Competition B – Calculator Team

Team Make-up: maximum 8 students;
2 freshmen; 2 freshman or sophomore; 2 freshman, sophomore, or junior; 2 students from any grade

Questions: 20

Time: 20 minutes

Format: Team members work together and submit one answer sheet

All battery operated calculators permitted, including CAS-type; students may use more than one calculator

Answers must be legible

Answers must be **rounded to 4 significant digits** unless otherwise indicated in the question**.** Answers must be simplified and in the **specific form if so stated. Except where noted, angles are in radians.**

Answers must be in standard notation. **Do not use scientific notation unless specified in the question**

Scoring: Correct answers are worth 5 points each; Maximum 100 points possible per team

Sample Regional Questions

1. The ordered pair (x, y) is a solution to the system $\begin{cases} 0.147x + 0.235y = 5.0192 \\ 1.38852x + 7.6y = 5.793482 \end{cases}$.

   Determine the value of y in that solution.

   Answer: –7.735

2. Determine the smaller of the roots for x when $\log(20x^2+18)-2=\log(3x+5)$.

   Answer: –1.464

3. Tom's Math teacher had each of the 20 students in class write four questions for the final exam. The teacher made sure all the students' questions were different and then promised that all twenty questions on this exam would be taken from the student written questions. Determine the probability that at least one of Tom's questions will be used. Express your answer as a common fraction reduced to lowest terms.

   Answer: $\frac{218789}{316316}$

4. Point A lies on the graph of $20x-17y+42=0$ and point C lies on the graph of $10x+24y+101=0$. These lines intersect at point B to form acute $\angle ABC$. Determine the degree measure of this $\angle ABC$. Express your answer as a decimal rounded to the nearest thousandth.

   Answer: 72.255

5. Let k and w be integers satisfying both inequalities $w \geq k^2 - 2$ and $w \leq 3 - k^2$. Determine *the number* of distinct possible ordered pairs (k, w) that exist.

Answer: 14

6. Let $f(x) = 4x^5 - 3x^4 - 72x^3 + 54x^2 + 320x - 240$. Let k be the largest positive zero and w be the smallest positive zero for $f(x)$. Compute the value of k^w.

Answer: 2.371

7. An eleven sided regular polygon is inscribed within a circle with radius of 11. Determine the area of the region inside the circle but outside of the eleven sided polygon.

Answer: 20.34

8. A parabolic function $f(x)$ contains the points $(3,5)$, $(7,21)$, and $(-1,53)$. The vertex of this parabola is also the center of a circle with radius 3. The point(s) of intersection of the graphs of this circle and this parabola may be denoted (k, w). Determine the sum of all the coordinates of these possible point(s) of intersection..

Answer: 19.52

9. $t_n = \sqrt{421 \cdot t_{(n-1)} + 3210000}$ and $t_1 = 999$. Find the value of t_{20}.

Answer: 2014

10. Susan is playing Theresa in a best of 7 games in a table tennis match. The first player to win 4 games will be the winner of the match. The probability that Theresa will win any particular game after the first game is constant. Susan now leads the match by a score of 1 game to none. If the probability that Theresa will win the match is now $\frac{2}{3}$, find the probability that Theresa will win any particular game from now on.

Answer: 0.6598 or .6598