2025 SAA

Fr/So 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X **5** pts. ea. = ☐

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. 909

2. 24

3. 5

4. 1:00 AM (Capitalization and periods optional.)

5. 7

6. $\frac{497}{9}$

7. 50

8. $\frac{3}{2}$

9. $(-1012,1013)$

10. 5

11. $2\sqrt{10}$

12. $\frac{2\sqrt{5}}{5}$

13. 12

14. $\frac{9}{5}$ (This value only.)

15. 20

16. 350

17. 7

18. -7

19. 19

20. 736

1. Test a smaller problem: $10^6 - 1 = 999999$ and observe that the sum of the digits is $6 \bullet 9 = 72$. That's enough to say that the answer is $101 \bullet 9 = 909$.

2. The intersection is at (2, 6) so the area is $\frac{1}{2} \bullet 8 \bullet 6 = 24$.

3. $b + \frac{b}{8} = 67.5 \Rightarrow 9b = 540$, so b = 60 and a peck basket holds 7.5 pounds.
The 200 pounds of wheat kernels will fill 3 bushel baskets (180 pounds) and 2 peck baskets (15 pounds), with 5 pounds left for the partially filled peck basket. Report 5.

4. We must have for round trips $40A = 30B = 64C$, which is $2^2 \bullet 5A = 2 \bullet 3 \bullet 5B = 2^6 C$. Thus each product must have 6 factors of 2 and one each of 3 and 5. That means that A = 48, B = 32, and C = 15. The total time for each boat is 960 minutes, or 16 hours, making the first common arrival time 1:00 AM.

5. The sum is $2^{2025}(1+2+4)$, so the smallest n is 7,

6. Choose the set {14, 14, 36, 37, 38, 88, 89, 90, 91}. The largest arithmetic mean is $\frac{497}{9}$.

7. Let y be the number of students taking both German and Chinese. Let z be the number taking both Latin and Chinese. The number taking two courses is $40 + 90 + y + z = 230$, so $y + z = 100$. The number taking Chinese or German or both is given to be 330, which is 150 + 210 – y, from the set relation $n(G\quad C) = n(G) + n(C) - n(G\quad C)$, so y = 30. This makes z = 70 and it gives the number taking Latin only to be 210 – 90 – 70 = 50. Report 50.

8. Solution by CS: Let r be the rower's rate in still water and c the speed of the stream.
The rower's upstream trip is start (S) to drop (D) to turnaround (T), which is $6 + 2(r - c)$ miles.
The float's time in the water is $\frac{6}{c}$ hours. The rower's time in the water following the drop is $2 + \frac{6 + 2(r-c)}{r+c}$ hours. To arrive at S at the same time we have $\frac{2r + 2c + 6 + 2(r-c)}{r+c} = \frac{6}{c}$.
Cross-multiply, do some cancellation, and get $4rc = 6r$, which gives $c = \frac{3}{2}$.

9. Starting at (0, 0) the successive turn points are (0, 1), (2, 1), (2, -2), (-2, -2), (-2, 3), (4, 3), (4, -4), (-4, -4), (-4, 5), and so on. The "northwest" corners are (0, 1), (-2, 3), (-4, 5), and so on. These fit the pattern $\left(\frac{n-1}{2}, \frac{n+1}{2}\right)$, where n is the number of units traveled upward to reach those corners. This can be seen by tracing the paths with a pencil and observing that the upward trip lengths are 1, 5, 9, 13, 17, 21, 25, ..., 2021, 2025. The sequence formula is 1 + 4k, k = 0 through 506. The ant's final trip is 2025 up and ends at (-1012, 1013).

10. Let the two-digit integer be $10t + u$. Then the sum $\frac{10t + u}{tu} = \frac{10}{u} + \frac{1}{t}$. $\frac{10}{u} + \frac{1}{t}$ is an integer for the (t, u) pairs (1, 10), (1, 2), (1, 5), (2, 4), (3, 6) and only those. Report 5.

11. Complete squares to get $(x-3)^2+(y+4)^2=25$. The distance from (10, 0) to (3, -4) is $\sqrt{65}$, which is the hypotenuse of a right triangle with legs 5 and $2\sqrt{10}$. Report $2\sqrt{10}$.

12. $AB = 1$, so the median $\tan\theta$ is $\frac{1}{2}$. In a $1-2-\sqrt{5}$ right triangle the longer leg is $\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$. Report $\frac{2\sqrt{5}}{5}$.

13. The discriminant $D=p^2-4q$ must be a square. $(2,1)\Rightarrow D=0, (3,2)\Rightarrow D=1, (4,3)\Rightarrow D=4,$
$(4,4)\Rightarrow D=0, (5,4)\Rightarrow D=9, (5,6)\Rightarrow D=1, (6,5)\Rightarrow D=16, (6,8)\Rightarrow D=4, (6,9)\Rightarrow D=0,$
$(7,6)\Rightarrow D=25, (8,7)\Rightarrow D=36, (9,8)\Rightarrow D=49$. That's 12 D's that are squares as required. All roots are integers, so report 12 as the number of equations that yield integer roots.

14. If the first two are equal, then $k = 3$, violating the triangle inequality. If the second and third are equal, then $k=\frac{9}{5}$, which satisfies the triangle inequality. If the first and third are equal, then $k = 1$, violating the triangle inequality. Report $\frac{9}{5}$.

15. Angle H is $\frac{(9-2)(180)}{9}=140$, which gives 20 for angle GIH. Report 20.

16. A simple trick is to put $B = 7$ and $A = 2$. Then 14 is 100 per cent of 14, which means that $\frac{2k}{7}=1$, making $k=3.5$. That's 350 per cent. Report 350.

17. The first possibility is $\frac{x+12}{x+2}\geq 3$, so $x+12\geq 3x+6\Rightarrow x\leq 3$. The other is $\frac{x+12}{x+2}\leq -3$, or $x+12\leq -3x-6\Rightarrow x\geq -\frac{9}{2}$. The successful integers are $-4,-3,-1,0,1,2,3$. Note that -2 is not allowed. Report 7.

18. Write the first as $x(x+y+1)=14$ and the second as $y(x+y+1)=28$ and divide to get $y=2x$. The first equation then becomes $3x^2+x-14=0=(3x+7)(x-2)$. The third quadrant intersection is then $\left(-\frac{7}{3},-\frac{14}{3}\right)$. Report the sum of the coordinates, -7.

19. Let r and s be solutions for x to $x^2+4x+\frac{k}{w}=0$. Then $r+s=-4$ and $rs=\frac{k}{w}=-4r-r^2$.

The quadratic formula gives $x=\frac{-4\pm\sqrt{16-\frac{4k}{w}}}{2}=\frac{-4\pm\sqrt{16-4(-4r-r^2)}}{2}=-2\pm\sqrt{(r+2)^2}$
$=-2\pm|r+2|$. The positive difference between the solutions is given to be 1, so $2|r+2|=1$.

Then $(r,s)=\left(-\frac{3}{2},-\frac{5}{2}\right)$ or $\left(-\frac{5}{2},-\frac{3}{2}\right)$. So $\frac{k}{w}=rs=\frac{15}{4}$, which checks in the given equation. Report 19.

20. The radius is 5 feet, so the total area to be painted is $50\pi\approx 150+7$, using 3.14 as an approximation to π. Then 32 cans of paint are needed since 31 will cover 155 square feet, falling short by about 2 square feet.. Report $32\bullet 23=736$. (Note: if different colors were used for front and back, the total number of cans is still 32 and the amount spent is still 736 dollars.)