2025 SA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = ☐

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1.	5	11.	44
2.	3	12.	5
3.	$\frac{13}{10}$	13.	40 (40% is incorrect.)
4.	176	14.	$\sqrt{2}$
5.	4	15.	$6\sqrt{3}$
6.	4π	16.	$\left(-2, 2\sqrt{3}\right)$
7.	3	17.	10
8.	7	18.	-5
9.	3	19.	11
10.	-8	20.	51

1. The product on the left gives $3x^2-6=42$ and $3xy-8=4$. The (x, y) pairs are (4, 1) and (-4, -1), so report 5.

2. Factor to get the rational expression $\frac{(x-3)(x^2-9)}{(x-3)(x+5)}=\frac{x^2-9}{x+5}$. This give the "hole" (3, 0). Report 3.

3. $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta=1+\sin 2\theta=1+\frac{69}{100}=\frac{169}{100}$. Since both $\sin\theta$ and $\cos\theta$ are positive, report $\frac{13}{10}$.

4. $k=3\sum_{n=0}^{10}n+\sum_{n=0}^{10}1=3\bullet\frac{10\bullet 11}{2}+11=176$.

5. $3^x(1+3)=324\Rightarrow 3^x=81\Rightarrow x=4$. Report 4.

6. The quadratic formula gives $\sin x=\frac{1\pm\sqrt{1+48}}{24}=\frac{1}{3},-\frac{1}{4}$. Sketch the graph of $y=\sin x$ on the interval $[0, 2\pi]$ along with the horizontal lines $y=\frac{1}{3}$ and $y=-\frac{1}{4}$ to see four intersections, one with $x = a$ near the y-axis, a second with $x=\pi-a$, a third with b just beyond $x=\pi$, and a fourth with $x=2\pi-(b-\pi)=3\pi-b$. Report the sum of these four values, 4π.

7. $\frac{7}{4}=1+\frac{3}{4}$; $\frac{49}{4}=12+\frac{1}{4}$; $\frac{343}{4}=85+\frac{3}{4}$; $\frac{2401}{4}=600+\frac{1}{4}\cdots$. The remainders alternate between 3 and 1. Since 31 is odd, report the remainder 3.

8. A is 3; $\underline{B}$ is 3 + 5 = 8; C is -3 + 5 = 2. Report $\frac{B}{C}+A=4+3=7$.

9. The given equation simplifies to $x^2+(1-k)x+1=0$. The discriminant $(1-k)^2-4$ is non-negative when $|1-k|\geq 2$, so the least positive value of k is 3. Report 3.

10. One obvious solution for x is 1. Synthetic division shows the 3 is a double-root and -4 is a simple root. So $a=-4, b=1, c=3$. Report -8.

11. Rationalize denominators to get $(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots+(\sqrt{2025}-\sqrt{2024})=\sqrt{2025}-1=44$. Report 44.

12. The slope of $\overline{NY}$ is 1/2 and the slope of $\overline{CY}$ is -2, so the triangle is a right triangle with hypotenuse $\overline{CN}$ measuring 10. This makes the radius of the circle 5. Report 5.

13. The numerator restricts the domain to [-3, 3], but the denominator disallows 2 and above. The domain is the half-closed, half-open interval [-3, 2), with 40% positive. Report 40.

14. The line has the equation $y-(-1)=\dfrac{-1-4}{4-9}(x-4)$, or $x-y-5=0$, which the form needed for the point-to-line distance formula, which yields $d=\dfrac{|1\bullet 5-1\bullet 2-5|}{\sqrt{1^2+(-1)^2}}=\dfrac{2}{\sqrt{2}}=\sqrt{2}$. Report $\sqrt{2}$.

15. The first length is $\dfrac{\log 17}{\log 11}$; the second is $\dfrac{2\log 13}{2\log 17}$; the third is $\dfrac{2\log 11}{\log 13}$. The product of these is 2. If the edge lengths of the cubes are *a, b, c*, the face diagonals have lengths $\sqrt{2}a, \sqrt{2}b, \sqrt{2}c$, and cube diagonal lengths $\sqrt{3}a, \sqrt{3}b, \sqrt{3}c$ with the product $3\sqrt{3}abc$. Since $abc = 2$, report $6\sqrt{3}$.

16. Since $x=r\cos\theta$ and $y=r\sin\theta$, we have $(x, y)=\left(4\bullet\dfrac{-1}{2}, 4\bullet\dfrac{\sqrt{3}}{2}\right)=(-2, 2\sqrt{3})$.

17. Let $\log_a b=x$. Then $\log_b a=\dfrac{1}{x}$ because $a^x=b\Rightarrow a=b^{1/x}$. The solutions to $x+\dfrac{1}{x}=\dfrac{5}{2}$ are $\dfrac{1}{2}$ and 2. So put $a=b^2$ and get $b^3=64, b=4, a=16$. Report the arithmetic mean, 10.

18. There's a trick here. Write each fraction in its equivalent form: $\dfrac{x+(n-1)}{x+n}=1-\dfrac{1}{x+n}$ and get $1-\dfrac{1}{x+2}+1-\dfrac{1}{x+8}=1-\dfrac{1}{x+4}+1-\dfrac{1}{x+6}$. The 1's cancel to give $\dfrac{1}{x+2}+\dfrac{1}{x+8}=\dfrac{1}{x+4}+\dfrac{1}{x+6}$. Then $\dfrac{2x+10}{x^2+10x+16}=\dfrac{2x+10}{x^2+10x+24}$. The numerators are the same for all *x* and the denominators differ by the constant 8, so the only solution is -5.

19. The side lengths are 6, $6r$ and $6r^2$ and Pythagoras gives $36+36r^2=36r^4$, or $r^4-r^2-1=0$. The positive solution for r^2 is $\dfrac{1+\sqrt{1+4}}{2}$ and the hypotenuse is $3(1+\sqrt{5})=3+3\sqrt{5}$. Report 11.

20. Evaluate by minors of the first row to get $(2i)(16+2i)-(-i)(4i-6)+3(i^2+12)$, which gives $32i-4+4i^2-6i+3(-1+12)=26i+25$. Report 51.