

Competition D – Junior-Senior 2 Person Team

Team Make-up: maximum 2 students, any grade levels

Questions: 10 (initial competition)

For regionals: additional questions are available for use in case of procedural issues with any of the initial questions

For state finals: ties are broken, if possible, by the number of “first correct” answers. If there are still ties, additional questions are available for tie-breakers, to replace questions with upheld appeals or for use in case of procedural issues with any of the initial questions

Time: 3 minutes per question

Format: Team members work together and submit one answer sheet

Questions 1-5: NO Calculators permitted

Questions 6-10: All battery operated calculators permitted, including CAS-type

Questions 11-15 (if needed): All battery operated calculators permitted, including CAS-type

Answers must be legible

Answers must be **exact** unless otherwise indicated in the question

Scoring: Correct answers submitted in the 1st minute are worth 6 points; correct answers submitted in the 2nd minute are worth 4 points; correct answers submitted in the 3rd minute are worth 3 points; the first team with the correct answer to each question earns a 2 point bonus.

Sample Regional Questions (NO CALCULATOR Questions 1-3, Calculator Permitted Questions 4-6)

1. Let $f(x) = x^2 - 2x - 3$ and $g(x) = x^2 + 3x - 4$. Determine the value of the sum $[f(g(2)) + g(f(2))]$.

Answer: 17

2. Let $10^{\log 3 + \log 2} = 2k$ and $(4^w)(8^{w+2}) = 2^{2w}$. Determine the sum $(k + w)$.

Answer: 1

3. Let k be the result of adding two times the product of the roots of $x^2 - 17x + 25 = 0$ to three times the sum of the roots of $x^2 - 17x + 25 = 0$. In right triangle $\triangle ABC$ with right angle at C , point D lies on \overline{AC} so that \overline{BD} is a median in the triangle. $\sin(\angle A) = \frac{3}{5}$.

$\cos \angle CDB = \frac{a\sqrt{b}}{c}$ in simplified radical form. Let $w = (a + b + c)$. Report as your answer the sum $(k + w)$.

Answer: 222

4. A sequence is defined by $a_n = 4(n-1)^2 + 3n - 2$. Determine the absolute value of the difference between a_{20} and a_{18} .

Answer: 294

5. When $(2x+y)^5$ is expanded, one of the terms is ax^3y^c . Let d be the length of the diameter in the circle $x^2 + y^2 - 4x + 10y - 7 = 0$. Determine the sum $(a+c+d)$.

Answer: 94

6. Find the sum of all distinct values of x such that the three terms $x+7$, $5x-6$, and $7x+2$ taken in **some order** form an arithmetic sequence. Express your answer as an improper fraction reduced to lowest terms.

Answer: $\frac{477}{40}$