2025 SAA

Jr/Sr 8 Person

School ANSWERS

(Use full school name – no abbreviations)

______ Correct X 5 pts. ea. = [ ]

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. 52
2. -12
3. $\sqrt{3}$
4. $\frac{7}{10}$
5. 4
6. $4\sqrt{3}-4i$
7. 3
8. $\frac{11}{9}$
9. $\frac{3}{2}$
10. 4049
11. 260
12. $\frac{5}{162}$
13. 40 (40% is incorrect.)
14. 1
15. $6\sqrt{3}$
16. 5
17. 10
18. -5
19. $\frac{\sqrt{21}}{2}$
20. 165

1. For an odd number of data values the median is the middle one, 59 in this case. All seven values are $59-3d, 59-2d, 59-d, 59, 59+d, 59+2d, 59+3d$. The range is $6d$, which is 78, so d is 13 and the values are 20, 33, 46, 59, 72, 85, 98. The lower quartile is the median of 20, 33 and 46, or 33. The upper quartile is the median of 72, 85 and 98, or 85. The IQR is 85 – 33 = 52.

2. The top expression has the value 3 when $x = 2$, so continuity demands $\frac{k}{3}\bullet 2+11=3$ as well. This makes k = -12.

3. The right side becomes $\frac{\cos^2(15°)+2\cos 15°\sin 15°+\sin^2(15°)}{\cos^2(15°)-\sin^2(15°)}$ when top and bottom are multiplied $\left(\cos(15°)+\sin(15°)\right)$. This becomes $\frac{1+\sin(30°)}{\cos(30°)}=\frac{1+1/2}{\sqrt{3}/2}=\sqrt{3}$. Report $\sqrt{3}$.

4. Observe three geometric series. The first has first term $\frac{1}{6}$ and ratio $\frac{1}{6}$ and sum $\frac{1/6}{1-1/6}=\frac{1}{5}$. The second has sum $\frac{1/3}{1-1/3}=\frac{1}{2}$. The third has sum $\frac{1/2}{1-1/2}=1$. Report $\frac{1}{5}-\frac{1}{2}+1=\frac{7}{10}$.

5. $3^x(1+3)=324 \Rightarrow 3^x=81 \Rightarrow x=4$. Report 4.

6. The magnitude of the given vector is 8. The rotation gives the image $4\sqrt{3}-4i$. Report that.

7. $\frac{7}{4}=1+\frac{3}{4}; \frac{49}{4}=12+\frac{1}{4}; \frac{343}{4}=85+\frac{3}{4}; \frac{2401}{4}=600+\frac{1}{4}\cdots$. The remainders alternate between 3 and 1. Since 31 is odd, report the remainder 3.

8. (Solution by CS) The equation $3x^3-2x^2+5x-3=0$ has solutions $3a+2, 3b+2, 3c+2$, whose sum, $6+3(a+b+c)$, is equal to $-\frac{-2}{3}=\frac{2}{3}$. This gives $a+b+c=-\frac{16}{9}$. The theorem about sum and product of roots suggests looking at the sum of the two-root products since the sought- after expression involves $ab + bc + ca$. So we compute this product sum and get $(3a+2)(3b+2)$ $+(3b+2)(3c+2)+(3c+2)(3a+2)$. This gives $9(ab+bc+ca)+12(a+b+c)+12$, and the theorem say that this equals $\frac{5}{3}$. We then have $9(ab+bc+ca)+12\left(-\frac{16}{9}\right)+12=\frac{5}{3}$. Solve this for the sought-after expression and get $(ab+bc+ca)=\left(\frac{5}{3}+\frac{64}{3}-\frac{36}{3}\right)\left(\frac{1}{9}\right)=\frac{11}{9}$. Report $\frac{11}{9}$.

9. Cancel to get $\frac{3x+1}{2x-1}$, with limit $\frac{3}{2}$ as x gets indefinitely large in either direction.

10. The first few terms are 1, 9, 25, 49, 81, 121, and so on, suggesting $(2n-1)^2$ for a_n. This can be verified by looking at successive differences and finding that all second differences are 8 and a quadratic function fits. So $a_{2025}=(4050-1)^2$. Report 4049.

11. The formula, found by (last resort) googling, gives $Sum = \frac{1}{n}\sum_{k=1}^{n^2} k = \frac{1}{8}\sum_{k=1}^{64} k = \frac{1}{8}\bullet\frac{64\bullet 65}{2} = 260$. Explanation from CS: The integers 1 through 64 have to be arranged into 8 rows (or columns) with equal sums, which explains the numerical computation shown in the first sentence since the sum of the first 64 integers is (64)(65)/2

12. Let the face letters temporarily be PA_1SCA_2L. It doesn't matter what the first top you inspect is because it's a successful start. The probability or seeing the remaining five letters on your subsequent looks is $\frac{5}{6}\bullet\frac{4}{6}\bullet\frac{3}{6}\bullet\frac{2}{6}\bullet\frac{1}{6} = \frac{20}{1296} = \frac{5}{324}$. But A_1 and A_2 are not really different, so double the result and report $\frac{5}{162}$. (Alternate solution by CS: If the cubes line up in the successful showing (PASCAL), the probability of that particular order is $\frac{1}{6}\bullet\frac{1}{3}\bullet\frac{1}{6}\bullet\frac{1}{6}\bullet\frac{1}{3}\bullet\frac{1}{6}$. There are $P(6,2) = \frac{6!}{2!}$ permutations like this, so the required probability is $\frac{1}{6}\bullet\frac{1}{3}\bullet\frac{1}{6}\bullet\frac{1}{6}\bullet\frac{1}{3}\bullet\frac{1}{6}\bullet\frac{6!}{2!} = \frac{5}{162}$.)

13. The numerator restricts the domain to [-3, 3], but the denominator disallows 2 and above. The domain is the half-closed, half-open interval [-3, 2), with 40% positive. Report 40.

14. The expression in the denominator symmetric because replacing x by its opposite gives the same value, so look at $x^2+2x+2=(x+1)^2+1$. The minimum value of this is 1 when x = -1, and this makes the given expression its maximum value, 1. Report 1.

15. The first length is $\frac{\log 17}{\log 11}$; the second is $\frac{2\log 13}{2\log 17}$; the third is $\frac{2\log 11}{\log 13}$. The product of these is 2. If the edge lengths of the cubes are a, b, c, the face diagonals have lengths $\sqrt{2}a, \sqrt{2}b, \sqrt{2}c$, and cube diagonal lengths $\sqrt{3}a, \sqrt{3}b, \sqrt{3}c$ with the product $3\sqrt{3}abc$. Since abc = 2, report $6\sqrt{3}$.

16. The symbol $\binom{n}{r}$ is the *combination* expression, which appears as $ncr(8,5)$,, for example, on many calculators and has the value $\frac{8!}{5!(8-5)!} = 56$ in this example. The equation $\binom{n}{k} = \binom{n}{r}$ has the least possible sum of k and r when n is 5 and k and r are 3 and 2 because the restriction r > 1. Report 5.

17. Let $\log_a b = x$. Then $\log_b a = \frac{1}{x}$ because $a^x = b \Rightarrow a = b^{1/x}$. The solutions to $x+\frac{1}{x}=\frac{5}{2}$ are $\frac{1}{2}$ and 2. So put $a=b^2$ and get $b^3=64, b=4, a=16$. Report the arithmetic mean, 10.

18. There's a trick here. Write each fraction in its equivalent form: $\frac{x+(n-1)}{x+n} = 1-\frac{1}{x+n}$ and get $1-\frac{1}{x+2}+1-\frac{1}{x+8} = 1-\frac{1}{x+4}+1-\frac{1}{x+6}$. The 1's cancel to give $\frac{1}{x+2}+\frac{1}{x+8} = \frac{1}{x+4}+\frac{1}{x+6}$. Then $\frac{2x+10}{x^2+10x+16} = \frac{2x+10}{x^2+10x+24}$. The numerators are the same for all x and the denominators differ by the constant 8, so the only solution is -5.

19. Simplify the problem with a horizontal shift and a vertical shift to get the foci at (10, 0) and (-10, 0) and the referred-to vertex at (4, 0). The equation of this hyperbola is $\frac{x^2}{4}-\frac{y^2}{b^2}=1$ because the relation among *a, b* and *c* for a hyperbola is $c^2=a^2+b^2$ and *c* = 10 and *a* = 4. This gives $b=\sqrt{100-16}=2\sqrt{21}$. The positive slope of one asymptote is thus $\frac{\sqrt{21}}{2}$.

20. Let $a=3x, b=5y, c=7z$.Then $3x=5y-1$ and $7z=5y+1$. Trying some values of *y* we find that 11 gives 55 for *b*, 54 for *a* and 56 for *c*. No smaller value of *y* works. Report 165.