2025 SAA

Pre-Calculus

Name ANSWERS

School ____________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = ☐

NOTE: All answers are to be written in accordance with the Acceptable Forms of Answers document. Exact answers are to be given. All rational answers that are not integers are to be written as simplified common or improper fractions. A problem's specific instructions for answer format takes precedence.

1. 0.466 OR .466
2. $(25,10)$
3. $\frac{49}{4}$
4. 5
5. $\frac{7}{50}$
6. $-\frac{5}{3}$ OR $\frac{-5}{3}$
7. 10
8. 3
9. 16.040 (Trailing zero necessary.)
10. 13
11. 3.761
12. $\sqrt{3}$
13. 333
14. $-\frac{35}{9}$ OR $\frac{-35}{9}$
15. $\frac{3}{16}$
16. -32
17. $\frac{11}{6}$
18. $4\sqrt{6}$
19. 3
20. $\frac{576}{25}$

1. The difference is approximately .46597. Report .466.

2. $A: m\times 20, B: 25\times n, C: p\times 10.$ To add A and B we must have m =25 and n = 20. To multiply this sum on the right by C we must have the number of rows of C equal to the number of columns of A and B, so 20 = p, but that's not needed here. The number of rows of A (25) and the number of columns of C (10) give the dimensions of the product D. Report (25, 10).

3. From $2\sin x=\frac{1}{\cos x}$ we get $2\sin x\cos x=1=\sin 2x$, so $x=\frac{\pi}{4}+n\pi$. The value of n that puts x in the interval $36\le x\le 40$ is 12. Report $k=\frac{49}{4}$.

4. Both expressions in parentheses have the value 1, so we have the difference 15 – 10 = 5.

5. $\frac{1}{6-8i}=\frac{1}{6-8i}\bullet\frac{6+8i}{6+8i}=\frac{6+8i}{6^2+8^2}=\frac{6}{100}+\frac{8}{100}i$. Report $\frac{6}{100}+\frac{8}{100}=\frac{7}{50}$.

6. It's –4 that is inadmissible for h, and that means $w=-2$. Then $h(x)=g(3x+2)=\frac{1}{3x+6}$. Solve $y=\frac{1}{3x+6}$ for x and get $x=\frac{1/y-6}{3}$. Reverse the letters and get $h^{-1}(x)=\frac{1}{3x}-2$. Report $h^{-1}(-2+3)=\frac{1}{3}-2=-\frac{5}{3}$.

7. By definition of an ellipse for every point P on the ellipse PF_1+PF_2 is a constant. The easiest point to use is (0, 5). A sketch shows that the sum is 10. Report 10.

8. The first equations are for the unit circle $x^2+y^2=1$. The second equations are for the line $y=5-\frac{3x}{4}$, or $3x+4y-20=0$. The distance from (0, 0) to this line is $\frac{|3\bullet 0+4\bullet 0-20|}{\sqrt{3^2+4^2}}=4$. This makes the distance from the circle to the line 3. Report 3.

9. $\vec{v}\bullet\vec{w}=\left|\vec{v}\right|\bullet\left|\vec{w}\right|\cos\theta$, where θ is the angle between the vectors. The left side is $200+600=800$. The right side is $\sqrt{400+625}\bullet\sqrt{100+576}\cos\theta$. These give $\cos\theta\approx .9610693$, so $\theta\approx 16.039943°$. Report 16,040.

10. A sketch shows that the circle closest to the origin passes through A, C and E. An equation of the plane containing these points is $x + y + z - 1 = 0$. The center of the circle is $R\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ and the radius is $AR=\sqrt{(2/3)^2+(1/3)^2+(1/3)^2}=\frac{\sqrt{6}}{3}$. The sum is $\frac{3+1\sqrt{6}}{3}$. Report 13.

11. The Law of Cosines gives $49=100+c^2-20c\bullet\cos 30°$, or $c^2-10\sqrt{3}c+51=0$. The smaller solution for c is 3.7612. Report 3.761.

12. $f\left(\frac{\pi}{3}\right)=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ and $f\left(-\frac{\pi}{3}\right)=\frac{1}{2}-\frac{\sqrt{3}}{2}i$. Report the vertical distance between these, $\sqrt{3}$.

13. DeMoivre's Theorem says that the absolute value of $\sqrt{z}$ is $\sqrt{10}$ and the two square roots of z have arguments $\left(\frac{306.870}{2}\right)°$ and $\left(180 \pm \frac{306.870}{2}\right)°$, with the sign depending on keeping both arguments in the range $0 \le \theta \le 360$, and this gives $153.435°$ and $333.435°$. Report 333.

14. The terms are $20, 20+d, \cdots, 20+19d$. The sum, 25, is equal to $200 + \frac{9 \bullet 10}{2}d$, which gives $d = -\frac{175}{45} = -\frac{35}{9}$.

15. Four of the sixteenth roots of 1 are $1+0i, 0+1i, -1+0i, 0-1i$. Each quadrant has three sixteenth roots of 1, so the probability is the number in Quadrant III divided by 16, $\frac{3}{16}$. This is a consequence of DeMoivre's Theorem.

16. Put $A = C = 1$ and $B = 0$ to get $(-2)^5 = -32$. (Expanding the expression gives many terms involving *A, B, C* with numerical coefficients. Those coefficients do not change, regardless of the values of *A, B* and *C*, so the chosen substitutions are valid and make the computation easy.)

17. Use partial fractions decomposition: $\frac{3}{(n+1)(n-2)} = \frac{A}{n+1} + \frac{B}{n-2}$, from which we get $3 = (n-2)A + (n+1)B$. Put n = 2, which we couldn't do before, and get B = 1. Put n = -1 and get A = -1. We then have $k = -\frac{1}{4} + 1 - \frac{1}{5} + \frac{1}{2} - \frac{1}{6} + \frac{1}{3} - \frac{1}{7} + \frac{1}{4} - \frac{1}{8} + \frac{1}{5} - \cdots$. This shows that the only numbers that do not get canceled are $1, \frac{1}{2}, \frac{1}{3}$. Report the sum of these, $\frac{11}{6}$.

18. The polar point $A\left(12, \frac{\pi}{6}\right)$ has rectangular coordinates $\left(12\cos\frac{\pi}{6}, 12\sin\frac{\pi}{6}\right) = (6\sqrt{3}, 6)$. Multiply the equation $r = 8\sin\theta$ by r to get $r^2 = 8r\sin\theta$, which is $x^2 + y^2 = 8y$ in rectangular coordinates. This is the circle with center $C(0, 4)$ and radius 4. Let B be a tangency point from A. The right triangle ABC has $AB^2 + BC^2 = AC^2$, or $AB^2 = (6\sqrt{3} - 0)^2 - (6-4)^2 = 96$. Report $4\sqrt{6}$.

19. The lines intersect at $A(2, 6)$. The first line meets the x-axis at $B(-4, 0)$. The line through B that is perpendicular to the second line has the equation $y - 0 = \frac{1}{2}(x+4)$. This line meets the second line at $C\left(\frac{16}{5}, \frac{18}{5}\right)$. Then triangle ABC has side lengths $AC = \frac{6\sqrt{5}}{5}$ and $BC = \frac{18\sqrt{5}}{5}$. Then $\tan A = \frac{18\sqrt{5}/5}{6\sqrt{5}/5} = 3$. Report 3.

20. The parametric equations give the ellipse with rectangular equation $\frac{x^2}{9} + \frac{y^2}{16} = 1$. The polar equation gives the two lines $y = x$ and $y = -x$. The Quadrant I intersection is $\left(\frac{12}{5}, \frac{12}{5}\right)$. This is a corner of a square with side length $\frac{24}{5}$ and area $\frac{576}{25}$.