2025 SAA

School ANSWERS

Calculator Team

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = [ ]

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Express answers rounded to four significant digits and write in standard notation unless otherwise specified in the question. Except where noted, angles are in radians.

1. -0.8037 OR $-.8037$
2. 0.5657 OR $.5657$
3. 9.145×10^{7}
4. 1
5. 0.7858 OR $.7858$
6. 3276
7. 2.392×10^{18}
8. 8.665
9. 21.91
10. 6.177
11. 0.3155 OR $.3155$
12. 2.348
13. 5.821×10^{807}
14. 32410
15. 7.608
16. -2855.538
17. 35.69
18. 16.37
19. 11.87
20. 4.198×10^{-2}

1. The evaluation is $-.80369$, so report $-.8037$.

2. The Law of Sines gives $\frac{\sin C}{20} = \frac{\sin 45°}{25}$, which gives $\sin C = .5657$.

3. In the usual ellipse notation, $c^2 = a^2 - b^2 = 93000000^2 - 92987001^2$, from which we get $c \approx 1554877.82$. At the closet point the distance is $a - c \approx 9.145 \times 10^7$. Report this number in this form.

4. The integer 2^n starts with $2, 4, 8, 16, 32, 64, 128, 256, 512 \cdots$. When expressed in base 7 all of the digits are multiples of powers of 7, including the zero-th power of 7 as the rightmost.
We look at the base 7 numerals for those listed: $2_7, 4_7, 11_7, 22_7, 44_7, 121_7, 242_7, 514_7, 1331_7, \cdots$. The sequence of unit digits here is 2, 4, 1, 2, 4, 1, $\cdots$. The third, sixth, ninth, and so on, have 1 as the units digit in base 7. Since 2025 is divisible by 3 the 2025^{th} in the list will also have 1 as the units digit. Report 1.

5. The first segment uses 75 seconds of walking and covers $\frac{(100)(36)}{39.37} = 91.4402$ meters.
The second segment uses 200 seconds of walking and covers $\frac{(200)(36)}{39.37} = 182.8804$ meters.
The third segment uses 40 seconds of walking and covers 40 meters. The total distance traveled is approximately 314.205 meters and uses (75 + 25 + 200 + 60 + 40) = 400 seconds. The average speed is about .7858 meters per second. Report .7858.

6. Think of a horizontal string of 28 boxes, three of which will contain plus signs, the others empty. For example, putting a plus sign in boxes, numbered from the left, 6, 9 and 20 corresponds to the ordered addends 5, 2, 10, 8 with sum 25. Plus signs in boxes 1, 7 and 19 represent the sum 0 + 5 + 11 + 9 = 25. The number of ordered addends is the number of ways to place the plus signs and is thus C(28, 3) = 3276.

7. $h(x) = 3\cos^2 x - 3\sin^2 x = 3\cos(2x)$. $h(3) = 3\cos(6) \approx 2.88051$ and $\left(h(3)\right)^{40} = 2.88051^{40} \approx 2.39211 \times 10^{18}$. Report 2.392×10^{18}.

8. Graph $y1(x) = \sqrt{x} + \sqrt{x+1}$ and $y2(x) = 1 + \sqrt{2x+1}$ and find the lone intersection where $x \approx 2.48588$. Report the product $(2.48588)(3.48588) \approx 8.665$.

9. Solve $50.17 + k = 2(14.13 + k) = 28.26 + 2k$ to get $k = 21.91$.

10. With x and y for the legs and $(12 - x - y)$ for the hypotenuse, we gen an equation that reduces to $72 - 12x - 12y + xy = 0$, from which $y = \frac{12x - 72}{x - 12}$. The area is $\frac{6x^2 - 36x}{x - 12}$. Your calculator gives the maximum value of this expression 6.17662 when x is 3.54172. Report 6.177.

11. $2^x < 3^y \Rightarrow y < \frac{x \log 2}{\log 3} = .63093x$. A square with corners (0, 0), (10, 0), (10, 10) and (0, 10) is separated by the line $y = .63093x$ into the successful region, the triangle with vertices (0, 0), (10, 0) and (10, 6.3092). The area of this triangle is $5(6.3093) = 31.546$. The probability that $y < x$ is .3155.

12. Use the coordinates $C(0,0)$, $B(2,0)$, $A(2,2)$, $D(0,2)$. Then P is (1.5, 0) and the center of the square is $Q(1,1)$. The perpendicular bisector of $\overline{AP}$ has the equation $y=-\frac{1}{4}x+\frac{23}{16}$. The perpendicular bisector of $\overline{AQ}$ has the equation $y=-x+3$. These lines meet at $O\left(\frac{25}{12},\frac{11}{12}\right)$. The required distance OD is $\sqrt{\left(\frac{25}{12}\right)^2+\left(\frac{13}{12}\right)^2}=2.348.$

13. $k=\sum_{n=5}^{2025} 2025\left(\frac{5}{2}\right)^{n-4}=2025\sum_{n=5}^{2025}\left(\frac{5}{2}\right)^{n-4}\rightarrow\sum_{n=5}^{2025}\left(\frac{5}{2}\right)^{n-4}=\sum_{n=1}^{2021}\left(\frac{5}{2}\right)^{n}=\dfrac{\frac{5}{2}\left(1-\left(\frac{5}{2}\right)^{2021}\right)}{1-\frac{5}{2}}=\frac{-5}{3}\left(1-\left(\frac{5}{2}\right)^{2021}\right).$

$\log\left(\frac{5}{2}\right)^{2021}=2021\log\frac{5}{2}\approx 804.2367575\rightarrow\left(\frac{5}{2}\right)^{21}\approx 10^{804.2367575}\approx$ 1.7248745×10^{804}. $2025\left[\frac{5}{3}\left[\left(\frac{5}{2}\right)^{2021}\right]\right]\approx 5821.451765\times 10^{804}$. Report 5.821×10^{807}

14. $\text{Log}(x)=2025\bullet\log(2025)\approx 6695.51068$, so $x\approx 10^{6695}\bullet 10^{.51068}\approx 3.24101$. Report 32410.

15. Make a sketch to see that the central angle A_1OA_2 is 144°. The Law of Cosines gives $A_1A_2=\sqrt{32(1-\cos 144°)}\approx 7.608.$

16. Evaluate the expression at x = -10.2 to get -2855.538 and report that number exactly.

17. The circles have centers and radii A(-4, 0), 4 and B(3,0), 3. The respective tangency points are P and Q and line $\overleftrightarrow{PQ}$ meets the x-axis at Z, which similar triangles show is (24,0). We then have $\sin(\angle QZB)=\frac{3}{21}$, giving $\angle QZB\approx 8.21321°$ and $\angle QBZ\approx 81.7868°$. Double this to get 45.437% of $9\pi+16\pi$ for the total area, 35.69.

18. From calculus we get that the slope of the tangent line at P is $\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ and an equation of the tangent line at P is $y=\frac{\sqrt{3}}{2}x+.04655$. This gives $A=(-.05375,0)$. The slope of the tangent line at Q is $\cos\left(\frac{8\pi}{3}\right)=-\frac{1}{2}$ and an equation of the tangent line at Q is $y=-\frac{1}{2}x+5.05482$. This gives $B=(10.10963,0)$. The two tangent lines intersect at $C(3.66630,3.22166)$. The area of triangle ABC is then $\frac{1}{2}(3.22166)(10.10963-(-.05375))\approx 16.3715$. Report 16.37.

19. Since the equation for the curve is a polynomial it's almost a certainty that the function we seek is also a polynomial. The degree isn't obvious, but the most likely are 2 and 3. For quadratic and cubic regression we use the given data pairs (0, 0), (2, 16), (.5, 5.875), (1.5, 14.625) and get $y1(x) = -3x^2 + 14.15x - .15$ for quadratic regression with a confidence measure of .998687, quite good. For cubic regression with the same data pairs we get $y2(x) = -1.0x^3 + 0x^2 + 12.0x + 0$ with the perfect confidence measure 1. Which one to choose? Both equations give exactly 11 when $x = 1$, but only $y2(x)$ gives the right values at the other given x-values, so choose cubic regression and get Report 11.87. (Note: The two graphs nearly coincide on the interval $0 < x < 2$.)

20. Because the period of $y = \sin(x)$ is 2π we need only find those values of x in the interval $0 \le x \le 2\pi$ that produce outputs between 0.6 and 0.7. One such interval, found from the graphs of $y = \sin x$ and $y = 0.6$ and $y = 0.7$, is approximately $.64350 \le x \le .77540$, with length .13190. The other is approximately $2.36620 \le x \le 2.49809$, with the same approximate length. This gives the probability of success approximately $\frac{.26379}{2\pi} \approx .041984$. Report this in the required form 4.198×10^{-2}.