# 2025 SA Pre-Calculus

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [    ]

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. $1.678$
2. $(25,10)$
3. $4$ (This value only.)
4. $7$
5. $\frac{7}{50}$
6. $51$
7. $10$
8. $-8$
9. $143.675$
10. $25$
11. $8.384$
12. $7$
13. $333$
14. $\frac{1}{5}$
15. $\frac{3}{16}$
16. $243$
17. $\frac{11}{6}$
18. $4\sqrt{6}$
19. $(-11,6)$
20. $3$

1. $k \approx .970492-(-.707107) \approx 1.677598$. Report 1.678.

2. The $row \times column$ dimensions give $(20 \times \underline{25)(k} \times \underline{w)(10} \times 24)$. The multiplications are possible only when $k = 25$ and $w = 10$. Report (25, 10).

3. The left side is $\log(k^2+21k)$, so $k^2+21k=10^2$. Report the only positive solution, 4.

4. $g(x)=1$ from a trigonometric identity, so we have $f(1)-g(14)=8-1=7$.

5. $\dfrac{1}{6-8i}=\dfrac{1}{6-8i}\bullet\dfrac{6+8i}{6+8i}=\dfrac{6+8i}{6^2+8^2}=\dfrac{6}{100}+\dfrac{8}{100}i$. Report $\dfrac{7}{50}$.

6. It's clear that $L = 2$, so we get $2-\left(2-\dfrac{1}{n}\right)<.02$, or $\dfrac{1}{n}<.02$, which holds when $n$ is 51 or higher. Report 51.

7. By definition of an ellipse for every point $P$ on the ellipse $PF_1+PF_2$ is a constant. The easiest point to use is (0, 5). A sketch shows that the sum is 10. Report 10.

8. The dot product must be 0 for perpendicularity, so $(20)(10)+25k=0$.Report $k=-8$.

9. The Law of Sines gives $\dfrac{\sin B}{20}=\dfrac{\sin(20.25°)}{25}$, so $\sin B \approx .276894$. Then $B \approx 16.07489°$ and $C \approx 143.675°$. Report 143.675.

10. The amplitude is 25. Everything else is irrelevant. Report 25.

11. Easy way: Set $y=ab^t$ and use $10=ab^0$ and $30=ab^2$. Divide to get $b^2=3,\ b=\sqrt{3},\ a=10.$ So Set $10\left(\sqrt{3}\right)^x=1000$ and get $x \approx 8.3836$. Report 8.384.

Or...use exponential regression with the two data points (0, 10) and (2, 30) and get the regression equation $y=10(1.732051)^x$. Set this equal to 1000 and solve to get $x \approx 8.3836$. (Note: Using two other data points such as (1, 10) and (3, 30) in which the inputs differ by 2 gives a different regression equation; in that case it's $y=5.773503(1.732051)^x$. Setting this equal to 1000 and solving gives $x \approx 9.3836$. Using (4, 10) and (6, 30) yields a regression equation which, when asked to find $x$ when $y$ = 1000, returns 12.3836. Since "initial" means starting at time = 0, not 1 or 4, a subtraction of 1 or 4 at the end of the solution for $x$ is necessary. There is just one answer. It's 8.3836, which rounds to 8.384.)

12. $\cos x=\dfrac{\sin x}{\cos x} \to \cos^2 x=\sin x \to 1-\sin^2 x=\sin x \to \sin^2 x+\sin x-1=0$. Using the quadratic formula, $\sin x=\dfrac{-1\pm\sqrt{1-4(-1)(1)}}{2}$. The least positive solution is $\dfrac{-1+\sqrt{5}}{2}$. $-1+1+5+2=7$. Report 7.

13. DeMoivre's Theorem says that the absolute value of $\sqrt{z}$ is $\sqrt{10}$ and the two square roots of $z$ have arguments $\left(\dfrac{306.870}{2}\right)°$ and $\left(180\pm\dfrac{306.870}{2}\right)°$, with the sign depending on keeping both arguments in the range $0\le\theta\le360$, and this gives $153.435°$ and $333.435°$. Report 333.

14. $\dfrac{20}{1-r} = 25 \to 20 = 25 - 25r \to r = \dfrac{1}{5}$

15. Four of the sixteenth roots of 1 are $1+0i, 0+1i, -1+0i, 0-1i$. Each quadrant has three sixteenth roots of 1, so the probability is the number in Quadrant III divided by 16, $\dfrac{3}{16}$. This is a consequence of DeMoivre's Theorem.

16. Put $A = B = 1$ to get $3^5 = 243$. (Expanding the expression gives many terms involving $A$ and $B$ with numerical coefficients. Those coefficients do not change, regardless of the values of $A$ and $B$, so the chosen substitutions are valid and make the computation easy.)

17. Use partial fractions decomposition: $\dfrac{3}{(n+1)(n-2)} = \dfrac{A}{n+1} + \dfrac{B}{n-2}$, from which we get $3 = (n-2)A + (n+1)B$. Put $n$ = 2, which we couldn't do before, and get $B$ = 1. Put $n$ = -1 and get $A$ = -1. We then have $k = -\dfrac{1}{4} + 1 - \dfrac{1}{5} + \dfrac{1}{2} - \dfrac{1}{6} + \dfrac{1}{3} - \dfrac{1}{7} + \dfrac{1}{4} - \dfrac{1}{8} + \dfrac{1}{5} - \cdots$. This shows that the only numbers that do not get canceled are $1, \dfrac{1}{2}, \dfrac{1}{3}$. Report the sum of these, $\dfrac{11}{6}$.

18. The polar point $A\left(12, \dfrac{\pi}{6}\right)$ has rectangular coordinates $\left(12\cos\dfrac{\pi}{6}, 12\sin\dfrac{\pi}{6}\right) = (6\sqrt{3}, 6)$. Multiply the equation $r = 8\sin\theta$ by $r$ to get $r^2 = 8r\sin\theta$, which is $x^2 + y^2 = 8y$ in rectangular coordinates. This is the circle with center $C(0, 4)$ and radius 4. Let $B$ be a tangency point from $A$. The right triangle $ABC$ has $AB^2 + BC^2 = AC^2$, or $AB^2 = (6\sqrt{3} - 0)^2 - (6-4)^2 = 96$. Report $4\sqrt{6}$.

19. $P(1) = (7, 0)$ and $P(-7) = (-17, 8)$. The point three-fourths of the way from the first of these two to the second is $P(-5) = (-11, 6)$.

20. $\{a_n\} = \left\{\dfrac{1}{2}, 2, \dfrac{9}{2}, 8, \dfrac{25}{2}\right\}$. $\lceil \{a_n\} \rceil = \{1, 2, 5, 8, 13\}$. $\lceil \{a_n\} \rceil - \lfloor \{a_n\} \rfloor = \{1, 0, 1, 0, 1\}$. Report 3.