2025 SAA

Algebra II

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. 1.127

2. 64

3. 4

4. 53130 (Comma use ok.)

5. 256

6. $\frac{1}{3}\sqrt{17}$ OR $\frac{\sqrt{17}}{3}$

7. 252

8. -4

9. 4860

10. $-2, -\frac{1}{2}$ OR $-2, \frac{-1}{2}$ (Must have both values, either order.)

11. $\frac{128}{3267}$

12. 81

13. $-\frac{195}{8}$ OR $\frac{-195}{8}$

14. 16384 (Comma use ok.)

15. 12

16. -12

17. 12

18. 1476

19. 38

20. $\frac{329}{1000}$

1. Calculator gives 1.1273, so report 1.127.

2. There are six numbers, each with an "in or out" possibility, so report $2^6 = 64$.

3. Clear fractions and get $A(x-5)+B(x+3)=3x+1$. Then $A+B=3$ and $-5A+3B=1$. The solution to these is $A=1, B=2$. Report 4.

4. The coefficient is $C(20, 25) = 53130$.

5. $k=\log(8^4)-\log(4^2)=\log\left(\frac{2^{12}}{2^4}\right)=\log\left(2^8\right)$. Report $k = 256$.

6. $27=x^3+3x^2y+3xy^2+y^3=11+3xy(x+y)=11+9xy$. This gives $y=\frac{16}{9x}$, $x+\frac{16}{9x}=3$, which gives $9x^2-27x+16=0$. The solutions for x are $\frac{27\pm\sqrt{27^2-4\bullet 9\bullet 16}}{18}=\frac{9\pm\sqrt{17}}{6}$. Report the absolute difference of these solutions, $\frac{\sqrt{17}}{3}$.

7. The four terms simplify to give $3\bullet 7\sqrt{3}+5\bullet 13\sqrt{2}+9\sqrt{3}+16\sqrt{2}=30\sqrt{3}+81\sqrt{2}$. Report $90+162=252$.

8. The center of both graphs is (1, -2). A translation puts the center at (0, 0) and gives the new equations $x^2+y^2=4$ and $|x|+|y|\geq 2$. The points that satisfy both of these are inside the circle with center (0, 0) and radius 2 and outside the square with two corners (2, 0) and (0, 2). The area of this region is $4\pi-8$, so report -4.

9. The sum is the sum of the first 100 positive integers minus the sum of the first 10. This gives the difference $\frac{100\bullet 101}{2}-\frac{19\bullet 20}{2}=5050-190=4860$.

10. $A^2=\begin{bmatrix} 1+2x & 0 \\ 0 & 2x+1 \end{bmatrix}$. $A^3=\begin{vmatrix} 1+2x & 2+4x \\ 2x^2+x & -2x-1 \end{vmatrix}$. The sum of the elements of A^3 is $2x^2+5x+2=(2x+1)(x+2)$. Report both solutions when this is equal to 0: -2 and $-\frac{1}{2}$.

11. $p(x)=11x^3-4x^2+0x+k$. Let the zeros be $r-d, r, r+d$. The "sum and product of roots" theorem gives $3r=\frac{4}{11}$, so $r=\frac{4}{33}$. Synthetic division gives $k-\frac{128}{99\bullet 33}=0$. Report $k=\frac{128}{3267}$.

12. Since $4!+1=5^2$, $5!+1=11^2$ and $7!+1=71^2$, but $6!+1$ is not a square, try $(5+7-4)!+k$ with a table using $y1(x)=8!+x$ and find that x = 81 gives 201^2. Report 81.

13. Complete squares to get $2\left(x^2-\frac{3}{2}x+\frac{9}{16}\right)-\frac{9}{8}+\left(y^2-7y+\frac{49}{4}\right)-\frac{49}{4}-11=2\left(x-\frac{3}{4}\right)^2+\left(y-\frac{7}{2}\right)^2-\frac{195}{8}$. Report the minimum value of this, $-\frac{195}{8}$.

14. The first five terms of the sequence are $1, 4, -32, -512, 16384$. Report 16384.

15. $A(x^2+2xy+y^2=49)$, $B(4x^2+4xy+y^2=100)$, $C(x^2+4xy+4y^2=121)$ show the following: equations B and C sum to give $5x^2+8xy+5y^2=221$. $5A$ gives $5x^2+10xy+5y^2=245$. Subtract to get $2xy=24$. Report 12.

16. W simplifies to $\dfrac{20a^{-9}b^{21}}{25a^{-8}b^{6}}$, which further simplifies to $\dfrac{4}{5}a^{-1}b^{15}$. Report $\dfrac{4}{5}\bullet(-1)(15)=-12$.

17. Let x be the integer being thought about. Then $x+\dfrac{1}{x}+\dfrac{1}{x+\dfrac{1}{x}}=\dfrac{x^4+3x^2+1}{x^3+x}$. This is $\dfrac{5}{2}$ when x = 1 and $\dfrac{29}{10}$ when x = 2. Larger values of x give too many digits, so report 2 + 9 + 1 + 0 = 12.

18. Long division*** is tedious but doable, and easier if you don't write all of the x terms. The quotient is $q(x)=x^2+23x+226$ and the remainder's constant term is $1024-(-226)=1250$, so $r(x)$ is second-degree and ends in 1250. Then $q(0)+r(0)=226+1250=1476$. Report 1476.

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$$\begin{array}{rrrr|rrrrrr}
 & & & & & 1 & 23 & 226 & & \\ \hline
1 & -3 & 3 & -1 & 1 & 20 & 160 & 640 & 1280 & 1024 \\
 & & & & 1 & -3 & 3 & -1 & 1280 & 1024 \\ \cline{5-10}
 & & & & & & \ddots & & & \\ \cline{7-10}
 & & & & & & 1250 & & 725 & 1250
\end{array}$$

19. $\dfrac{20+25i}{4-3i}\bullet\dfrac{4+3i}{4+3i}=\dfrac{80-75+160i}{16+9}=\dfrac{5+160i}{25}=\dfrac{1+32i}{5}$. Report 38.

20. $P(Animals\ in\ 3)=\dfrac{3}{5}\bullet\dfrac{1}{2}\bullet\dfrac{2}{5}=\dfrac{3}{25}$. For a 4-game Animals championship we need $WWLW$, $WLWW$, or $LWWW$. The respective probabilities are $\dfrac{3}{5}\bullet\dfrac{1}{2}\bullet\dfrac{3}{5}\bullet\dfrac{11}{20}=\dfrac{99}{1000}$, $\dfrac{2}{5}\bullet\dfrac{1}{2}\bullet\dfrac{2}{5}\bullet\dfrac{11}{20}=\dfrac{329}{1000}$. and $\dfrac{2}{5}\bullet\dfrac{1}{2}\bullet\dfrac{2}{5}\bullet\dfrac{11}{20}=\dfrac{44}{1000}$.

Report the sum of these four probabilities, $\dfrac{329}{1000}$.