a point A on horizontal level ground to the top of a 60.31 foot vertical flagpole is $32.33°$. Determine the number of feet in the distance from A to the bottom of the flagpole. Express your answer as a decimal rounded to four significant digits.

13. Determine the numeric area of the largest rectangle inscribed under the curve $y = e^{\left(-x^2\right)}$ and in the first and second quadrant. (That is, two vertices on the curve and one side lies on the x-axis.) Express your answer as a decimal rounded to four significant digits.

14. Determine the smallest positive integer y such that all the zeroes for x of the polynomial: $x^3 - yx^2 - 135x$ are integers and such that all the zeroes for w of the polynomial $3w^2 - 2yw - 135$ are integers. Express your answer as that integer.

15. In a clinic, 20% of the patients have the bird flu. A new test for this flu has been developed that gives a positive result 98% of the time if the patient has the flu and 40 % of the time if the patient does not have the flu. Patient "A" gets a positive test result. Determine the probability Patient "A" actually has the flu. Express your answer as a decimal rounded to four significant digits.

16. Tom, an out-of-shape old man, and Tim, a fit young man, are one mile apart on a straight road. They begin at the same time and walk toward each other. Tim's constant rate is 4 miles per hour. Tom's constant rate is 2 miles per hour, but he stops to rest for one minute after each 3 minutes of walking. Determine the number of minutes after starting that it will take for them to meet. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

17. Let $a_1 = 0.0001$, $a_2 = 0.0002$, and $a_3 = 0.0003$. For integers $n \geq 4$, $a_n = a_{n-3} + a_{n-2} + a_{n-1}$. Determine the value of $\sum_{n=1}^{350} a_n$. Express your answer rounded to four significant digits and in scientific notation.

18. Determine the solution(s) (x, y), when $y > 0$, for the system $\begin{cases} 2x + 3y^2 = 17 \\ 5x^2 + 2y = 12 \end{cases}$. Express your answer as those ordered pair(s) with decimal entries rounded to hundredths.

19. Determine the first five digits (first five digits of the number reading left to right) in the integer 2017^{2017}. Express your answer as a five digit integer with the correct digits left to right.

20. Determine the distance between the planes $5.27x - 3.19y + 4.11z + 23.52 = 0$ and $5.27x - 3.19y + 4.11z - 18.11 = 0$. Express your answer as a decimal rounded to four significant digits.

2017 SAA

Calculator Team

School ANSWERS

(Use full school name – no abbreviations)

______ **Correct X 5 pts. ea. =** [ ]

Note: All answers must be written legibly. Exact answers are required unless otherwise specified in the question. Answers must be simplified and in the specific form if so stated. Except where noted, angles are in radians. No units of measurement are required.

1. 13595.84 ("dollars or "$" optional.)
2. 3.963 (Must be this decimal.)
3. 17.94 (Must be this decimal, "miles per hour" or "mph" optional.)
4. 5144.882 (Must be this decimal.)
5. 10.657 (Must be this decimal.)
6. 37.85 (Must be this decimal.)
7. $(2,4,25)$ (Must be this ordered triple.)
8. 0.7768 OR .7768 (Must be this decimal.)
9. 804 (Must be this integer, "integers" optional.)
10. 1905.299 (Must be this decimal.)
11. 6278 (Must be this integer.)
12. 95.29 (Must be this integer, "ft." or "feet" optional.)
13. 0.8578 OR .8578 (Must be this decimal.)
14. 6 (Must be this integer.)
15. 0.3798 OR .3798 (Must be this decimal.)
16. $\frac{32}{3}$ (Must be this reduced improper fraction, "minutes" optional.)
17. 4.822×10^{88} (Must be this decimal and in scientific notation.)
18. $(-1.18, 2.54)$, $(1.23, 2.20)$ (Must have both ordered pairs in either order, with these decimals.)
19. 39065 (Must be this integer, comma use optional.)
20. 5.622 (Must be this decimal.)

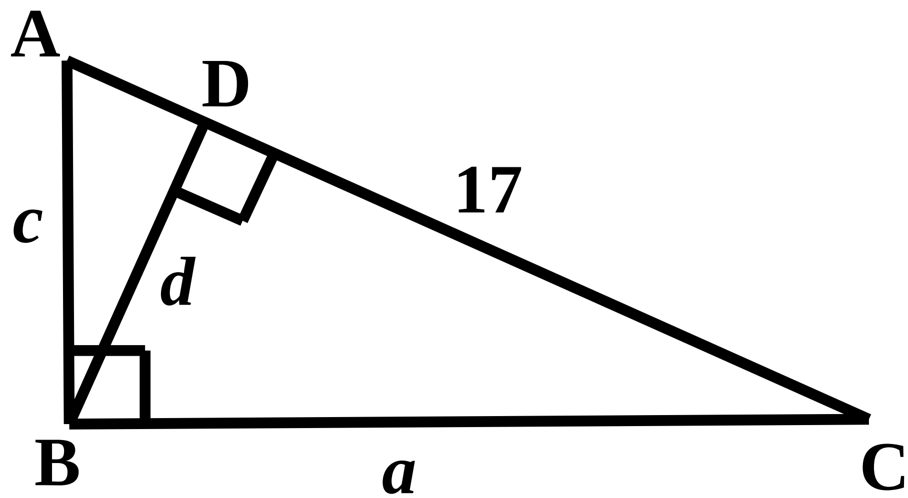
ICTM Math Contest

Freshman – Sophomore

2 Person Team

Division AA

1. Determine the sum of all distinct positive integers between 8 and 16 inclusive that can be expressed in one and only one way as the sum of at least two consecutive positive integers.

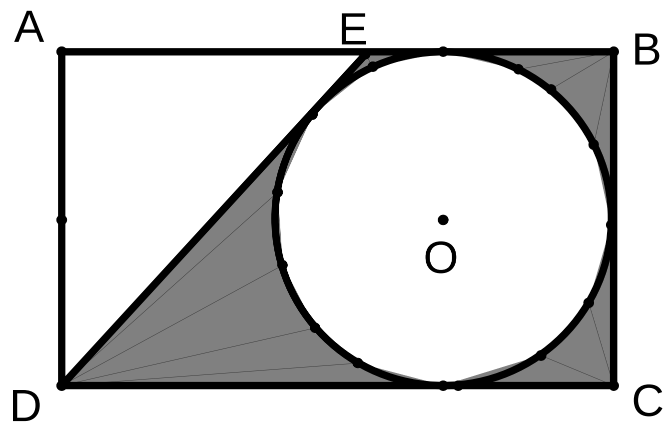


2. In right ΔABC with $\overline{AB} \perp \overline{BC}$ and D on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$, $CD = 17,\ AC = b = 20$ and $BD = d$. Determine the value of $\left(a^2 + b^2 + c^2 + d^2\right)$.

3. Line ℓ is parallel to the line $2x+6y=15$ and contains the point $(-2,-11)$. Line m is perpendicular to the line containing points $(-3,5)$ and $(-2,10)$ and passes through the point $(-2,3)$. The point (k,w) is the point of intersection of lines ℓ and m. Determine the sum $(k+w)$.

4. Determine the area of ΔABC with coordinate points $A(0,0)$, $B\left(-\sqrt{16},\frac{80}{16}\right)$ and $C\left(12^2-\left[2\sqrt{35}\right]^2,\sqrt[4]{625}\right)$.

5. Point O is the origin. Points A and B are the coordinates of the x-intercept and y-intercept for the graph of $7x+24y-168=0$. The area inside the circle containing O, A, and B, but outside ΔOAB is $\dfrac{k\pi+w}{p}$ where k, w, and p are relatively prime integers and $p>0$. Determine the value of $(k+w+p)$.



6. $ABCD$ is a rectangle with $AD = 6$ and E lies on $\overline{AB}$ so that $\angle AED = 45°$. Circle O is inscribed in trapezoid $EBCD$. Determine the numeric area of the shaded region. Express your answer as a decimal rounded to four significant digits.

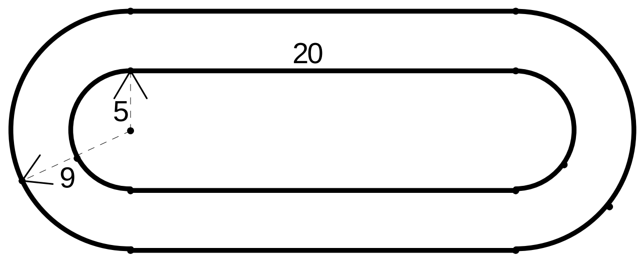
7. Let (k, w) be the unique solution to the system

$$\begin{cases} 5x + 2y = 10.71 \\ 13x - y = 38.51 \end{cases}$$

Let p° be the smallest angle of a triangle whose angles are in the ratio $210:1024:2017$.

Determine the sum $(k + w + p)$. Express your answer as a decimal rounded to the nearest thousandth.

8. Let $k = 3a(4b)$ when $3ab$ equals $(2ab - 2)$. Let w equal $4y$ times $2y$ when x equals 13 and y equals $2x$. Determine the sum $(k + w)$.



9. A slot car track with two slots that are formed by two rectangles with length 20 meters and semi-circular ends of radii 5 and 9 meters as shown. Two cars start and stop at the same time and place and run a complete lap. Let k be the additional distance traveled by the car in the outer slot. The rate of travel of the outside car is w times the rate of the car on the inside slot. Determine the sum $(k+w)$ as a decimal rounded to four significant digits

10. A sequence of concentric circles is formed such that each whole circle has ***area*** in the ratio $25:9$ when compared to the previous whole circle. The circle represented by $x^2 + y^2 + 6x - 4y - 2011 = 0$ is the 5th such circle. Determine the ***radius*** of the first circle. Report your answer as a decimal rounded to the nearest thousandth of a unit.

2017 SAA

School **ANSWERS**

Fr/So 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer		**Score** (to be filled in by proctor)
1. 60		
2. 851		
3. –83		
4. 20		
5. 293		
6. 15.18	(Must be this decimal.)	
7. 12.737	(Must be this decimal.)	
8. 5384		
9. 26.48	(Must be this decimal.)	
10. 5.831	(Must be this decimal.)	

TOTAL SCORE: ____________

(*enter in box above)

Extra Questions:

11. ____________

12. ____________

13. ____________

14. ____________

15. ____________

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

ICTM Math Contest

Junior – Senior

2 Person Team

Division AA

1. Let $k = \log_8 8192$ and $w = \sum_{n=1}^{\infty} 3\left(\frac{1}{9}\right)^n$. Determine the product (kw). Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2. For the system of equations $\begin{cases} 3x - Ay = 3 \\ -4x + 5y = -1 \end{cases}$, let k be the value of A such that the system is inconsistent.
Let w be the value of A such that this system represents perpendicular lines.
Determine the product (kw).
Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

3. $g(x)$ is the polynomial whose zeros are the negative reciprocals of the zeros of $f(x) = 3x^3 + x^2 - 4x + 2.$

Let k be the product of the zeros of $g(x)$.

Let w be the value $\lim_{x\to\infty}\left[\log_3(4x+5) - \log_3(12x-7)\right]$

Determine the sum $(k + w)$.

Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. Determine the *sum* of the distinct real root(s) for x, $x \neq 0$, when

$$\left(x+\frac{1}{x}\right)^4 - 8\left(x+\frac{1}{x}\right)^3 + 24\left(x+\frac{1}{x}\right)^2 - 32\left(x+\frac{1}{x}\right) + 16 = 0$$

Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

5. Consider angles in degree measure θ, $0 \le \theta < 360°$. Let k be the sum of all such θ when $10\cos^2\theta - 9\cos\theta + 2 = 0$. Let w be the sum of all such θ when $10\sin^2\theta - 9\sin\theta + 2 = 0$. Determine the sum $(k + w)$.

6. Let k and w be positive integers such that $k \geq 1$ and $w > 1$. Let $A = \{2, 3, 4, 5, \cdots, n, \cdots, 299\}$ Determine the *sum* of all **distinct** members of A that are in the form kw^3.

7. The hyperbolas $x^2 - \frac{y^2}{9} = 1$ and $y^2 - x^2 = 1$ meet in four points that determine a convex quadrilateral. Determine the exact perimeter of this quadrilateral.

8. In degree mode, let $s(x) = Sin^{-1}x$ (the inverse sine function) and $t(x) = \tan x$.

Determine the exact value of $s\left(t\left(s\left(t\left(s\left(t(30)\right)\right)\right)\right)\right)$.

9. A van with n teens, $n > 3$, ordered the following items at a fast-food drive-thru: x sandwiches at \$3.50 each, where $n \le x \le 2n$; y orders of French fries at \$1.80 each, where $0 < y \le n$; and n drinks at \$2.70 each. The total bill, not including tax, was \$60.20. Determine the sum of all possible integral values of n.

10. Determine the least area of a rectangle with horizontal and vertical sides that encloses the entire graph of the polar equation $r = 20(1-2\cos\theta)$ with θ in radians. Express your answer rounded to the nearest integer.

2017 SAA

School ANSWERS

Jr/Sr 2 Person Team

(Use full school name – no abbreviations)

Total Score (see below*) = ☐

NOTE: Questions 1-5 only are *NO CALCULATOR*

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer		**Score** (to be filled in by proctor)
1.	$\frac{13}{8}$	(Must be this reduced improper fraction.)	
2.	-9		
3.	$\frac{1}{2}$	(Must be this reduced common fraction.)	
4.	1		
5.	1080	("degrees" or "°" optional.)	
6.	7565		
7.	$6+2\sqrt{5}$ OR $2\left(3+\sqrt{5}\right)$ OR $2\sqrt{5}+6$ OR $2\left(\sqrt{5}+3\right)$	(Or exact simplified equivalent.)	
8.	90	("degrees" or "°" optional.)	
9.	14		
10.	4400	(Must be this integer.)	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. ______

12. ______

13. ______

14. ______

15. ______

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

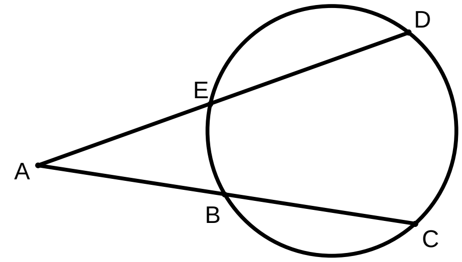
Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

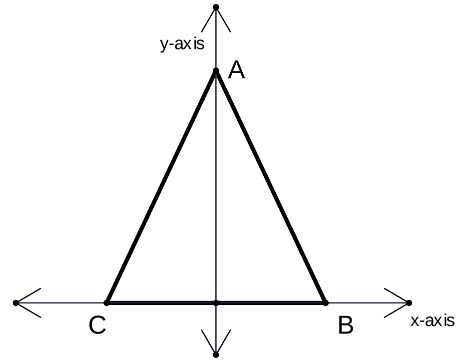
1. All values are in dollars. The value of a mug is one-third the value of a jug. The value of a jug is equal to four-fifths the value of a pot. A pot costs \$15. Determine the number of dollars for the cost of a mug.

2. Let $k = ANS$. The numeric area of rectangle $ABCD$ is 16. $AD = k - 2$. Determine the numeric perimeter of rectangle $ABCD$.

3. Let $k = ANS$. In the diagram shown, but not drawn to scale, B , C , D , and E lie on the circle. $AE = k$, $ED = 7$, and $AB = 15$. Determine the length of $\overline{BC}$.



4. Let $k = ANS$. Isosceles ΔABC (not drawn to scale) has vertex A on the positive y-axis and base $\overline{BC}$ on the x-axis. $BC = 4$ and the altitude of ΔABC from A has length k. ΔABC is rotated $180°$ about the y-axis. The numeric volume of the resulting three-dimensional solid is $w\pi$. Determine the value of w.



1. Determine the exact larger solution for $2x^2 - 7x - 5 = 0$.

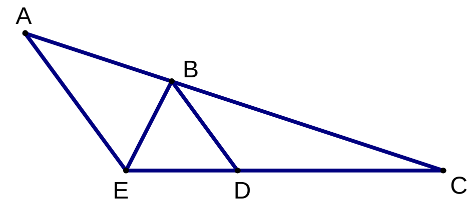
2. $ANS = \dfrac{a+\sqrt{b}}{c}$ or can be simplified to that form. Let $k = a+b+c$. Determine ***the number*** of prime number factors of k.

3. Let $k = ANS$. In rectangle $ABCD$, diagonal $BD = k$ and $\angle DBC = 30°$. Determine the exact numeric area of rectangle $ABCD$.

4. Let $k = ANS$. Each side of a regular hexagon has length k. Determine the exact numeric area of this regular hexagon.

1. Determine the slope of the line perpendicular to the line whose equation is $3\left(x-y-19\right)=16y$. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2. Let $k = ANS$. y varies jointly as x and $\sqrt{z}$. $y = 38$ when $x = k$ and $z = 9$. Determine the value of y when $x = -7$ and $z = 16$.



3. Let $k = ANS$. In ΔACE, $\overline{AE} \parallel \overline{BD}$, $\overrightarrow{EB}$ bisects $\angle AEC$, $EC = k$, and $AE = 42$. Determine the length DC.

4. Let $k = ANS$. The line $x + y = 4$ is tangent to a circle given by the equation $\left(x-9\right)^2 + \left(y-3\right)^2 = k$. Determine the exact y-coordinate of the point of tangency.

1. Sally's rectangular garden is twice as long as it is wide. The width of the garden is 6 feet. Determine the number of feet in the perimeter of the garden.

2. Let $k=\sqrt{ANS}$. Determine the solution (x,y) for the system $\begin{cases} 40x - 3y = 21 \\ ky - 76x = 18 \end{cases}$. Report as your answer the y-coordinate of this ordered pair (x,y).

3. Let $k = ANS + 16$. A certain convex polygon has k diagonals. Determine the number of sides in this polygon.

4. Let $k = ANS$. In an equilateral triangle with sides of length k, the radius of the inscribed circle is $\frac{x\sqrt{y}}{z}$ when completely reduced and written in simplified radical form where x, y, and z are integers. Determine the sum $(x+y+z)$.

1. It takes 5 minutes for Will to work 2 similar calculus problems. At this rate, determine the number *of seconds* it will take him to work 8 similar calculus problems.

2. Let $k = ANS$. Let $100x^2 + 700x + k = 0$. Determine the sum and the product of the solutions for this equation. Report as your answer the smaller of these two quantities.

3. Let $k = |ANS|$. k is the difference between the squares of the lengths of the two legs of a right triangle. k is also the sum of lengths of these same two legs. Determine the numerical area of this triangle.

4. Let $k = ANS$. The apothem of a regular hexagon is k units long. Determine the exact numeric area of this hexagon.

2017 SAA FR/SO RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. 4 ("dollars" or "$" optional.)
2. 20
3. 21
4. 28

ROUND 2

1. $\frac{7+\sqrt{89}}{4}$ OR $\frac{7}{4}+\frac{\sqrt{89}}{4}$ (Or exact simplified and reduced equivalent.)
2. 2
3. $\sqrt{3}$ (Must be this exact answer.)
4. $\frac{9\sqrt{3}}{2}$ OR $4.5\sqrt{3}$ (Must be this answer or simplified and reduced equivalent.)

ROUND 3

1. $-\frac{19}{3}$ OR $\frac{-19}{3}$ (Must be this reduced improper fraction.)
2. 56
3. 32
4. -1

EXTRA ROUND 4

1. 36
2. 193
3. 22
4. 17

EXTRA ROUND 5

1. 1200 (“seconds” or “sec.” optional.)
2. -7
3. 6
4. $72\sqrt{3}$ (Must be this exact answer.)

1. Determine the smallest solution for the equation $x^3 + 5x^2 - x - 5 = 0$.

2. Let $k = ANS$. Determine the exact numerical area of the polygon enclosed by $\begin{cases} -2 \le x \le 3 \\ |y| \le |k| \end{cases}$.

3. Let $k = ANS$. Let n be a positive integer. The sum of the first n positive odd integers is $2(k)$. Determine the value of n.

4. Let $k = ANS$. On January 1, 2017, Jeff took out a Certificate of Deposit (CD) with a deposit of $\$200.00$. The CD was for a term of k years and earned an Annual Percentage Rate of interest of 3% compounded annually. Determine the value of the CD at the end of k years. Report your answer in standard dollars & cents notation rounded to the nearest cent.

1. Determine the exact value of $\left[\left(3^{\frac{1}{3}}\right)\left(2^{\frac{3}{2}}\right)\left(2^{\frac{5}{2}}\right)\left(3^{\frac{5}{3}}\right)\right]^{\frac{1}{2}}$.

2. Let $k = ANS$. Determine the value for x such that $\log(5x-12)-\log(x-k)=\log(x+6)$.

3. Let $k = ANS$. Determine the degree measure for θ, $0° \le \theta \le 180°$, so that $\sin(\theta + k°) = \cos(180° - \theta)$. Report your answer as a decimal rounded to the nearest tenth of a degree.

4. Let $k = ANS$. Let $f(x) = e^{(x+2)}$. Determine the value of $f^{(-1)}(k)$. Express your answer as a decimal rounded to the nearest thousandth.

1. Determine the smallest integer in the domain of $f\left(x\right)=\sqrt{2x+3}+\sqrt{2-3x}$.

2. Let $k = ANS$. Let $y = k$ be the equation of the directrix of a parabola Determine the coordinates (x, y) of the vertex of the parabola if the focus of the parabola is $(3, -5)$. Report your answer as the ordered pair (x, y).

3. $ANS = (a, b)$. In ΔABC, $A = (-1, 4)$, $B = (9, 11)$, and $C = (a, b)$. Determine the degree measure of $\angle BAC$. Report your answer rounded to the nearest degree.

4. Let $k = ANS$. In ΔABC , $AB = k$, $AC = 50$, and $\angle BAC = 56°$. In ΔACD , $\angle CAD = 62°$ and $\angle ACD = 45°$. $\angle BAD = 118°$. Determine the numerical area of polygon $ABCD$. Report your answer rounded to the nearest integer.

1. Let $A=\begin{bmatrix} 2.5 & -1.5 \\ 2 & -1 \end{bmatrix}$. Determine the value of the determinant of A^{-1}. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

2. Let $k = ANS$. Determine the ordered triple that is the solution for the system: $\begin{cases} x - y + 2z = 7 \\ 3x + 2y - z = -10 \\ -x + 3y + z = -k \end{cases}$.

3. ANS will be an ordered triple of real numbers, (a,b,c). Point P has coordinates $(2,3,4)$, point A has coordinates $(x,4,-2)$, point O has coordinates $(0,0,0)$, and point B has coordinates (a,b,c). Determine the value of x such that the vectors equivalent to $\overrightarrow{PA}$ and $\overrightarrow{OB}$ are perpendicular. Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. Let $k = |ANS|$. All angles are in radians and all inverses are the respective inverse functions. Determine the exact value of the expression

$$k\left[\left(\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)-\left(\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)+\left(\text{Tan}^{-1}(-1)\right)-\left(\text{Cot}^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)\right].$$

1. Aune, Barb, Cindy and Ella are going to sit at a large, empty, circular table for math team practice. Determine the number of distinct ways they can be seated around the table.

2. Let $w = ANS$. $f(x) = \sqrt{(x+1)(x+2)(x+3)+25}$ and $g(x) = x(x-3)+(x+1)(x-2)$.

Determine the value of the expression $\dfrac{22(f(w))}{g(w)}$.

3. Let $k = ANS$. Determine the value of x if matrix $A = 3\begin{bmatrix} k & 1 \\ 2 & -3 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ 2 & x \end{bmatrix}$ and the determinant of A is 37 (alternately written $\det A = 37$.) Express your answer as an integer or as a common or improper fraction reduced to lowest terms.

4. Let $w = |ANS|$. Determine the value of the sum $\sum_{x=10}^{100}(4x - w)$.

2017 SAA JR/SR RELAY COMPETITION PROCTOR ANSWER SHEET

ROUND 1

1. -5
2. 50
3. 10
4. 268.78 (Must be this decimal, "$" or "dollars" optional.)

ROUND 2

1. 12
2. 15
3. 127.5 (Must be this exact decimal, "°" or "degrees optional.)
4. 2.848 (Must be this decimal.)

ROUND 3

1.
1. -1 (Must be this integer.)
2. $(3,-3)$ (Must be this ordered pair.)
3. 95 ("°" or "degrees optional.)
4. 2785 (Must be this integer.)

EXTRA ROUND 4

1. 2
2. $(-1,-2,3)$ (Must be this ordered triple.
3. -18
4. -18π (Must be this exact value.)

EXTRA ROUND 5

1. 6 ("ways" optional.)
2. 11
3. -5 (Must be this integer.)
4. 19565