2025 SA
Geometry

Name ANSWERS

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. $16\sqrt{2}$
2. Always (Must be this whole word, capitalization optional.)
3. $5\sqrt{5}$
4. $\frac{120}{13}$
5. 85
6. $\frac{\sqrt{3}}{9}$
7. $\frac{32}{3}$
8. 120
9. 4
10. 25
11. 6.9
12. $\frac{59}{10}$
13. 19.7
14. 48
15. 5.889
16. $3\sqrt{3}$
17. 8
18. $3+2\sqrt{2}$ OR $2\sqrt{2}+3$
19. 42
20. 80

1. $r^2 = 128 = 2^7 \Rightarrow r = 8\sqrt{2}$. The circumference is $16\sqrt{2}\pi$. Report $16\sqrt{2}$.

2. The statement is a well-known theorem. Report *Always*.

3. $AE = 2 \bullet 5\sqrt{2}$. The Law of Cosines in triangle ABE gives $BE^2 = 5^2 + (10\sqrt{2})^2 - 2 \bullet 5 \bullet 10\sqrt{2} \bullet \cos 45° = 25 + 200 - 100\sqrt{2} \bullet \frac{1}{\sqrt{2}} = 125$. Report $BE = 5\sqrt{5}$.

4. $AC = 26$. The similarity relation gives $\frac{\textit{long leg}}{\textit{hypotenuse}} = \frac{24}{26} = \frac{BD}{10}$, giving $BD = \frac{120}{13}$.

5. Let x be arc DA's degree measure. Then $\frac{125 + x}{2} = 105$ and $x = 85$.

6. The area of the triangle is $\frac{(x^2/9)\sqrt{3}}{4} = \frac{x^2\sqrt{3}}{36}$. The area of the circle is $\pi \bullet \frac{x^2}{4\pi^2} = \frac{x^2}{4\pi}$. $\frac{\textit{Area of triangle}}{\textit{Area of circle}} = \frac{\sqrt{3}}{36} \bullet 4\pi = \frac{\sqrt{3}}{9}\pi$. Report $\frac{\sqrt{3}}{9}$.

7. $(12 + OC)^2 - OA^2 = (12 + OC)^2 - OC^2 = 20^2 \Rightarrow OC = \frac{32}{3}$.

8. When the long leg of a right triangle is $\sqrt{3}$ times the short leg, the larger acute angle is $60°$. This makes arc BD equal to $120°$. Report 120.

9. The secant theorem says that the product of the lengths of a secant segment and its external part is constant. So here we have $(2a - x)a = (6x)x = (2x + 4)(x + 1)$. The latter two lead to the equation $2x^2 - 3x - 2 = 0$, with one positive solution, $x = 2$. Then $(2a - 2)a = 6 \bullet 2^2$, or $a^2 - a - 12 = 0$, with the positive solution $a = 4$. Report 4.

10. Draw a picture with diameter length $AB = d$, C on $\overline{AB}$ and $AC = 1$, H on the semi-circle with $CH = 7$. We then have three similar right triangles with $\frac{AH}{AC} = \frac{\sqrt{49+1}}{1} = \frac{d}{\sqrt{49+1}}$. This gives $d = 50$ and makes the highest point on the semi-circle have length $d/2 = 25$.

11. The difference in the volumes of spheres is $\frac{4}{3}\pi r^3 - \frac{4}{3}\pi(r - 4.5)^3 = 425\pi$, which leads to the equation $13.5r^2 - 60.75r - 227.625 = 0$. Solve this by your favorite method to get one positive solution, approximately 6.93. Report 6.9.

12. Let $\overline{OB}$ be perpendicular to the diameter at the center O, with B the center of the short chord and A between O and B on the long chord. Let C be one end of the short chord. Let D be on the long chord and between O and C. With r being the radius we have two right triangles and the equations $r^2 = 4^2 + (5 + x)^2$ and $r^2 = 10^2 + x^2$. Equate the right sides and get the equation $100 = 16 + 25 + 10x$. Report the solution for x, $\frac{59}{10}$.

13. The base area is 15. In one second the height increases by 12/15. In 9 seconds the increase is 36/5 = 7.2. The new height of the water is 12.5 + 7.2 = 19.7.

14. Let the other side length of the trapezoid be x and the common height be h. We then have $30h = \frac{1}{2}h(12+x)$, which reduces to $60 = 12 + x$. Report 48.

15. $\frac{1}{6}\pi r^2 = \frac{r^2\sqrt{3}}{4} + \pi \Rightarrow \left(2\pi - 3\sqrt{3}\right)r^2 = 12\pi \Rightarrow r^2 \approx 34.680746 \Rightarrow r \approx 5.889.$

16. The diagonal length is $\sqrt{6}$ so the edge length is $\sqrt{6/2} = \sqrt{3}$ and the volume is $3\sqrt{3}$.

17. Let $x = BP$ and $y = AP$. Then $\frac{x}{4} = \frac{QR}{y}$, so $QR = \frac{xy}{4} = PS$. In similar triangles QRC and ASC we have $\frac{QC}{AC} = \frac{QR}{AS}$, or $\frac{x+16}{12} = \frac{xy/4}{(xy/4)-y} = \frac{xy}{xy-4y} = \frac{x}{x-4}$. From $\frac{x+16}{12} = \frac{x}{x-4}$ we get $x^2 + 12x - 64 = 12x$. This makes x = 8. Report 8.

18. The small circle has the equation $(x-1)^2 + (y-1)^2 = 1$. The symmetry line $y = x$ meets this circle where $2x^2 - 4x + 2 = 1$, or $2x^2 - 4x + 1 = 0$. The solutions for x are $\frac{4 \pm \sqrt{16-8}}{4}$, also written as $1 + \frac{\sqrt{2}}{2}$ and $1 - \frac{\sqrt{2}}{2}$. With $O = (0,0)$ the points where $y = x$ meets the small circle are $A\left(1 - \frac{\sqrt{2}}{2}, 1 - \frac{\sqrt{2}}{2}\right)$ and $B\left(1 + \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}\right)$.

The distance from O to the closer intersection point is $\sqrt{2\left(1 - \frac{\sqrt{2}}{2}\right)^2}$. The distance from O to the farther intersection point is $\sqrt{2\left(1 + \frac{\sqrt{2}}{2}\right)^2}$. The ratio of these numbers is $\frac{larger}{smaller} = 2\sqrt{2} + 3$. The dilation (expansion) with center O and this scale factor will map the smaller circle to the larger circle and the larger radius will be $2\sqrt{2} + 3$.

19. The figure is a trapezoid with bottom base $\overline{AB}$ of length 100 and side lengths $AA' = 100$ and $BB' = h$. The parallel to $\overline{AB}$ through B' meets $\overline{AA'}$ at C. Then triangle $A'B'C$ is a right triangle with $B'C = 100$, $A'C = 100 - h$, and angle $A' = 60°$. This makes $\tan 30° = \frac{100-h}{100} = \frac{\sqrt{3}}{3} = 1 - \frac{h}{100}$. This gives $h = 100\left(1 - \frac{\sqrt{3}}{3}\right) \approx 42.26$. Report 42.

20. Let the base angle measures be x. The vertex angle is $180 - 2x$. The average of $180 - 2x$ and x is $\frac{180-x}{2} = 70$ and the average of x and x is x = 70. The possible x's are 70 and 40. The least possible sum of two angle measures in the triangle is 80. Report 80.