2025 SA	Name_	ANSWERS
Algebra I	School	
Correct X 2 pts. ea. =		(Use full school name – no abbreviations)

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1	27	11	705	
2	$\frac{9}{2}$	12	6	
3	60	13	8	
4	- 3	14	300	
5	4	15	20	
6	13	16	$\frac{19}{31}$	
7	(4, 6)	17	- 52	
8	$\frac{15}{2}$	18	154	
9	13	19	(1, 5, 2)	
10	-10	20	11, 12	(Must have both values, either order.)

I.C.T.M. 2025 State Algebra 1 Solutions - Divisions 1A, 2A

- 1.  $2x+10 = 4+3x-21 \Longrightarrow x = 27$ .
- 2.  $2^{6+12} = 2^{4k} \implies k = \frac{18}{4} = \frac{9}{2}$ .

3. Let x be Joe's number of cards. Then Jack has  $\frac{3}{4}x$  and Jesse has  $\frac{3}{4}x + 45$ . Then we

have  $x = \frac{2}{3} \left( \frac{3}{4} x + 45 \right) = 30 + \frac{1}{2} x$ . This gives x = 60.

4.  $\frac{k}{3} + \frac{4}{5} = \frac{5k+12}{15}$  and this is less than  $-\frac{1}{90}$ . Then  $5k+12 < -\frac{15}{90} = -\frac{1}{6}$ , so  $5k < -\frac{1}{6} - \frac{72}{6} = -\frac{73}{6}$ . Then  $k < -\frac{73}{30}$ . The largest integer x is thus -3.

5. 
$$(x+3)^2 - (x+5)(x+1) = k \Rightarrow x^2 + 6x + 9 - (x^2 + 6x + 5) = k$$
, so report 4 for k

- 6.  $3k + 4 = 25 \implies k = 7$ . Then 2k 1 = n = 13.
- 7. The first equation gives 2 = -a + b. The second gives 2 = 2a b. Add to get a = 4 and b = 6. Report (4, 6).
- 8.  $-\frac{k}{3} = \frac{-1}{2/5} = -\frac{5}{2}$ , so  $k = \frac{15}{2}$ .
- 9.  $|x+4| \le 6 \Rightarrow -6 \le x+4 \le 6 \Rightarrow -10 \le x \le 2$ . Report 13.

10.  $-2x^2 + 12x + b = -2(x^2 - 6x) + b = -2(x^2 - 6x + 9) + 18 + b = -2(x - 3)^2 + 18 + b$ . This has its greatest value 8 when x = 3. Then b = -10.

11.  $324_x = 3x^2 + 2x + 4$ .  $245_8 = 2 \cdot 64 + 4 \cdot 8 + 4 = 165$ . Equate to get  $3x^2 + 2x - 161 = 0 = (3x + 23)(x - 7)$ . Thus x = 7 and  $2025_7 = 2 \cdot 343 + 2 \cdot 7 + 5 = 705_{10}$ . Report 705.

12. Factor the product and get  $\frac{5(2x-1)(x+5)}{3(3x+2)(2x-1)} \bullet \frac{12(3x+2)}{25(x-5)(x+5)}$  and equate to  $\frac{4}{5}$ . Reduce to  $\frac{5(x+5)}{3(3x+2)} \bullet \frac{12(3x+2)}{25(x-5)(x+5)} = \frac{4}{5}$ . Reduce further to get  $\frac{4}{5} \bullet \frac{1}{x-5} = \frac{4}{5}$ . This is satisfied if and only if x = 6. Report 6.

13.  $\frac{3^{2k}}{3^{4w}} = \frac{1}{9} \Rightarrow 3^{4w} = 3^{2k+2} \Rightarrow 2k+2 = 4w$ .  $\frac{2^{4k}}{2^{6w}} = \frac{4}{1} \Rightarrow 2^{4k} = 2^{6w+2} \Rightarrow 2k = 3w+1$ . Then 4w-2 = 3w+1 and w = 3, which makes k = 5. Report 8.

14. This is tedious unless your calculator has a factoring tool to give  $155520 = 2^7 \cdot 3^5 \cdot 5$  and  $324000 = 2^5 \cdot 3^4 \cdot 5^3$ . These give  $L = 2^7 \cdot 3^5 \cdot 5^3$  and  $G = 2^5 \cdot 3^4 \cdot 5$ . Then  $\frac{L}{G} = 2^2 \cdot 3 \cdot 5^2 = 300$ .

15. 2x + y = 17 - k and 3x - y = 8 give 5x = 25 - k. k = 20 gives x = 1. Report 20.

16. 
$$P(red) = \frac{12}{31} \Rightarrow P(blue \text{ or } green) = 1 - \frac{12}{31} = \frac{19}{31}$$

17. Multiply the first equation by  $\frac{4}{3}$  to get  $\frac{16/3}{x+1} + \frac{4}{y-1} = \frac{2}{3} = \frac{8}{12}$ . Subtract this from the second given equation to get  $\frac{k-16/3}{x+1} = \frac{5}{12}$ . Multiplying by 12 gives -64+12k = 5x+5, from which we get  $x = \frac{12k-69}{5}$ . Report 12-69+5=-52.

18.  $\frac{a/b}{4b/a} = \frac{a^2}{4b^2} = k \Rightarrow \sqrt{k} = \frac{a}{2b} = \frac{a}{22}$  since *b* should be put at 11 to minimize *a*. Then  $\frac{a}{22} = 4n+3$  for some non-negative integer *n*. Put *n* = 1 and get *a* = 154. Report 154. (Note: *n* = 0 gives the non-integer  $\sqrt{k} = \frac{3}{2}$ .)

19. Since all coefficients are powers of 2, try synthetic division with the possible root 2 and find that 2 is indeed a root that yields the reduced equation  $x^4 + 2x^3 - 12x^2 - 32z - 64$ . Trying again with 2 is a failure. Trying 4 succeeds and leaves the reduced equation  $x^3 + 6x^2 + 12x + 16 = 0$ . There's no point trying 4 again, but - 4 is a root by substitution, and synthetic division leaves the reduced equation  $x^2 + 2x + 4 = 0$ , which is not factorable. The factors, then, are (x-2), (x+4) and  $(x^2 + 2x + 4)$ . The sum of these factors is  $x^2 + 5x + 2$ . Report (1, 5, 2).

20.  $\frac{x+4}{x-2} = 2 + \frac{6-x}{15}$ . Multiply this equation by 15(x-2) to get  $15x+60 = 30x-60-x^2+8x-12$ . Simplify and get  $x^2 - 23x + 132 = 0 = (x-11)(x-12)$ . Report both 11 and 12.