2025 SA
Algebra II

Name ANSWERS

School _____
(Use full school name – no abbreviations)

_____ **Correct X 2 pts. ea. =** ☐

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. 1.175

2. 8

3. -4

4. 2002

5. 256

6. $\sqrt{2}$

7. 252

8. 4935

9. 8

10. $\frac{3}{10}$

11. 20160 (Comma use ok.)

12. $(4,5,7)$

13. $-\frac{195}{8}$ OR $\frac{-195}{8}$

14. -512

15. 12

16. 63

17. 12

18. 1476

19. 92

20. $\frac{81}{125}$

I.C.T.M. 2025 State Algebra 2 – Divisions 1A, 2A

1. Calculator gives 1.17462. Report 1.175.

2. The three numbers are "in or out" of a subset. That's 2^3 subsets. Report 8.

3. Let *r* be the third zero. Then the "sum and product of the roots" theorem gives $4+r=-a,\ 1\bullet 3+1\bullet r+3\bullet r=b,\ 1\bullet r\bullet 3=-c.$ Subtract the first from the second to get $b+a=3+4r-4-r=3r-1.$ With the given equation we get $3r-1=-13,$so report $-4.$

4. It's $C(14,5)=2002.$

5. $k=\log(8^4)-\log(4^2)=\log\left(\frac{2^{12}}{2^4}\right)=\log\left(2^8\right).$ Report *k* =256.

6. $(x+y)^3=x^3+3x^2y+3xy^2+y^3=5+3xy(x+y),$ or $8=5+6xy$ and $xy=\frac{1}{2}.$ This gives $x(2-x)=\frac{1}{2},$so $x^2-2x+\frac{1}{2}=0$ and $x=\frac{2\pm\sqrt{4-2}}{2}=1\pm\frac{\sqrt{2}}{2}.$ Then either $x=1+\frac{\sqrt{2}}{2}$ and $y=1-\frac{\sqrt{2}}{2}$ of vice versa. Either way we have $\left|x-y\right|=\sqrt{2}.$

7. The four terms simplify to give $3\bullet 7\sqrt{3}+5\bullet 13\sqrt{2}+9\sqrt{3}+16\sqrt{2}=30\sqrt{3}+81\sqrt{2}.$ Report $90+162=252.$

8. It's $\frac{99\bullet 100}{2}-\frac{5\bullet 6}{2}=4950-15=4935.$

9. Clear fractions and get $A(x-7)+B(x+4)=11x-22.$ Then $A+B=11$ and $-7A+4B=-22.$ Multiply the first equation by 7 and add: $11B=55.$ Then *B* = 5 and *A* = 6. Report 18 – 10 = 8.

10. The decades have 4, 4, 2, 2, 3, 2, 2, 3, 2, 1. Report $\frac{3}{10}.$

11. $9x-39<94\Rightarrow x<\frac{133}{9}=\textit{fourteen}+\frac{7}{9}.$ The number of permutations is $\frac{8!}{2!}=20160.$

12. $4!+1=5^2, 5!+1=11^2, 7!+1=71^2.$ Report (4, 5, 7).

13. Complete squares to get $2\left(x^2-\frac{3}{2}x+\frac{9}{16}\right)-\frac{9}{8}+\left(y^2-7y+\frac{49}{4}\right)-\frac{49}{4}-11=2\left(x-\frac{3}{4}\right)^2+\left(y-\frac{7}{2}\right)^2-\frac{195}{8}.$ Report the minimum value of this, $-\frac{195}{8}.$

14. The first few terms are $1, 2, -16, -512.$ Report $-512.$

15. $A(x^2+2xy+y^2=49),\ B(4x^2+4xy+y^2=100),\ C(x^2+4xy+4y^2=121)$ show the following: equations *B* and *C* sum to give $5x^2+8xy+5y^2=221.$ 5*A* gives $5x^2+10xy+5y^2=245.$ Subtract to get $2xy=24.$ Report12.

16. $W = \dfrac{6a^{-16}b^{6}}{8a^{-10}b^{20}} = \dfrac{3}{4}a^{-6}b^{-14}$. Report $\dfrac{3}{4}(-6)(-14) = 63$.

17. Let x be the integer being though about. Then $x + \dfrac{1}{x} + \dfrac{1}{x + \frac{1}{x}} = \dfrac{x^4 + 3x^2 + 1}{x^3 + x}$. This is $\dfrac{5}{2}$ when $x = 1$ and $\dfrac{29}{10}$ when $x = 2$. Larger values of x give too many digits, so report 12.

18. Long division*** is tedious but doable, and easier if you don't write all of the x terms. The quotient is $q(x) = x^2 + 23x + 226$ and the remainder's constant term is $1024 - (-226) = 1250$, so $r(x)$ is second-degree and ends in 1250. Then $q(0) + r(0) = 226 + 1250 = 1476$. Report 1476.

***

$$
\begin{array}{rrrr|rrrrrr}
 & & & & 1 & 23 & 226 & & & \\
\hline
1 & -3 & 3 & -1 & 1 & 20 & 160 & 640 & 1280 & 1024 \\
 & & & & 1 & -3 & 3 & -1 & 1280 & 1024 \\
\cline{5-10}
 & & & & & & \ddots & & & \\
\cline{7-9}
 & & & & & & 1250 & 725 & 1250 &
\end{array}
$$

19. $\dfrac{16 - 20i}{4 + ki} \bullet \dfrac{4 - ki}{4 - ki} = \dfrac{(64 - 20k) - (80 + 16k)i}{16 + k^2} = \dfrac{a + bi}{25}$. Then $k = \pm 3$. If k is 3 we have $\dfrac{4 - 128i}{25}$, which fails the condition $a + b > 0$. Use $k = -3$ to get $\dfrac{124 - 32i}{25}$, which works. Report 92.

20. $P(D\ wins\ 0) = \left(\dfrac{3}{5}\right)^3 = \dfrac{27}{125}$. $P(D\ wins\ 1) = 3 \bullet \left(\dfrac{3}{5}\right)^2 \left(\dfrac{2}{5}\right) = \dfrac{54}{125}$. Add and report $\dfrac{81}{125}$.