2025 SAA
Geometry

Name ___ANSWERS___

School ________________

(Use full school name – no abbreviations)

______ **Correct X 2 pts. ea. =** ☐

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. 22

2. Sometimes (Must be this whole word, capitalization optional.)

3. $-\frac{4}{3}, 1$ OR $\frac{-4}{3}, 1$ (Must have both values, either order.)

4. $\frac{50}{13}$

5. 85

6. 17

7. $\frac{32}{3}$

8. 3

9. 300

10. 1200

11. $\frac{3}{4}$

12. 16

13. 19.7

14. $3\sqrt{10}$

15. 5.889

16. $\frac{4}{5}$

17. 8

18. $3+2\sqrt{2}$ OR $2\sqrt{2}+3$

19. $\frac{65}{2}$

20. 15

1. $\dfrac{121}{\pi} = \pi r^2 \Rightarrow r = \dfrac{11}{\pi} \Rightarrow c = 22.$

2. It's not always true, but it is when the transversal is perpendicular to the parallel lines. Report Somethmes.

3. The three slopes are $\dfrac{4-2k}{3k-1}, \dfrac{6k-4}{5-3k}, \dfrac{4k}{4} = 1.$ The first two of these are equal when cross-multiplying gives $12k^2 + 4k - 16 = 0,$ whose roots are 1 and $-\dfrac{4}{3}$. The three points are collinear, hence not on a circle, when k is either of these values, so report both 1 and $-\dfrac{4}{3}$.

4. $\dfrac{AD}{10} = \dfrac{10}{26} \Rightarrow x = \dfrac{50}{13}.$

5. Let x be arc DA's degree measure. Then $\dfrac{125+x}{2} = 105$ and $x = 85$.

6. Let $x = BA = FA$. Then $(1+x)^2 + (2+x)^2 = 3^2 \Rightarrow x^2 + 3x - 2 = 0,$ so $x = \dfrac{-3+\sqrt{9+8}}{2} = \dfrac{-3+1\sqrt{17}}{2}$. Report $-3+1+17+2 = 17.$

7. $(12+OC)^2 - OA^2 = (12+OC)^2 - OC^2 = 20^2 \Rightarrow OC = \dfrac{32}{3}.$

8. Draw an xy-plane diagram with $O = (0,0), B = (0,2), A = (2,0), C = (c,0), P = (4,0),$ where A and B are the points where the sphere meets the positive axes, and C is the foot of the perpendicular from the tangency point T. Then $TC = r$, the radius of the circle traced on the sphere by the tangents from P. Since $OT = 2$ and $OP = 4$, $PT = 2\sqrt{3}$ and the tangents of angles OPT and OTC are $\dfrac{2}{2\sqrt{3}}$ and $\dfrac{c}{r}$, respectively, and are equal, giving $c = \dfrac{r}{\sqrt{3}}$. In right triangle OTC we then have $\left(\dfrac{r}{\sqrt{3}}\right)^2 + r^2 = 2^2,$giving $r = \sqrt{3}$ and $c = 1$. Report 3 for k.

9. The given lengths make $\angle A = 30°, \angle AOB = 60° = arc\ EB$. This gives $arc\ DB = 120°$ and $arc\ BDE = 120° + 180° = 300°$. Report 300.

10. The solid is the union of two right circular cones, both with base radius r and heights x and $25-x$. The sum of the volumes is thus $\dfrac{1}{3}\pi r^2(x+25-x) = \dfrac{25}{3}\pi r^2$. The area of the triangle defining the region revolved is $\dfrac{1}{2}(25r) = \dfrac{1}{2}(15 \bullet 20),$ making $r = 12$. Report $\dfrac{25}{3} \bullet 144 = 1200.$

11. A picture shows that the ratio of the side lengths is $\sqrt{3}:2,$ a shortening that makes the ratio of the areas 3:4. Report $\dfrac{3}{4}$.

12. $ABCD$ has area 8. A picture shows four pairs of congruent triangles, doubling the area to 16.

13. The base area is 15. The volume at t = 0 is $\frac{1}{2}(3)(5)(25)=187.5$. Adding $(12)(9)=108$ results in a volume of 295.5. The new height is $\frac{295.5}{15}=19.7$

14. The diagonals of the of the rhombus have lengths 2x and 4x, which makes its area $4x^2$ and the diagonal lengths $2\sqrt{10}$ and $4\sqrt{10}$. Report the median of these, $3\sqrt{10}$.

15. $\frac{1}{6}\pi r^2=\frac{r^2\sqrt{3}}{4}+\pi \Rightarrow \left(2\pi-3\sqrt{3}\right)r^2=12\pi \Rightarrow r^2 \approx 34.680746 \Rightarrow r \approx 5.889.$

16. Let O be the center of the circle, r the radius and s the side length of the square. Then $r^2=s^2+\frac{1}{4}s^2=\frac{5}{4}s^2$ and $s=r\sqrt{\frac{4}{5}}=r\sqrt{k}$. Report $\frac{4}{5}$ for k.

17. Let $x=BP$ and $y=AP$. Then $\frac{x}{4}=\frac{QR}{y}$, so $QR=\frac{xy}{4}=PS$. In similar triangles QRC and ASC we have $\frac{QC}{AC}=\frac{QR}{AS}$, or $\frac{x+16}{12}=\frac{xy/4}{(xy/4)-y}=\frac{xy}{xy-4y}=\frac{x}{x-4}$. From $\frac{x+16}{12}=\frac{x}{x-4}$ we get $x^2+12x-64=12x$. This makes x = 8. Report 8.

18. The small circle has the equation $(x-1)^2+(y-1)^2=1$. The symmetry line y = x meets this circle where $2x^2-4x+2=1$, or $2x^2-4x+1=0$. The solutions for x are $\frac{4\pm\sqrt{16-8}}{4}$, also written as $1+\frac{\sqrt{2}}{2}$ and $1-\frac{\sqrt{2}}{2}$. With $O=(0,0)$ the points where y = x meets the small circle are $A\left(1-\frac{\sqrt{2}}{2},1-\frac{\sqrt{2}}{2}\right)$ and $B\left(1+\frac{\sqrt{2}}{2},1+\frac{\sqrt{2}}{2}\right)$.

The distance from O to the closer intersection point is $\sqrt{2\left(1-\frac{\sqrt{2}}{2}\right)^2}$. The distance from O to the farther intersection point is $\sqrt{2\left(1+\frac{\sqrt{2}}{2}\right)^2}$. The ratio of these numbers is $\frac{larger}{smaller}=2\sqrt{2}+3$. The dilation (expansion) with center O and this scale factor will map the smaller circle to the tangent larger circle and the radius of the larger circle will be $2\sqrt{2}+3$.

19. The slope of $\overleftrightarrow{AB}$ is the same as the line in question, $\frac{5}{12}$. The height from A(or B) to the line $5x-12y-3=0$ is $\frac{|5\bullet 2-12\bullet 6-3|}{\sqrt{5^2+12^2}}=\frac{65}{13}=5$. The area of triangle ABC is $\frac{1}{2}\bullet 5\bullet\sqrt{12^2+5^2}=\frac{65}{2}$.

20. For the small triangle the Triangle Inequality gives 1 < k < 9. The large triangle gives $9<k^2<41$. The latter inequality is contained in and more restrictive than the former and gives $k^2=16, 25, 36$. Report the sum of the k values, 4 + 5 + 6 = 15.